
CS580: Radiometry

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(윤성의)

Course URL:
<http://sglab.kaist.ac.kr/~sungeui/GCG>

KAIST



Announcements

- **Final project**
 - **Make a team of two or three members**
 - **Think about which papers you want to implement**
 - **Present papers related to it**
 - **Declare your members at the Noah board by Apr.-4**

- **Scope of papers for presentation**
 - **Papers published since 2011**
 - **ACM SIGGRAPH, SIG. Asia, or**
 - **Any top-tier conf./journal (e.g., CVPR); get a permission from the instructor**

Announcements

- **Declare two papers at the Noah board by Apr-11**
 - **First come, first served**
- **Present a single talk covering those two papers**
 - **Give a intro and related work**
 - **Discuss main methods and results of those two papers**
 - **Discuss their pros/cons**
- **Talk time: 35 min**
 - **25 min for the talk and 10 min for Q&A**

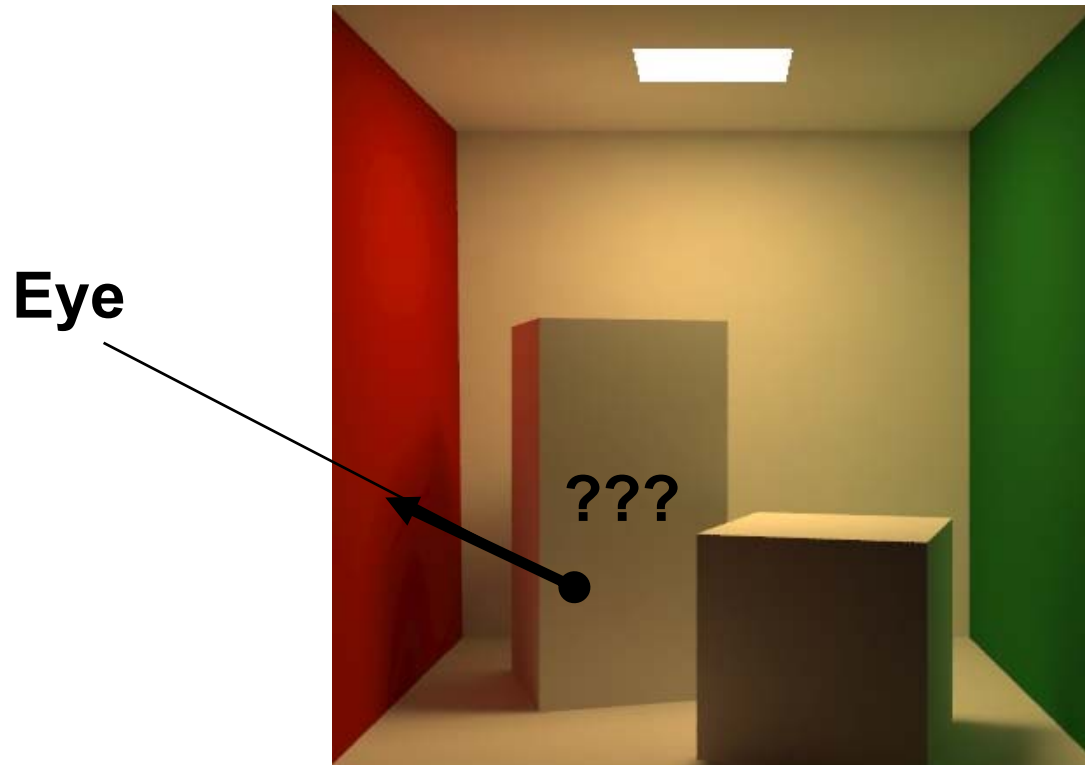
Tentative schedule

- **4/26: mid-term exam**
- **4/28: student presentations (SP) 1**
- **5/3: SP 2**
- **5/10: SP 3**
- **5/12: SP 4**
- **5/24: mid-project presentation I**
- **5/26: II**
- **5/31: SP 5**
- **6/2: SP 6**
- **6/7: SP 7**
- **6/9: SP 8**
- **6/14: Final project I**
- **6/16: II**

Class Objectives

- **Know terms of:**
 - **Hemispherical coordinates and integration**
 - **Various radiometric quantities (e.g., radiance)**
 - **Basic material function, BRDF**

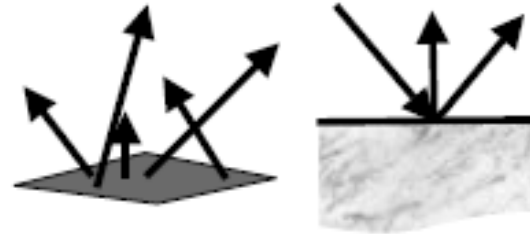
Motivation



Light and Material Interactions

- **Physics of light**
- **Radiometry**
- **Material properties**

- **Rendering equation**



From kavita's slides

Models of Light

- **Quantum optics**
 - **Fundamental model of the light**
 - **Explain the dual wave-particle nature of light**
- **Wave model**
 - **Simplified quantum optics**
 - **Explains diffraction, interference, and polarization**
- **Geometric optics**
 - **Most commonly used model in CG**
 - **Size of objects \gg wavelength of light**
 - **Light is emitted, reflected, and transmitted**

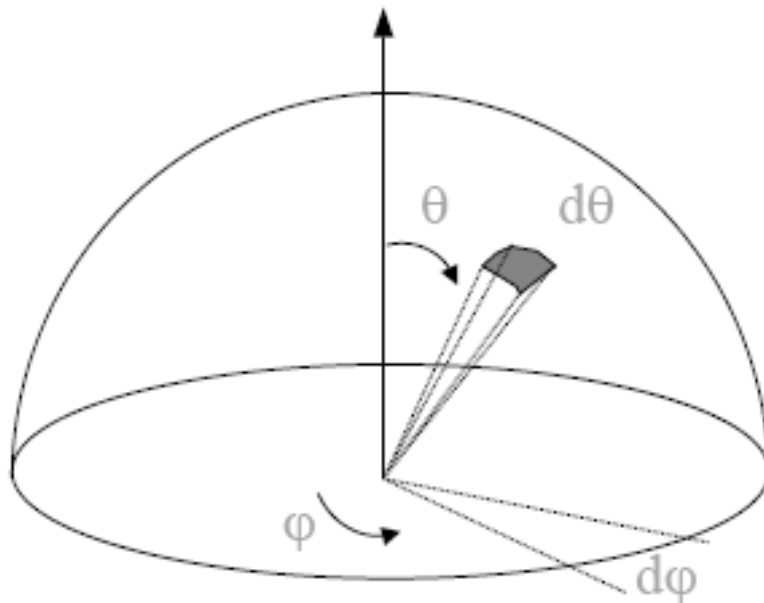


Radiometry and Photometry

- **Photometry**
 - **Quantify the perception of light energy**
- **Radiometry**
 - **Measurement of light energy: critical component for photo-realistic rendering**
 - **Light energy flows through space, and varies with time, position, and direction**
 - **Radiometric quantities: densities of energy at particular places in time, space, and direction**

