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# CS580: Monte Carlo Ray Tracing:

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**KAIST**

The KAIST logo consists of the letters 'KAIST' in a bold, blue, sans-serif font. Below the text is a light blue, horizontal oval shape that serves as a shadow or base for the letters.

# Talk Evaluations

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- **Do not evaluate based on:**
  - English
  - Relative ranking among presenters
  
- **Evaluate based on:**
  - His/her message
  - His/her understanding, and delivery
  - Achievement of the team for the project presentation

# Project Guidelines: Project Topics

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- **Any topics related to the course theme are okay**
  - **You can find topics by browsing recent papers**

# Expectations

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- **Mid-term project presentation**
  - **Introduce problems and explain why it is important**
  - **Give an overall idea on the related work**
  - **Explain what problems those existing techniques have**
  - **(Optional) explain how you can address those problems**
  - **Explain roles of each member**

# Expectations

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- **Final-term project presentation**
  - **Cover all the materials that you talked for your mid-term project**
  - **Present your ideas that can address problems of those state-of-the-art techniques**
  - **Give your qualitatively (or intuitive) reasons how your ideas address them**
  - **Also, explain expected benefits and drawbacks of your approach**
  - **(Optional) backup your claims with quantitative results collected by some implementations**
  - **Explain roles of each members**

# A few more comments

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- **Start to implement a paper, if you don't have any clear ideas**
  - **While you implement it, you may get ideas about improving it**
- **Utilize any existing materials (codes) with proper ack.**

# Project evaluation sheet

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You name:

ID:

Score table: higher score is better.

Speaker	Novelty of the project and idea (1 ~ 5)	Practical benefits of the method (1 ~ 5)	Completeness level of the project (1 ~ 5)	Total score (3 ~ 15)	Role of each student is clear and well balanced? (Yes or No)
XXX					
YYY					

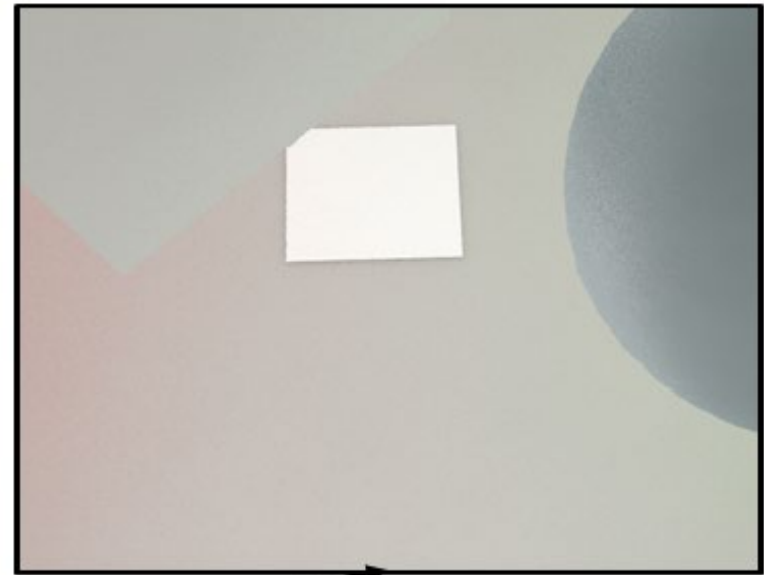
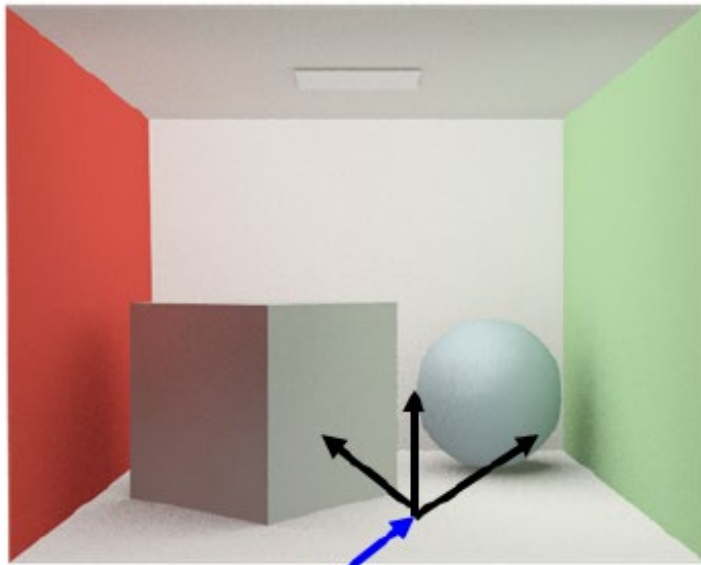
# Class Objectives

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- **Understand a basic structure of Monte Carlo ray tracing**
  - **Russian roulette for its termination**
  - **Path tracing**
  - **Biased techniques**



# Rendering Equation

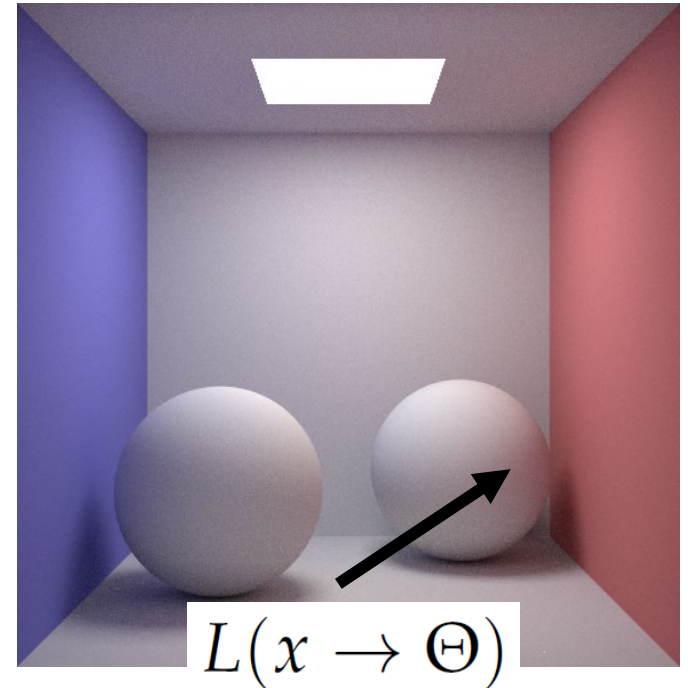


Incoming radiance on the hemisphere

$$L_r(x \rightarrow \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \rightarrow \Theta) \cos \theta_x dw_{\Psi}$$

# Evaluation

- **To compute**  $L(x \rightarrow \Theta)$  :
  - **Check**  $L_e(x \rightarrow \Theta)$
  - **Evaluate**  $L_r(x \rightarrow \Theta)$



$$L_r(x \rightarrow \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \rightarrow \Theta) \cos \theta_x d\omega_{\Psi}$$

# Evaluation

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- Use Monte Carlo
- Generate random directions on hemisphere  $\Psi$  using pdf  $p(\Psi)$

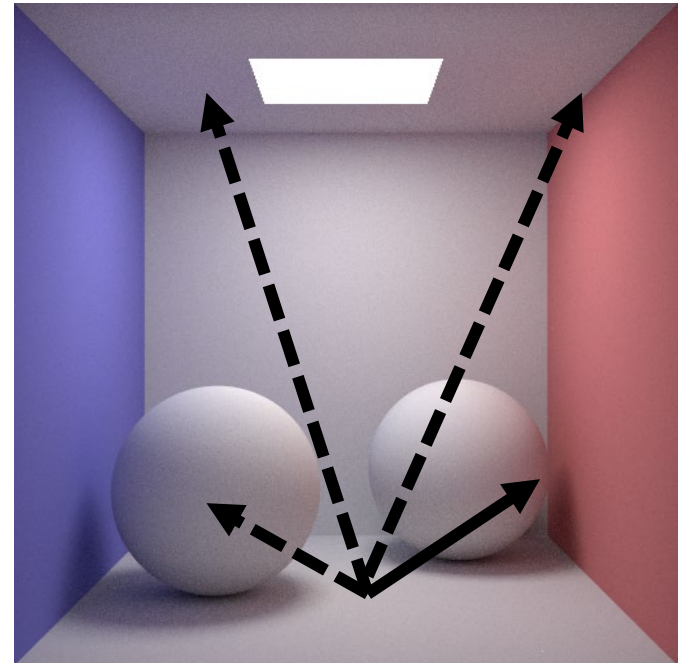
$$L_r(x \rightarrow \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \rightarrow \Theta) \cos \theta_x d\omega_{\Psi}$$

$$\hat{L}_r(x \rightarrow \Theta) = \frac{1}{N} \sum_{i=1}^N \frac{L(x \leftarrow \Psi_i) f_r(x, \Psi_i \rightarrow \Theta) \cos \theta_x}{p(\Psi_i)}$$

- How about  $L(x \leftarrow \Psi_i)$  ?

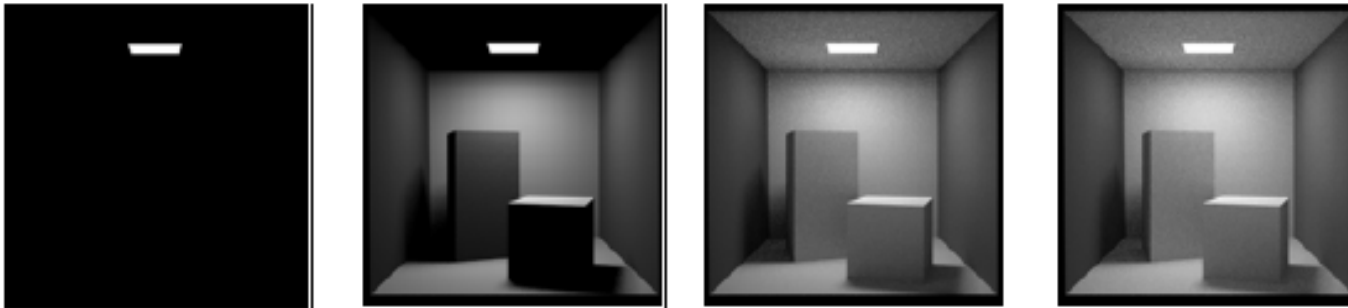
# Evaluation

- How about  $L(x \leftarrow \Psi_i)$  ?
- Perform ray casting backward
- Compute radiance from those visible points to  $x$ 
  - Assume reciprocity
- Recursively perform the process
  - Each additional bounce supports one more indirect illumination



# When to end recursion?

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From kavita's slides

- **Contributions of further light bounces become less significant**
  - Max recursion
  - Some threshold for radiance value
- **If we just ignore them, estimators will be biased**