Hemispheres

- Hemisphere
 - Two-dimensional surfaces
- Direction
 - Point on (unit) sphere

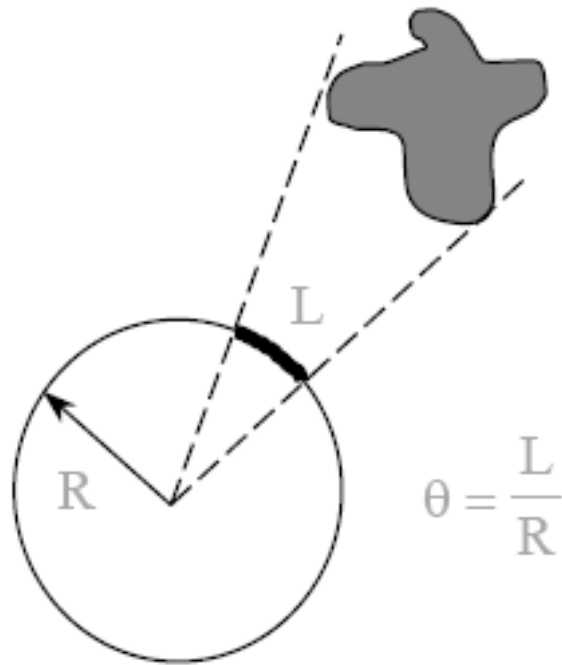


$$\theta \in [0, \frac{\pi}{2}]$$
$$\varphi \in [0, 2\pi]$$

From kavita's slides

Solid Angles

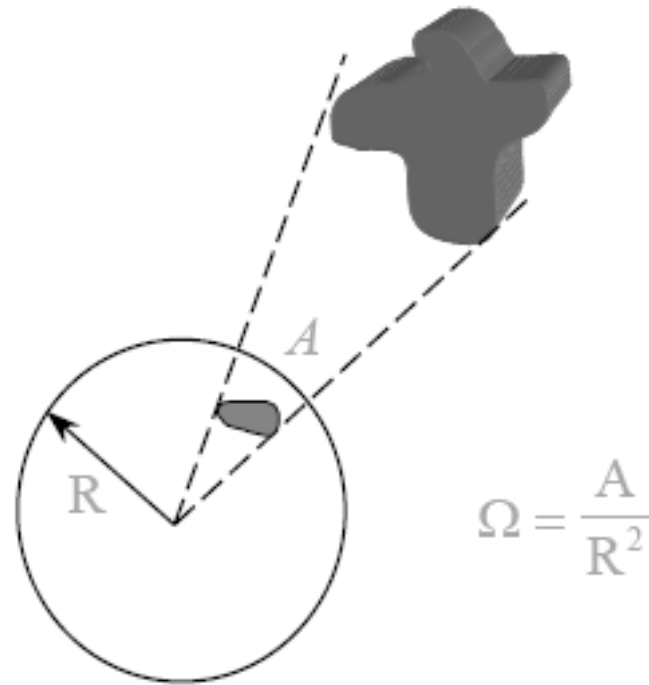
2D



$$\theta = \frac{L}{R}$$

**Full circle
= 2π radians**

3D

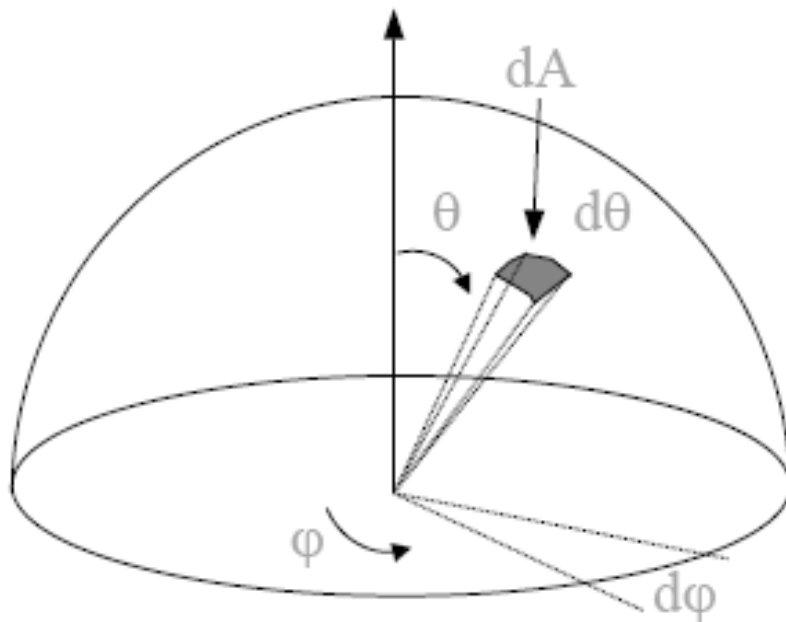


$$\Omega = \frac{A}{R^2}$$

**Full sphere
= 4π steradians**

Hemispherical Coordinates

- Direction, Θ
 - Point on (unit) sphere

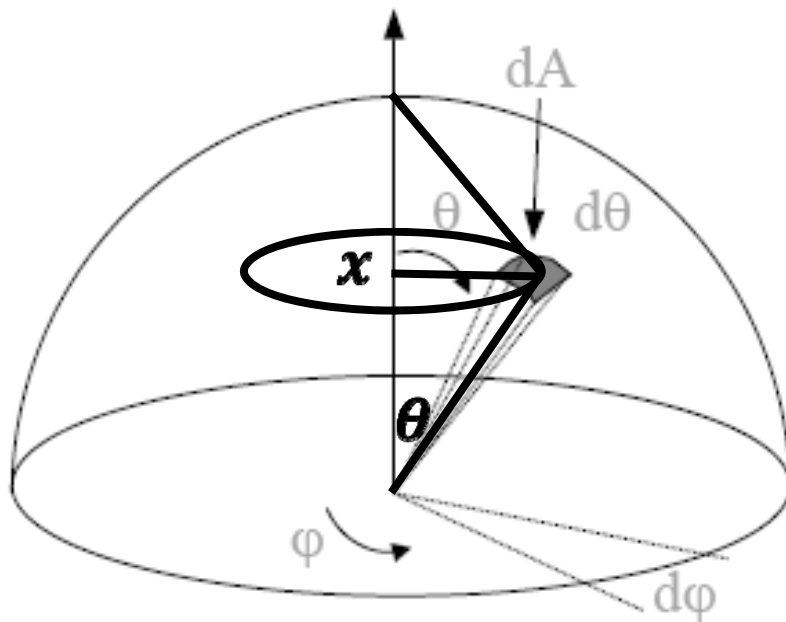


$$dA = (r \sin \theta d\varphi)(r d\theta)$$

From kavita's slides

Hemispherical Coordinates

- Direction, \ominus
 - Point on (unit) sphere



$$\sin \theta = \frac{x}{r},$$
$$x = r \sin \theta$$

$$dA = (r \sin \theta d\phi)(r d\theta)$$

From kavita's slides

Hemispherical Coordinates

- **Differential solid angle**

$$d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\varphi$$

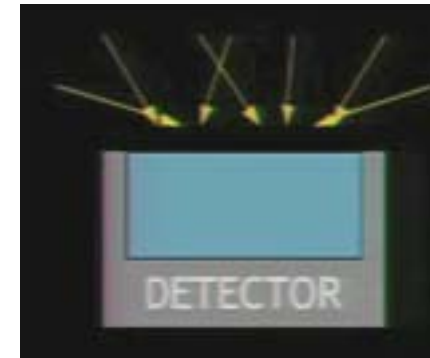
Hemispherical Integration

- Area of hemisphere:

$$\begin{aligned}\int_{\Omega_x} d\omega &= \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin\theta d\theta \\ &= \int_0^{2\pi} d\varphi [-\cos\theta]_0^{\pi/2} \\ &= \int_0^{2\pi} d\varphi \\ &= 2\pi\end{aligned}$$