# Russian Roulette

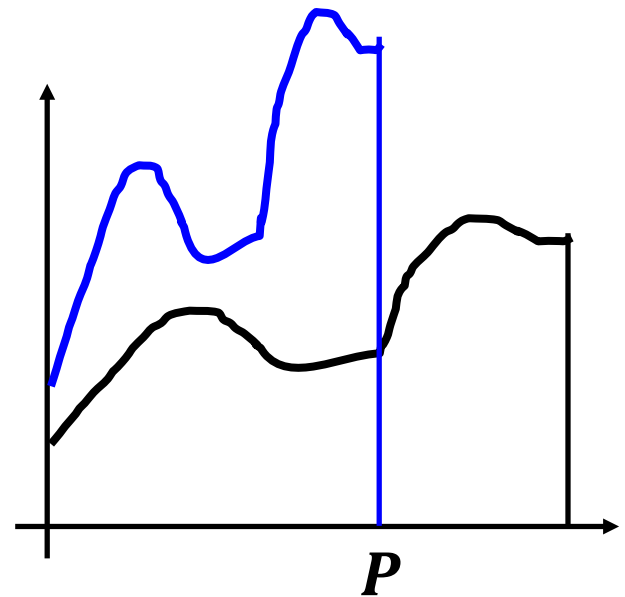
- **Integral: Substitute  $y = Px$**

$$I = \int_0^1 f(x) dx = \int_0^P \frac{f(y/P)}{P} dy.$$

- **Estimator**

$$\hat{I}_{\text{roulette}} = \begin{cases} \frac{f(x_i)}{P} & \text{if } x_i \leq P, \\ 0 & \text{if } x_i > P. \end{cases}$$

- **Variance?**



# Russian Roulette

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- **Pick absorption probability,  $\alpha = 1-P$** 
  - **Recursion is terminated**
  
- **P is commonly to be equal to the reflectance of the material of the surface**
  - **Water: 7%**
  - **Snow: 65%**

# Algorithm so far

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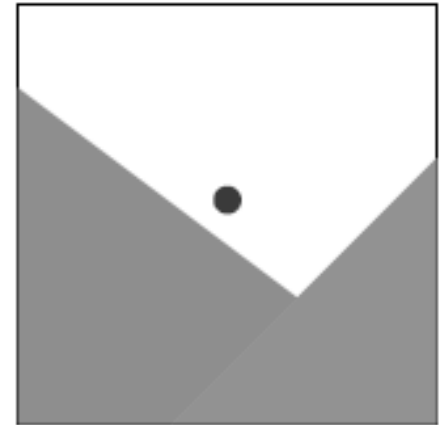
- **Shoot primary rays through each pixel**
- **Shoot indirect rays, sampled over hemisphere**
- **Terminate recursion using Russian Roulette**



# Pixel Anti-Aliasing

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- **Compute radiance only at the center of pixel**
  - **Produce jaggies**
- **We want to evaluate using MC**
- **Simple box filter**
  - **The averaging method**

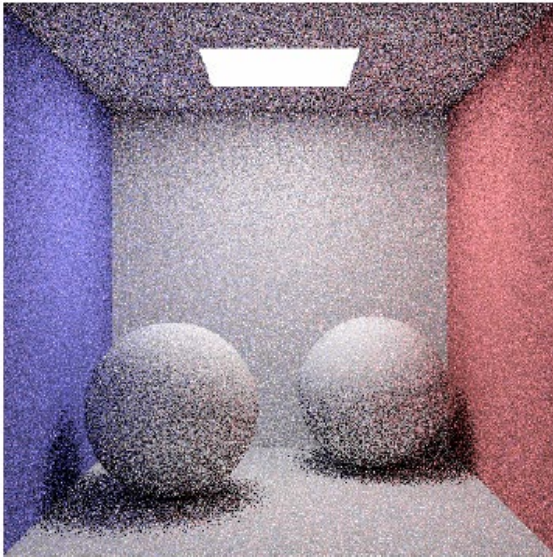


# Stochastic Ray Tracing

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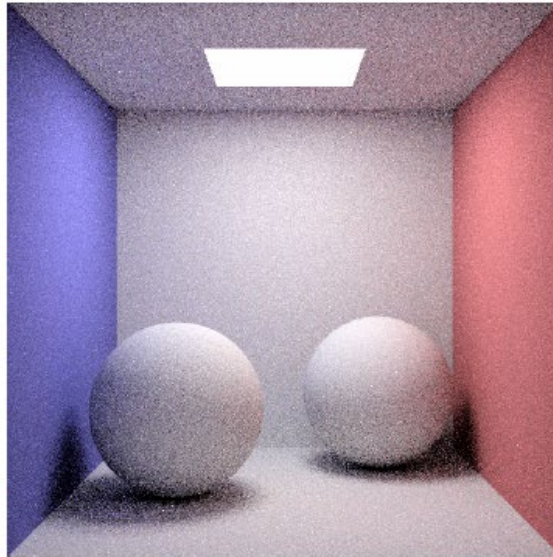
- **Parameters**
  - **Num. of starting ray per pixel**
  - **Num. of random rays for each surface point (branching factor)**
- **Path tracing**
  - **Branching factor = 1**

# Path Tracing

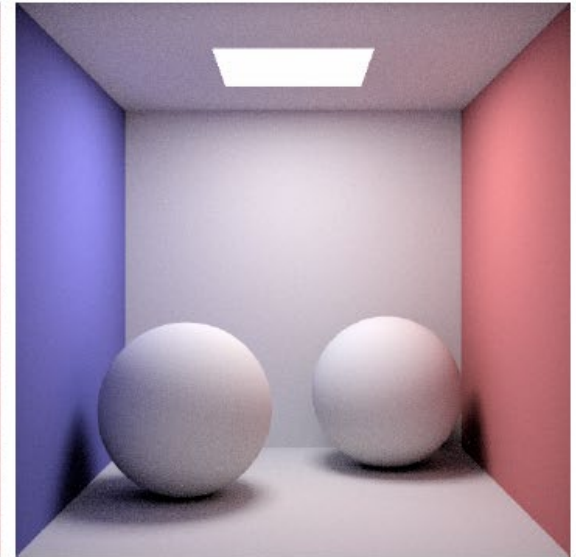


1 spp

(samples per pixel)



4 spp



16 spp

- **Pixel sampling + light source sampling folded into one method**

# Algorithm so far

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- **Shoot primary rays through each pixel**
- **Shoot indirect rays, sampled over hemisphere**
  - **Path tracing shoots only 1 indirect ray**
- **Terminate recursion using Russian Roulette**

# Performance

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- **Want better quality with smaller # of samples**
  - **Fewer samples/better performance**
  - **Quasi Monte Carlo: well-distributed samples**
  - **Adaptive sampling**
- **See my book, if you are interested**

# Some Example

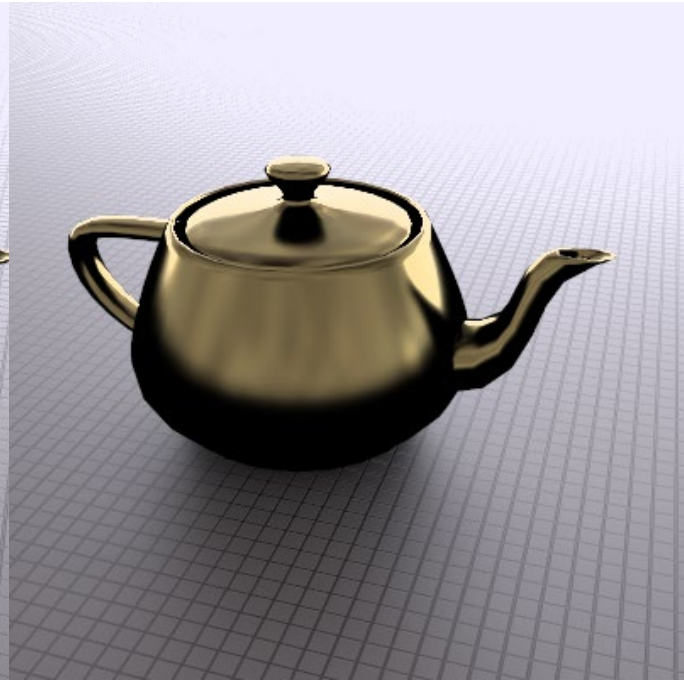
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**Uniform sampling  
(64 samples per pixel)**



**Adaptive sampling**

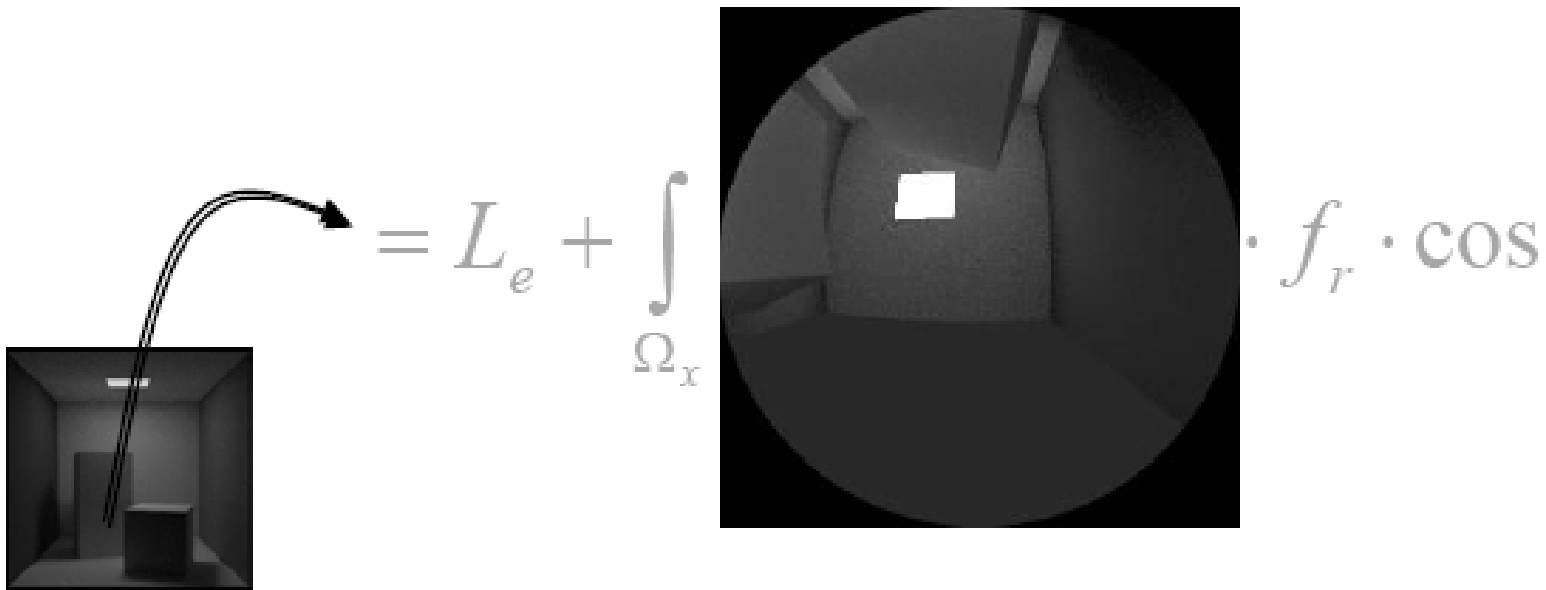


**Reference**

# Importance Sampling

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos(\Psi, n_x) \cdot d\omega_\Psi$$