Energy

- **Symbol: Q**
 - **# of photons in this context**
 - **Unit: Joules**

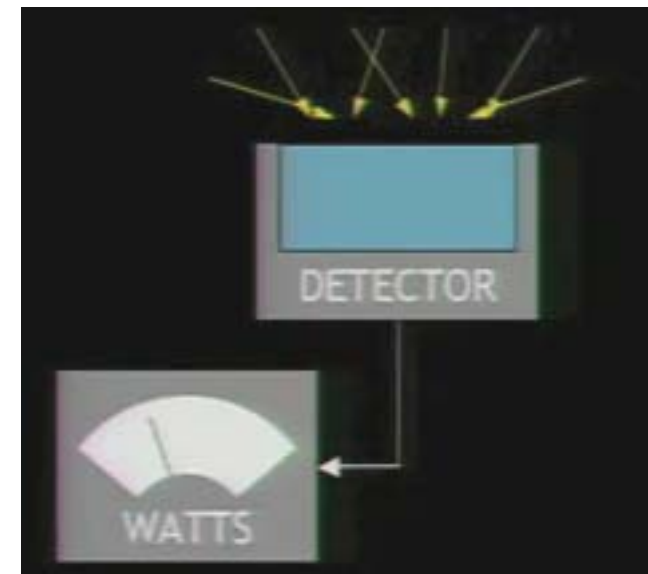


From Steve Marschner's talk

Power (or Flux)

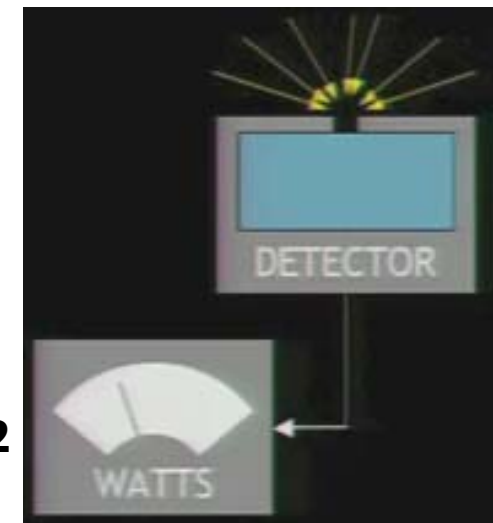
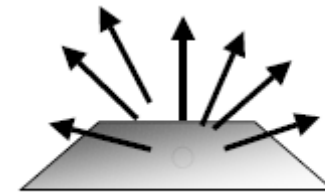
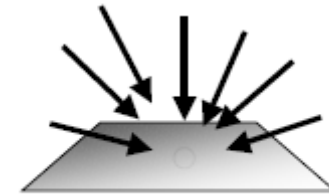
- **Symbol, P or Φ**
 - Total amount of energy through a surface per unit time, dQ/dt
 - Radiant flux in this context
 - Unit: Watts (=Joules / sec.)
 - Other quantities are derivatives of P

- **Example**
 - A light source emits 50 watts of radiant power
 - 20 watts of radiant power is incident on a table



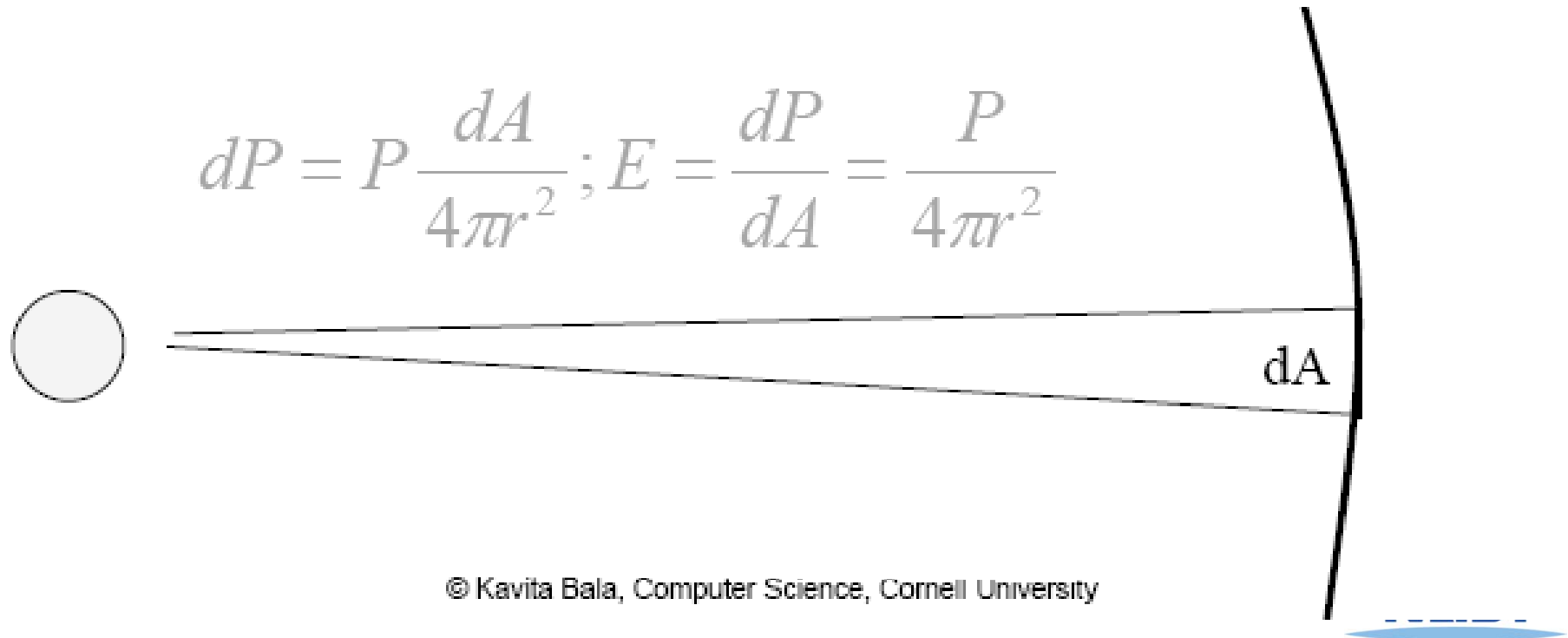
Irradiance

- **Incident radiant power per unit area (dP/dA)**
 - Area density of power
- **Symbol: E , unit: W/m^2**
 - Area power density existing a surface is called radiance exitance (M) or radiosity (B)
- **For example**
 - A light source emitting 100 W of area $0.1 m^2$
 - Its radiant exitance is $1000 W/m^2$