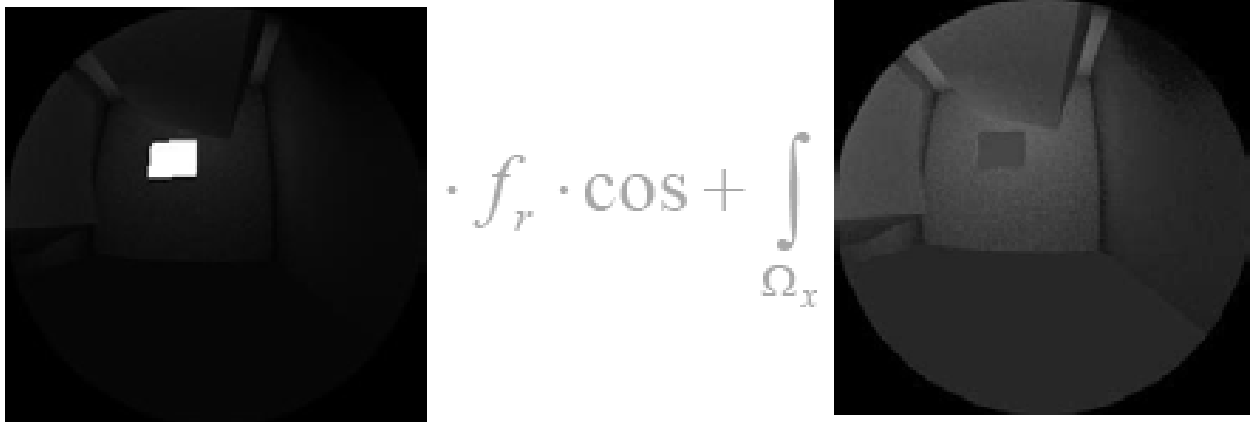
Radiance from light sources + radiance from other surfaces



# Importance Sampling

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$$L(x \rightarrow \Theta) = L_e + L_{direct} + L_{indirect}$$

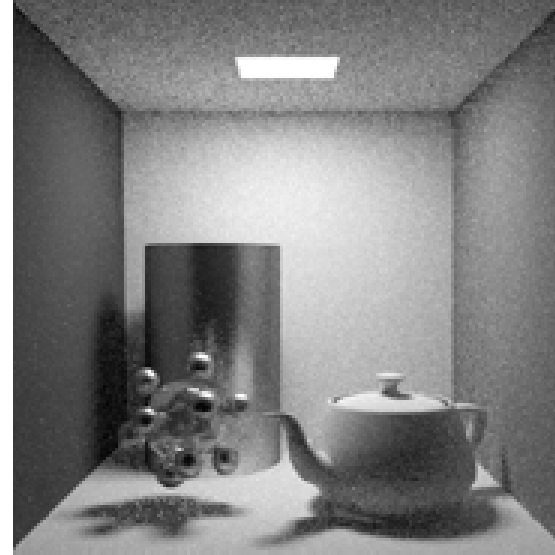
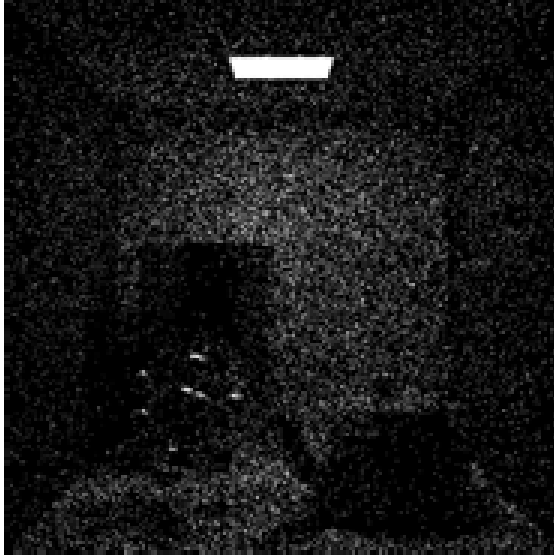
$$= L_e + \int_{\Omega_x} \text{img}_1 \cdot f_r \cdot \cos + \int_{\Omega_x} \text{img}_2 \cdot f_r \cdot \cos$$


- So ... sample direct and indirect with separate MC integration



# Comparison

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From kavita's slides

- **With and without considering direct illumination**
  - **16 samples / pixel**