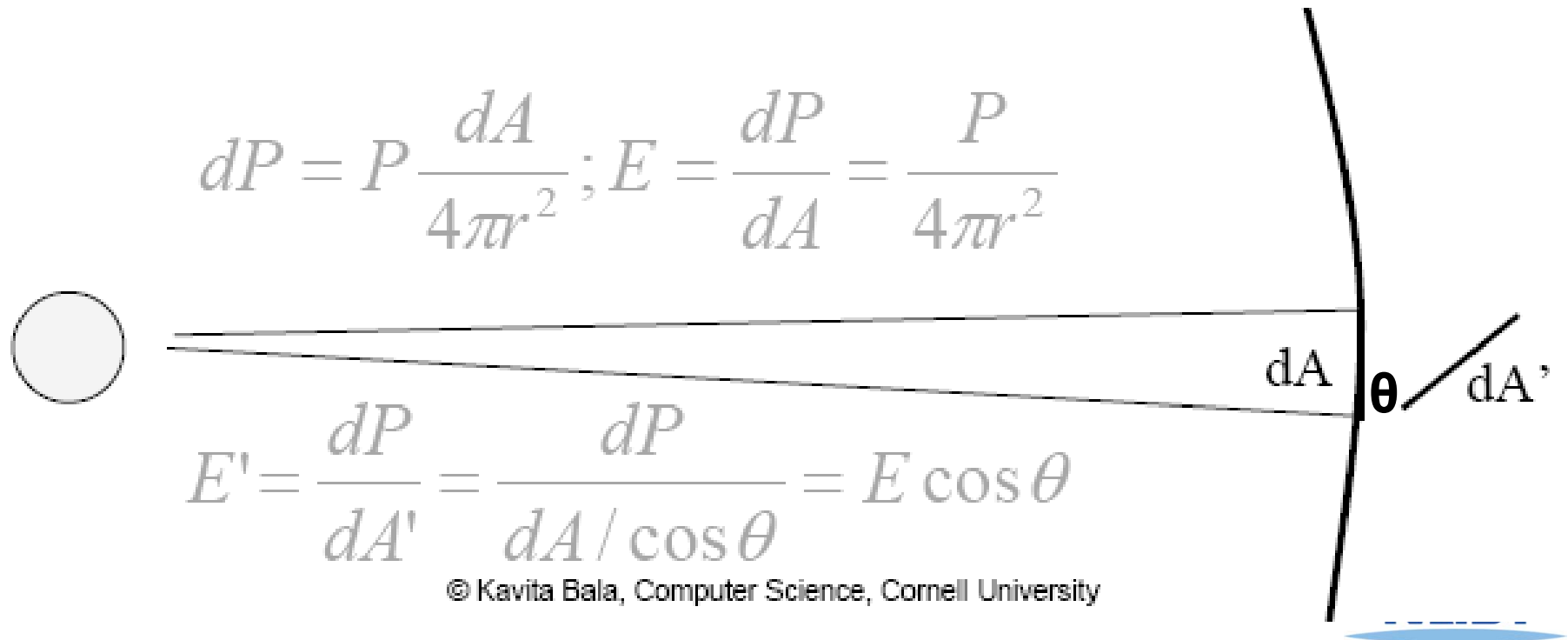
Irradiance Example

- **Uniform point source illuminates a small surface dA from a distance r**
 - **Power P is uniformly spread over the area of the sphere**



Irradiance Example

- **Uniform point source illuminates a small surface dA from a distance r**
 - **Power P is uniformly spread over the area of the sphere**



Radiance

- **Radiant power at x in direction θ**
 - $L(x \rightarrow \Theta)$: 5D function
 - Per unit area
 - Per unit solid angle

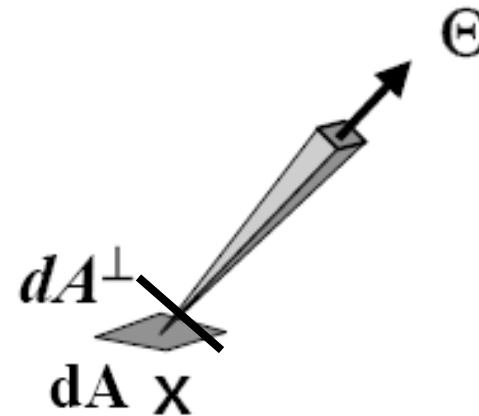


- **Important quantity for rendering**

Radiance

- Radiant power at x in direction Θ
 - $L(x \rightarrow \Theta)$: 5D function
 - Per unit area
 - Per unit solid angle

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^\perp d\omega_\Theta}$$

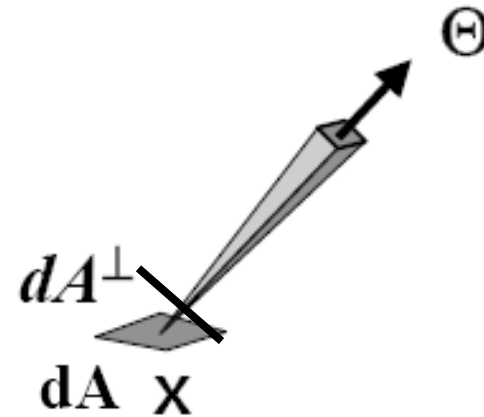


- Units: Watt / (m^2 sr)
- Irradiance per unit solid angle
- 2nd derivative of P
- Most commonly used term

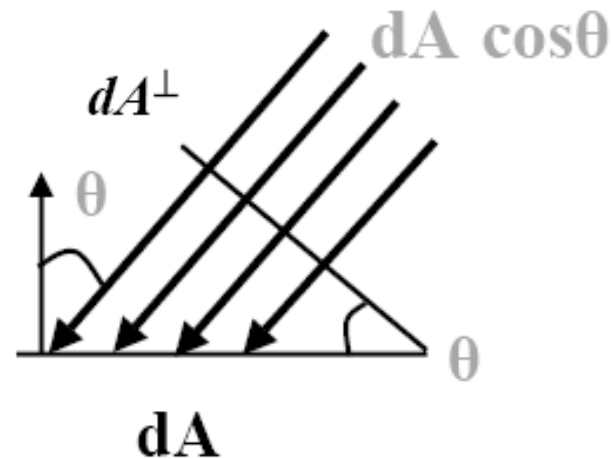
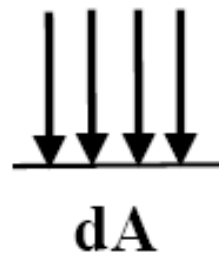
Radiance: Projected Area

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^\perp d\omega_\Theta}$$

$$= \frac{d^2 P}{d\omega_\Theta dA \cos \theta}$$

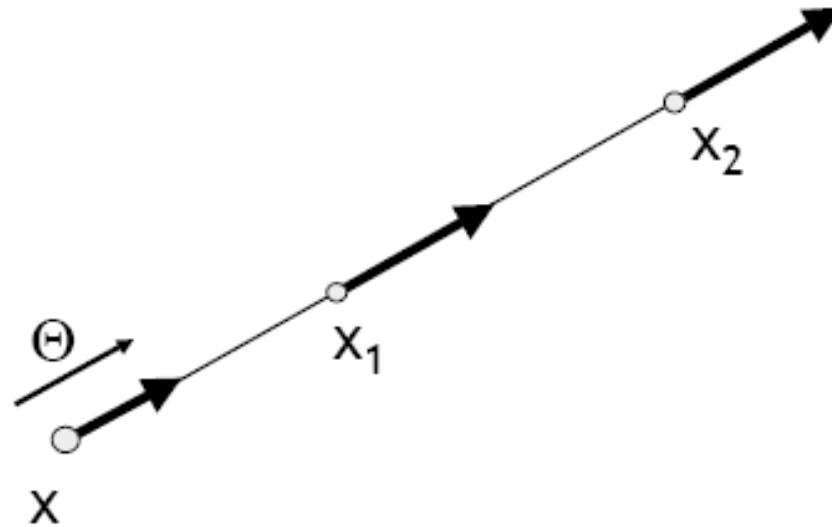


- Why per unit projected surface area



Properties of Radiance

- **Invariant along a straight line (in vacuum)**



From kavita's slides

Invariance of Radiance

We can prove it based on the assumption the conservation of energy.

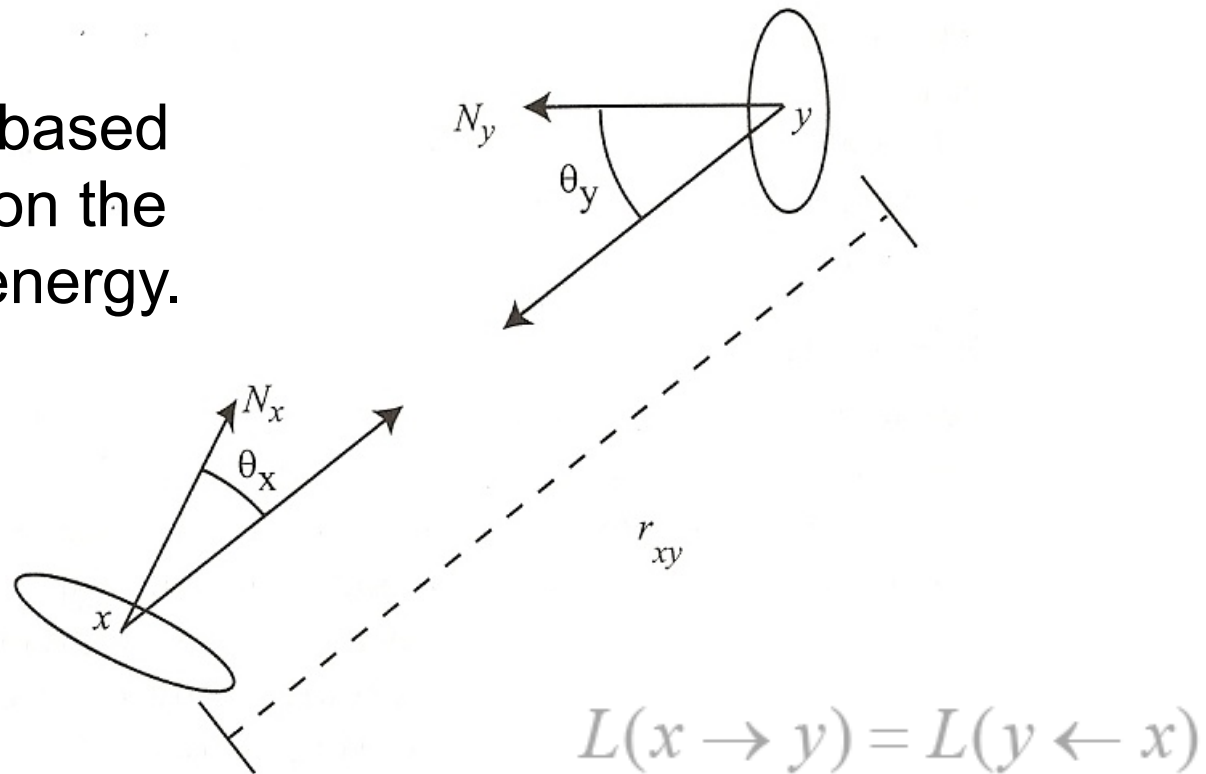
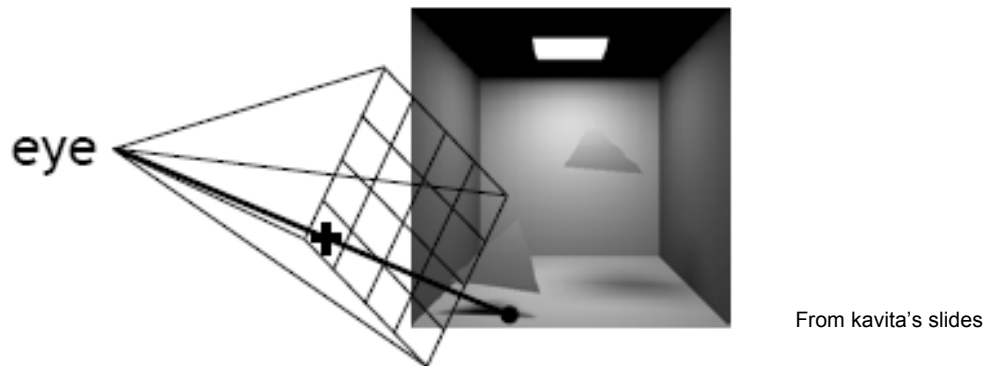


Figure 2.3. Invariance of radiance.

Sensitivity to Radiance

- **Responses of sensors (camera, human eye) is proportional to radiance**



- **Pixel values in image proportional to radiance received from that direction**

Relationships

- Radiance is the fundamental quantity

$$L(x \rightarrow \Theta) = \frac{d^2P}{dA^\perp d\omega_\Theta}$$

- Power:

$$P = \int_{\text{Area}} \int_{\text{Solid Angle}} L(x \rightarrow \Theta) \cdot \cos \theta \cdot d\omega_\Theta \cdot dA$$

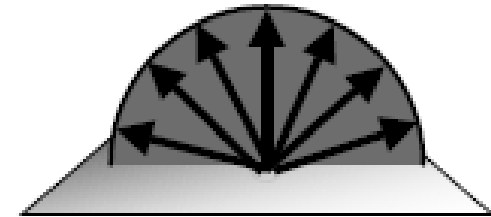
- Radiosity:

$$B = \int_{\text{Solid Angle}} L(x \rightarrow \Theta) \cdot \cos \theta \cdot d\omega_\Theta$$

Example: Diffuse emitter

- Diffuse emitter: light source with equal radiance everywhere

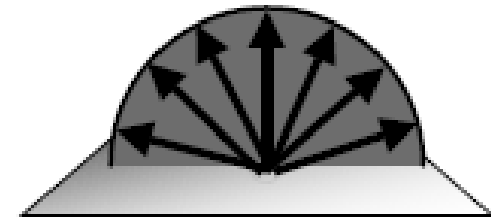
$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^\perp d\omega_\Theta}$$



Example: Diffuse emitter

- Diffuse emitter: light source with equal radiance everywhere

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^\perp d\omega_\Theta}$$

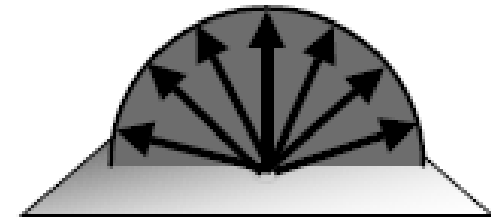


$$P = \int_{\text{Area}} \int_{\text{Solid Angle}} L(x \rightarrow \Theta) \cdot \cos \theta \cdot d\omega_\Theta \cdot dA$$

Example: Diffuse emitter

- Diffuse emitter: light source with equal radiance everywhere

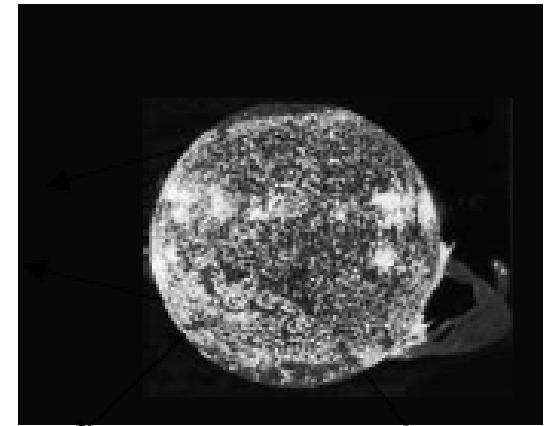
$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^\perp d\omega_\Theta}$$



$$\begin{aligned} P &= \int_{\substack{\text{Area} \\ \text{Angle}}} \int_{\substack{\text{Solid} \\ \text{Angle}}} L(x \rightarrow \Theta) \cdot \cos \theta \cdot d\omega_\Theta \cdot dA \\ &= L \int_{\substack{\text{Area} \\ \text{Area}}} dA \int_{\substack{\text{Solid} \\ \text{Angle}}} \cos \theta \cdot d\omega_\Theta \\ &= L \cdot \text{Area} \cdot \pi \end{aligned}$$