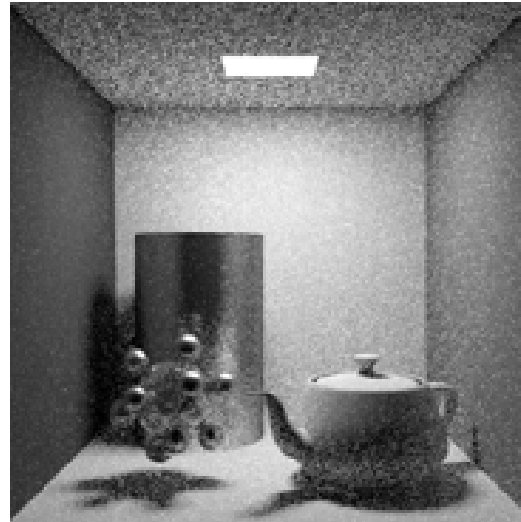
# Rays per pixel

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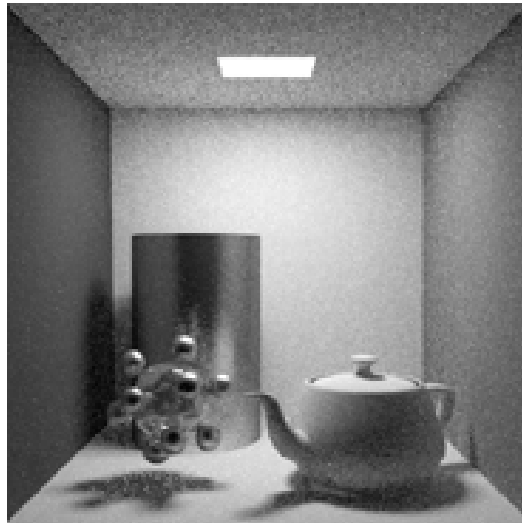
1 sample/  
pixel



4 samples/  
pixel



16 samples/  
pixel



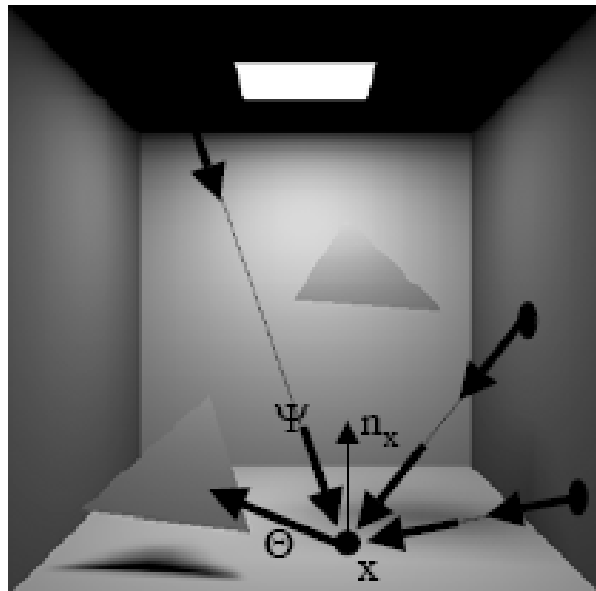
256 samples/  
pixel



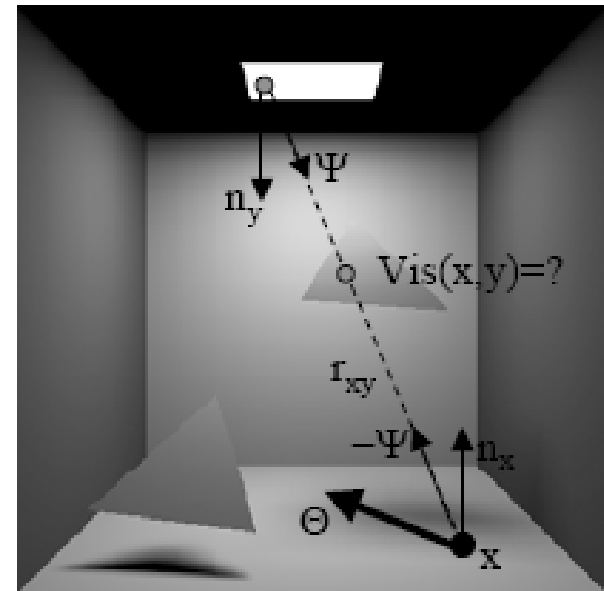
# Direct Illumination

$$L(x \rightarrow \Theta) = \int_{A_{source}} f_r(x, -\Psi \leftrightarrow \Theta) \cdot L(y \rightarrow \Psi) \cdot G(x, y) \cdot dA_y$$

$$G(x, y) = \frac{\cos(n_x, \Theta) \cos(n_y, \Psi) Vis(x, y)}{r_{xy}^2}$$



hemisphere integration



area integration

# Estimator for direct lighting

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- Pick a point on the light's surface with pdf

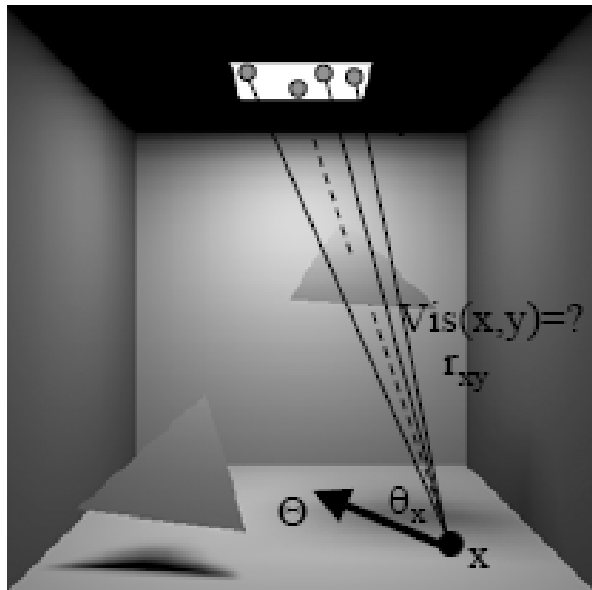
$$p(y)$$

- For N samples, direct light at point x is:

$$E(x) = \frac{1}{N} \sum_{i=1}^N \frac{f_r L_{source} \frac{\cos \theta_x \cos \theta_{\bar{y}_i}}{r_{x\bar{y}_i}^2} Vis(x, \bar{y}_i)}{p(\bar{y}_i)}$$