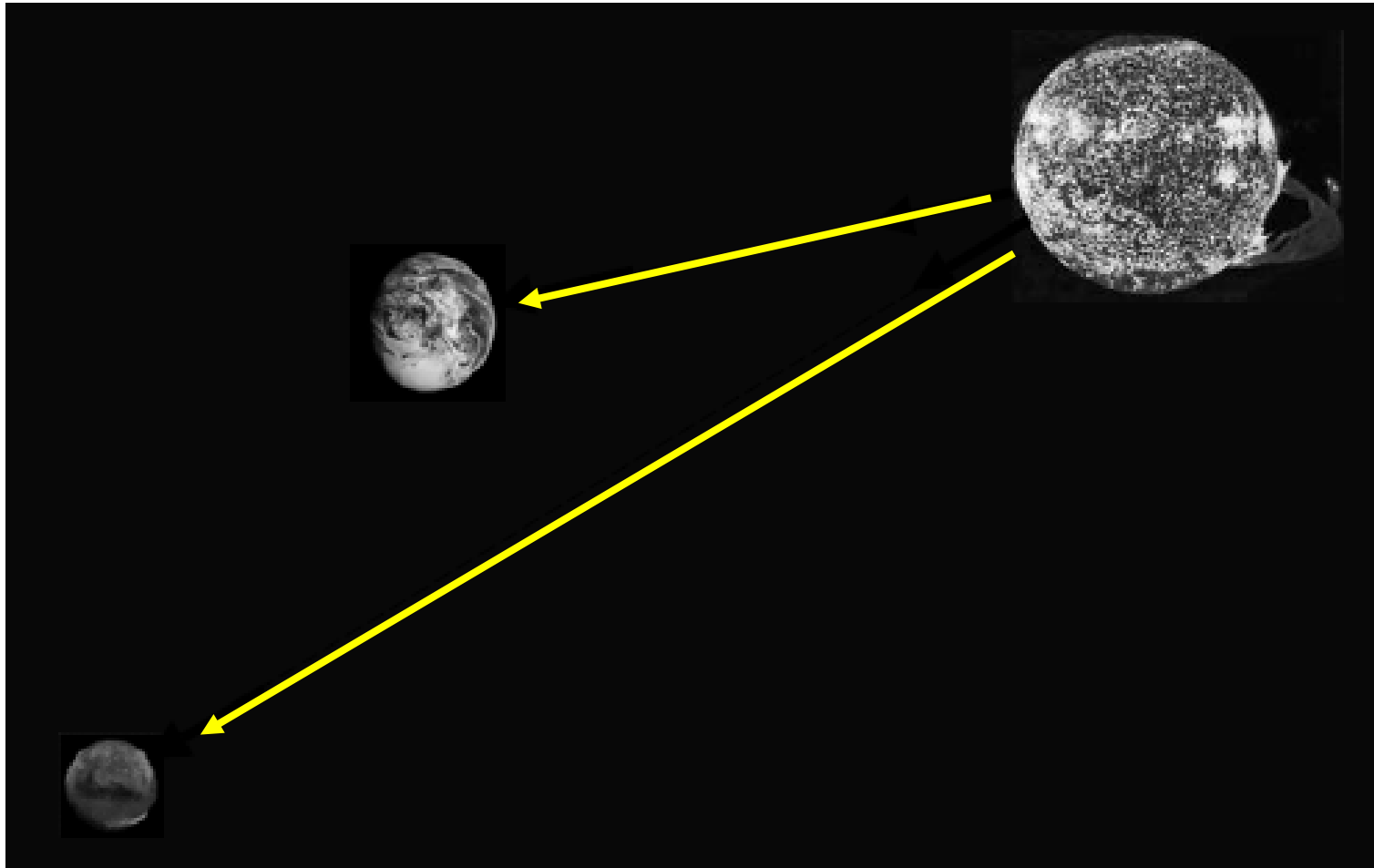
Sun Example: radiance

- Power: $3.91 \times 10^{26} \text{ W}$
- Surface Area: $6.07 \times 10^{18} \text{ m}^2$



- Power = Radiance \cdot Surface Area $\cdot \pi$
- Radiance = Power / (Surface Area $\cdot \pi$)
- Radiance = $2.05 \times 10^7 \text{ W/ m}^2 \cdot \text{sr}$

Sun Example



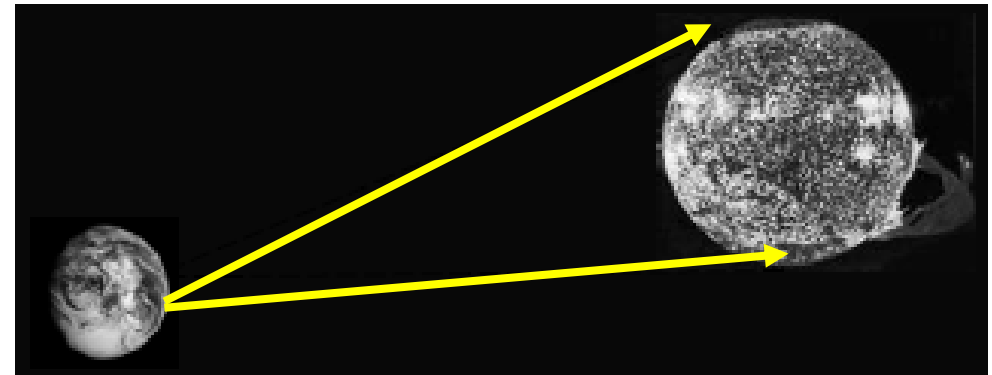
Same radiance on Earth and Mars?

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Sun Example: Power on Earth

- Power reaching earth on a 1m² square:

$$P = L \int_{\text{Area}} dA \int_{\text{Solid Angle}} \cos\theta \cdot d\omega_{\odot}$$

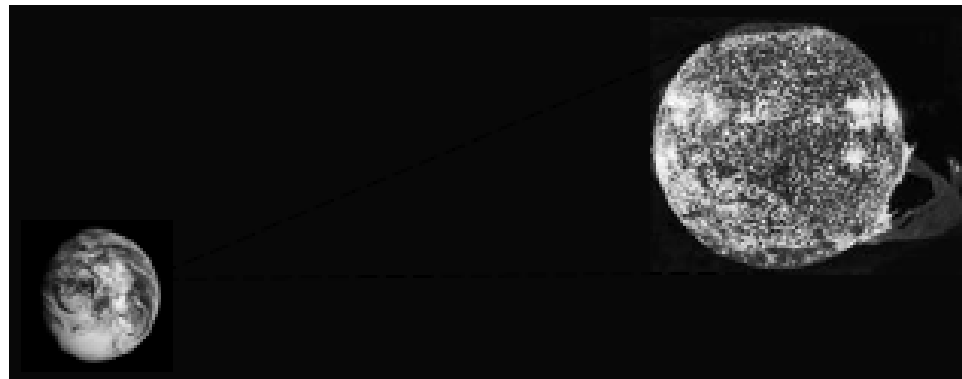


- Assume $\cos\theta = 1$ (sun in zenith)

$$P = L \int_{\text{Area}} dA \int_{\text{Solid Angle}} d\omega_{\odot}$$

Sun Example: Power on Earth

Power = Radiance.Area.Solid Angle

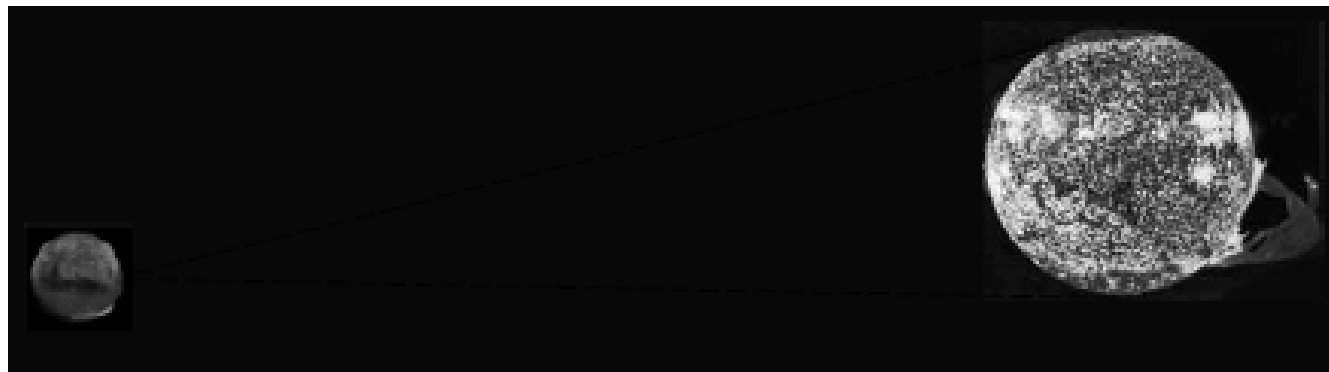


$$\begin{aligned}\text{Solid Angle} &= \text{Projected Area}_{\text{Sun}} / (\text{distance}_{\text{earth_sun}})^2 \\ &= 6.7 \cdot 10^{-5} \text{ sr}\end{aligned}$$

$$\begin{aligned}P &= (2.05 \times 10^7 \text{ W/ m}^2\cdot\text{sr}) \times (1 \text{ m}^2) \times (6.7 \cdot 10^{-5} \text{ sr}) \\ &= 1373.5 \text{ Watt}\end{aligned}$$

Sun Example: Power on Mars

Power = Radiance.Area.Solid Angle

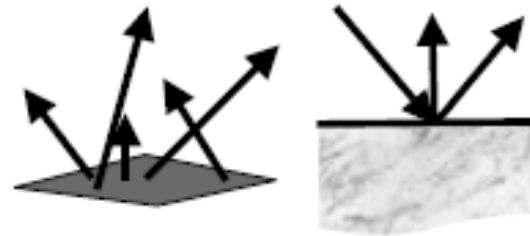


$$\begin{aligned}\text{Solid Angle} &= \text{Projected Area}_{\text{Sun}} / (\text{distance}_{\text{mars_sun}})^2 \\ &= 2.92 \cdot 10^{-5} \text{ sr}\end{aligned}$$

$$\begin{aligned}P &= (2.05 \times 10^7 \text{ W/ m}^2 \cdot \text{sr}) \times (1 \text{ m}^2) \times (2.92 \cdot 10^{-5} \text{ sr}) \\ &= 598.6 \text{ Watt}\end{aligned}$$

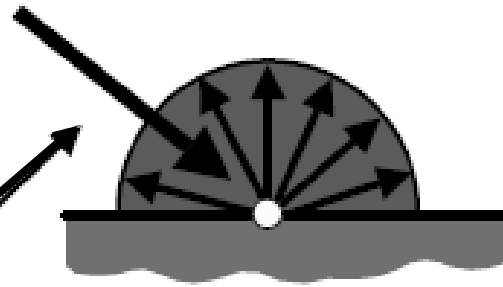
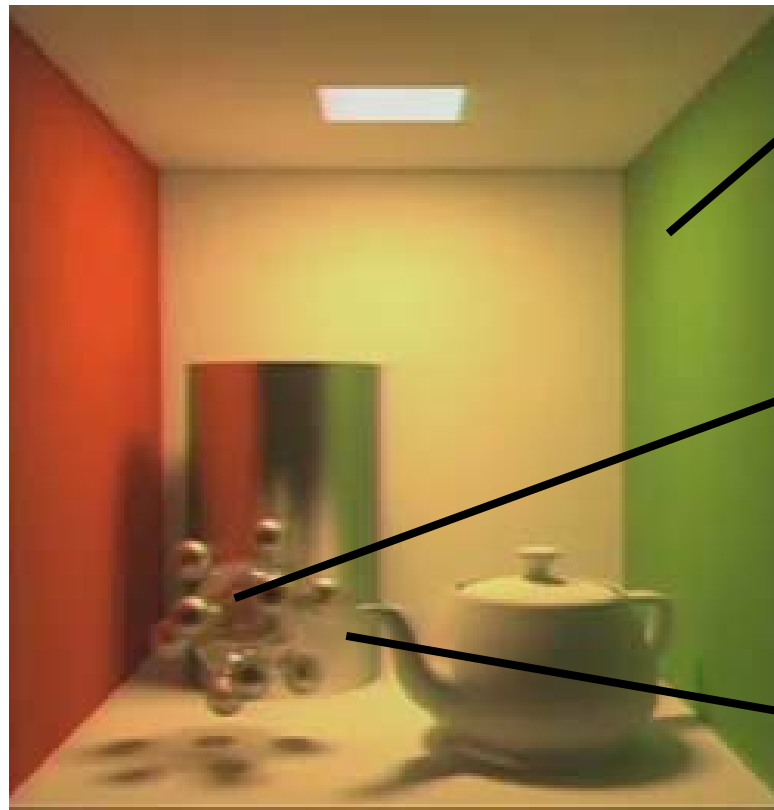
Light and Material Interactions

- Physics of light
- Radiometry
- **Material properties**
- Rendering equation



From kavita's slides

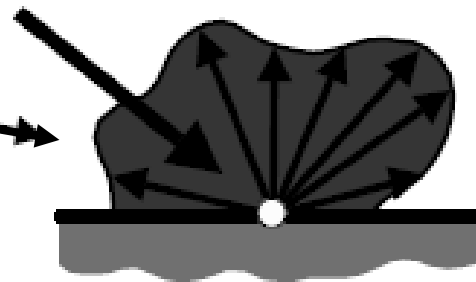
Materials



**Ideal diffuse
(Lambertian)**



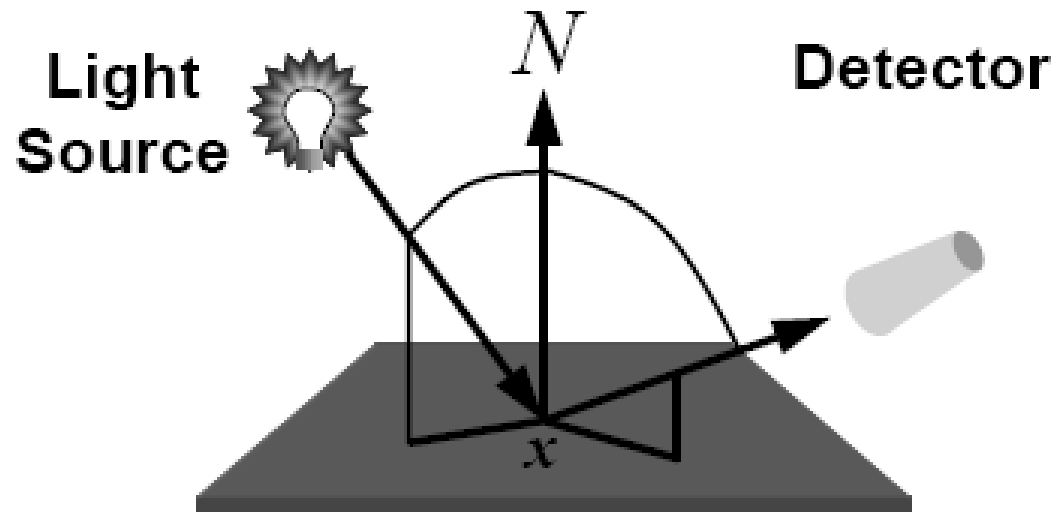
Ideal specular



Glossy

From kavita's slides

Bidirectional Reflectance Distribution Function (BRDF)