# Generating direct paths

- Pick surface points  $y_i$  on light source
- Evaluate direct illumination integral



$$\langle L(x \rightarrow \Theta) \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f_r(\dots)L(\dots)G(x, y_i)}{p(y_i)}$$

# PDF for sampling light

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- Uniform

$$p(y) = \frac{1}{Area_{source}}$$

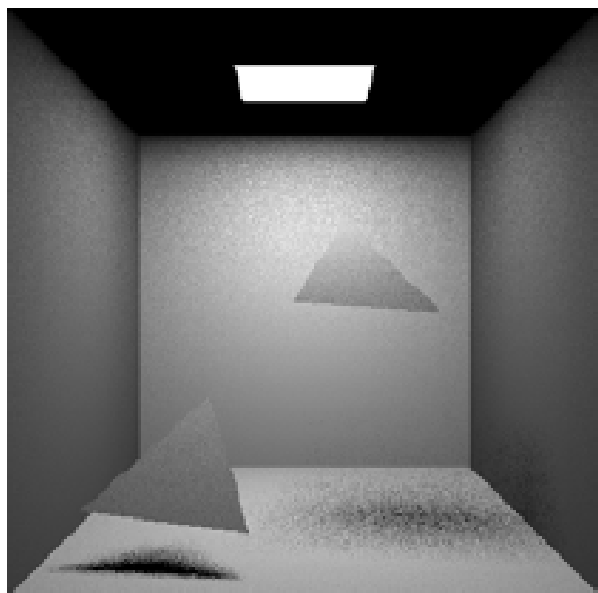
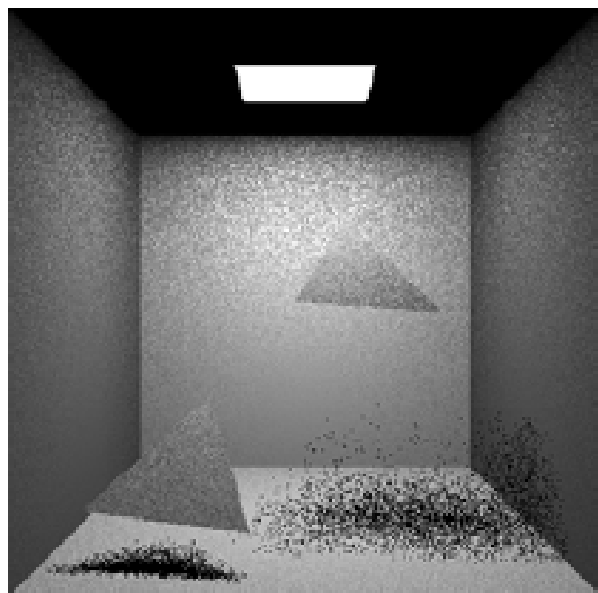
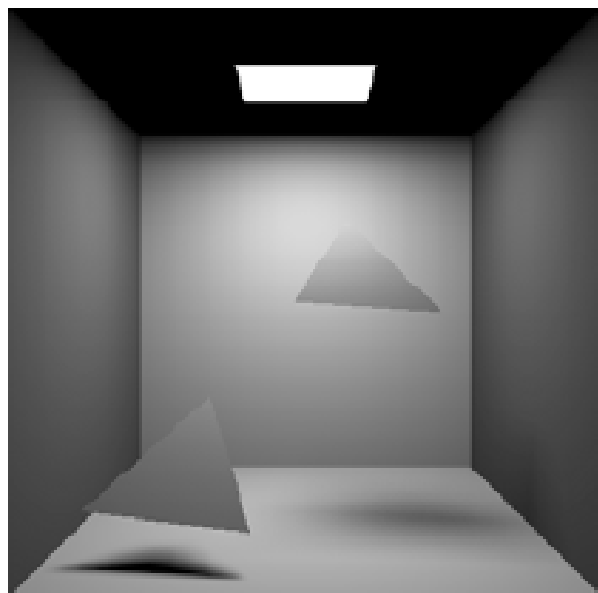
- Pick a point uniformly over light's area
  - Can stratify samples

- Estimator:

$$E(x) = \frac{Area_{source}}{N} \sum_{i=1}^N f_r L_{source} \frac{\cos \theta_x \cos \theta_{\bar{y}_i}}{r_{x\bar{y}_i}^2} Vis(x, \bar{y}_i)$$