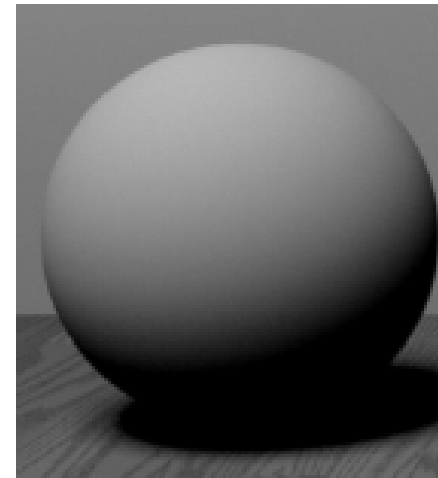
$$f_r(x, \Psi \rightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{dE(x \leftarrow \Psi)} = \frac{dL(x \rightarrow \Theta)}{L(x \leftarrow \Psi) \cos(N_x, \Psi) d\omega_\Psi}$$

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BRDF special case: ideal diffuse

Pure Lambertian

$$f_r(x, \Psi \rightarrow \Theta) = \frac{\rho_d}{\pi}$$



$$\rho_d = \frac{\text{Energy}_{out}}{\text{Energy}_{in}} \quad 0 \leq \rho_d \leq 1$$

Properties of the BRDF

- Reciprocity:

$$f_r(x, \Psi \rightarrow \Theta) = f_r(x, \Theta \rightarrow \Psi)$$

- Therefore, notation: $f_r(x, \Psi \leftrightarrow \Theta)$
- Important for bidirectional tracing

Properties of the BRDF

- Bounds:

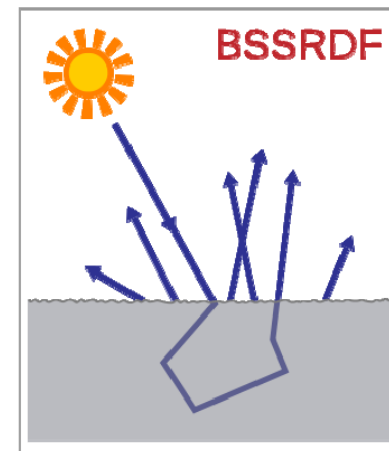
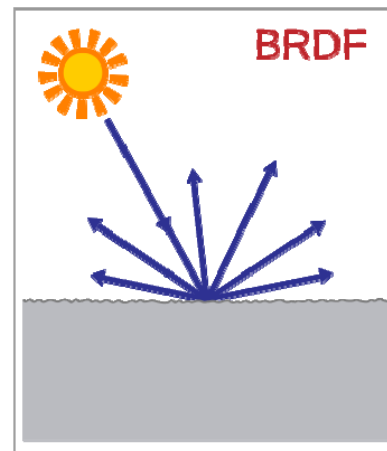
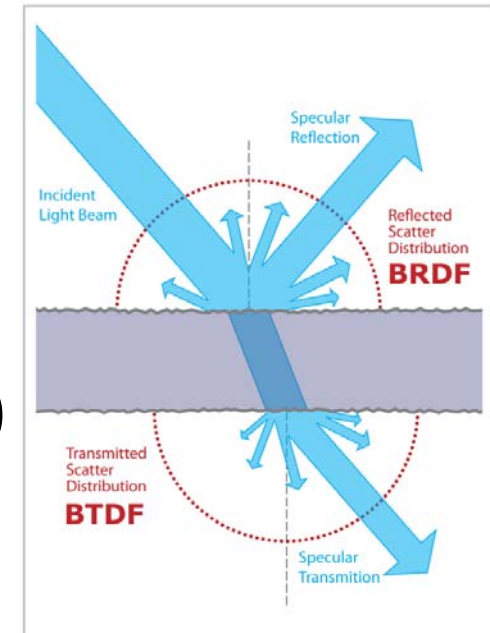
$$0 \leq f_r(x, \Psi \leftrightarrow \Theta) \leq \infty$$

- Energy conservation:

$$\forall \Psi \int_{\Theta} f_r(x, \Psi \leftrightarrow \Theta) \cos(N_x, \Theta) d\omega_{\Theta} \leq 1$$

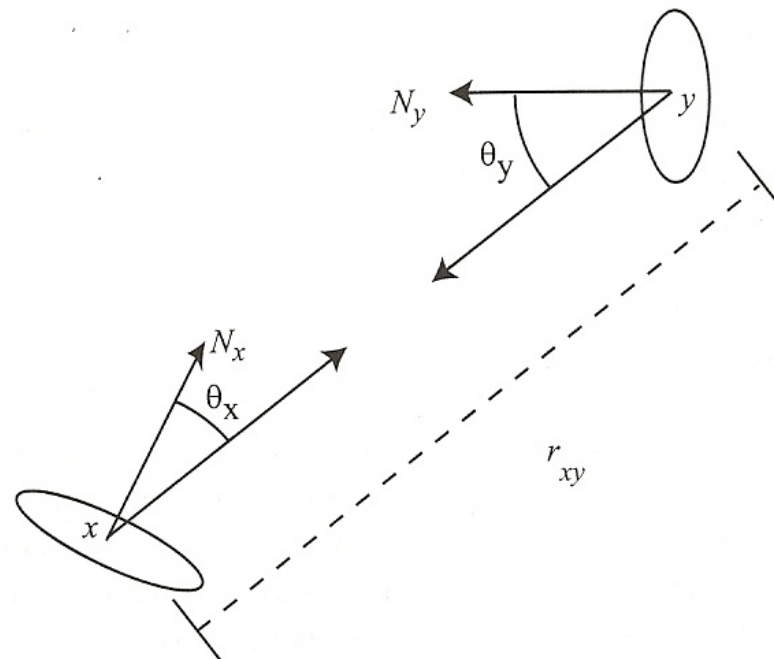
Other Distribution Functions: BxDF

- **BSDF (S: Scattering)**
 - The general form combining BRDF + BTDF (T: Transmittance)
- **BSSRDF (SS: Surface Scattering)**
 - Handle subsurface scattering



Homework 1

- Prove the invariance
 - Utilize the energy conservation



$$L(x \rightarrow y) = L(y \leftarrow x)$$

Figure 2.3. Invariance of radiance.

Class Objectives were:

- **Know terms of:**
 - **Hemispherical coordinates and integration**
 - **Various radiometric quantities (e.g., radiance)**
 - **Basic material function, BRDF**

Homework

- **Go over the next lecture slides before the class**
- **Watch 2 SIGGRAPH videos and submit your summaries every Tue. class**
 - **Just one paragraph for each summary**

Example:

Title: XXX XXXX XXXX

Abstract: this video is about accelerating the performance of ray tracing. To achieve its goal, they design a new technique for reordering rays, since by doing so, they can improve the ray coherence and thus improve the overall performance.

Any Questions?

- **Come up with one question on what we have discussed in the class and submit at the end of the class**
 - **1 for already answered questions**
 - **2 for typical questions**
 - **3 for questions with thoughts**
- **Submit questions at least four times before the mid-term exam**
 - **Multiple questions for the class is counted as only a single time**

Next Time

- **Rendering equation**