# More points ...

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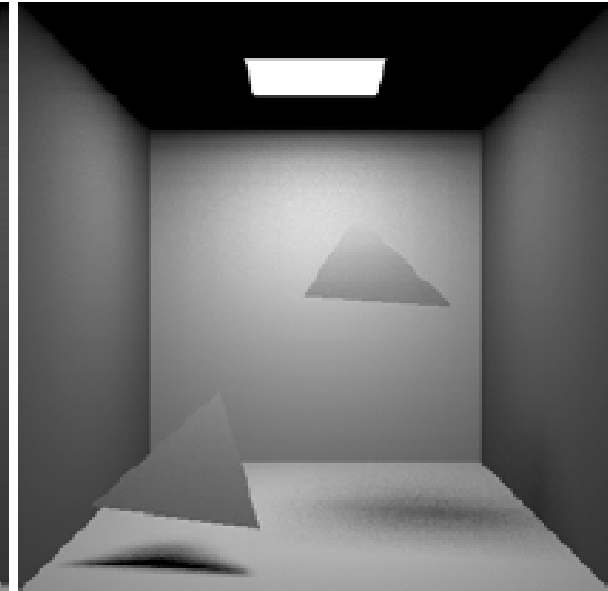
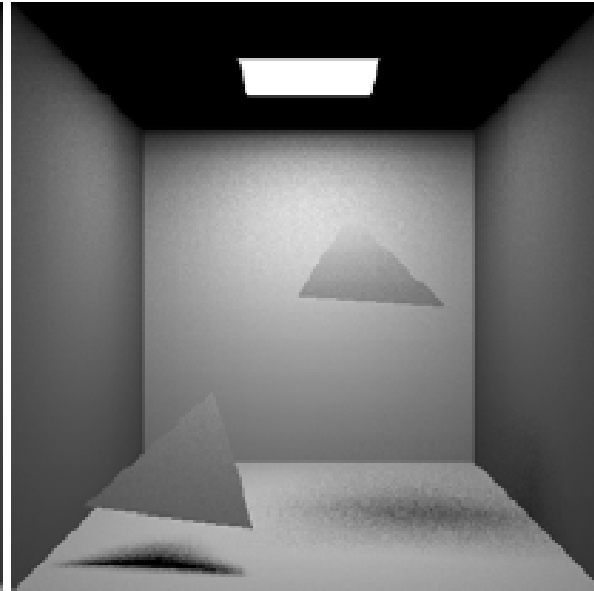
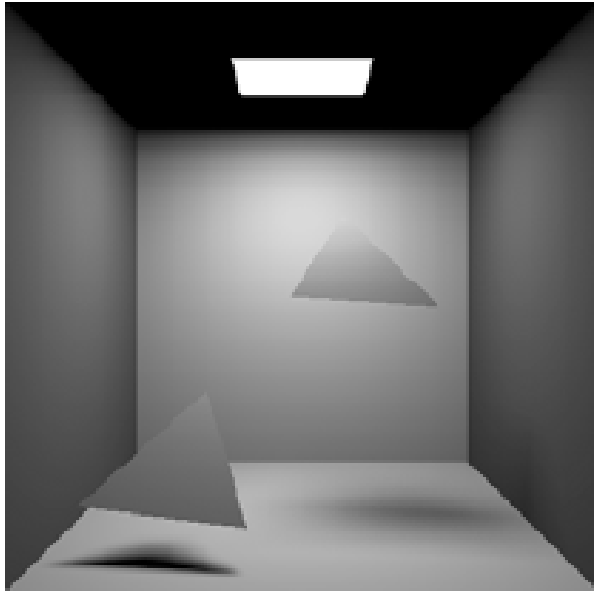
1 shadow ray

9 shadow rays

$$E(x) = \frac{Area_{source}}{N} \sum_{i=1}^N f_r L_{source} \frac{\cos \theta_x \cos \theta_{\bar{y}_i}}{r_{x\bar{y}_i}^2} Vis(x, \bar{y}_i)$$

# Even more points ...

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36 shadow rays

100 shadow rays

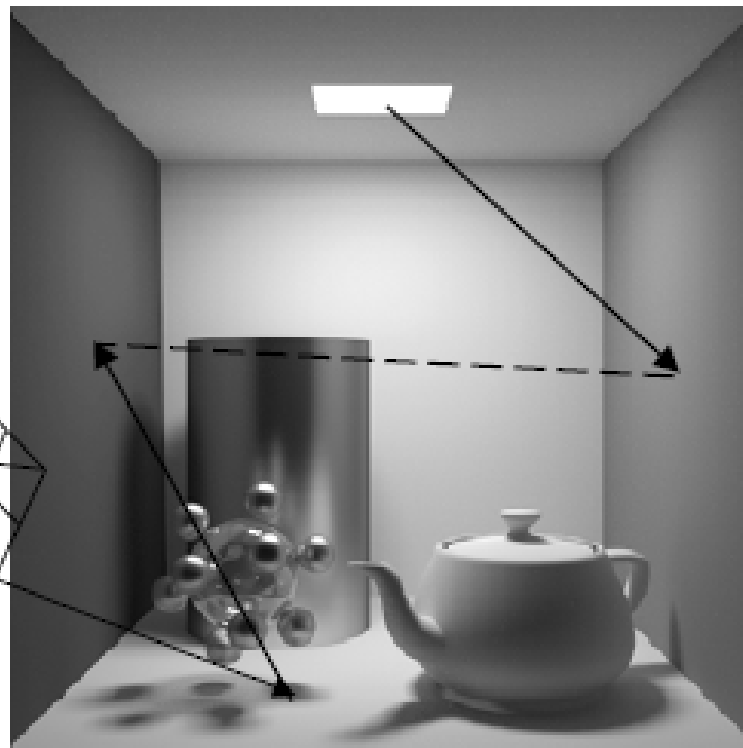
$$E(x) = \frac{Area_{source}}{N} \sum_{i=1}^N f_r L_{source} \frac{\cos \theta_x \cos \theta_{\bar{y}_i}}{r_{x\bar{y}_i}^2} Vis(x, \bar{y}_i)$$



# Bidirectional Path Tracing

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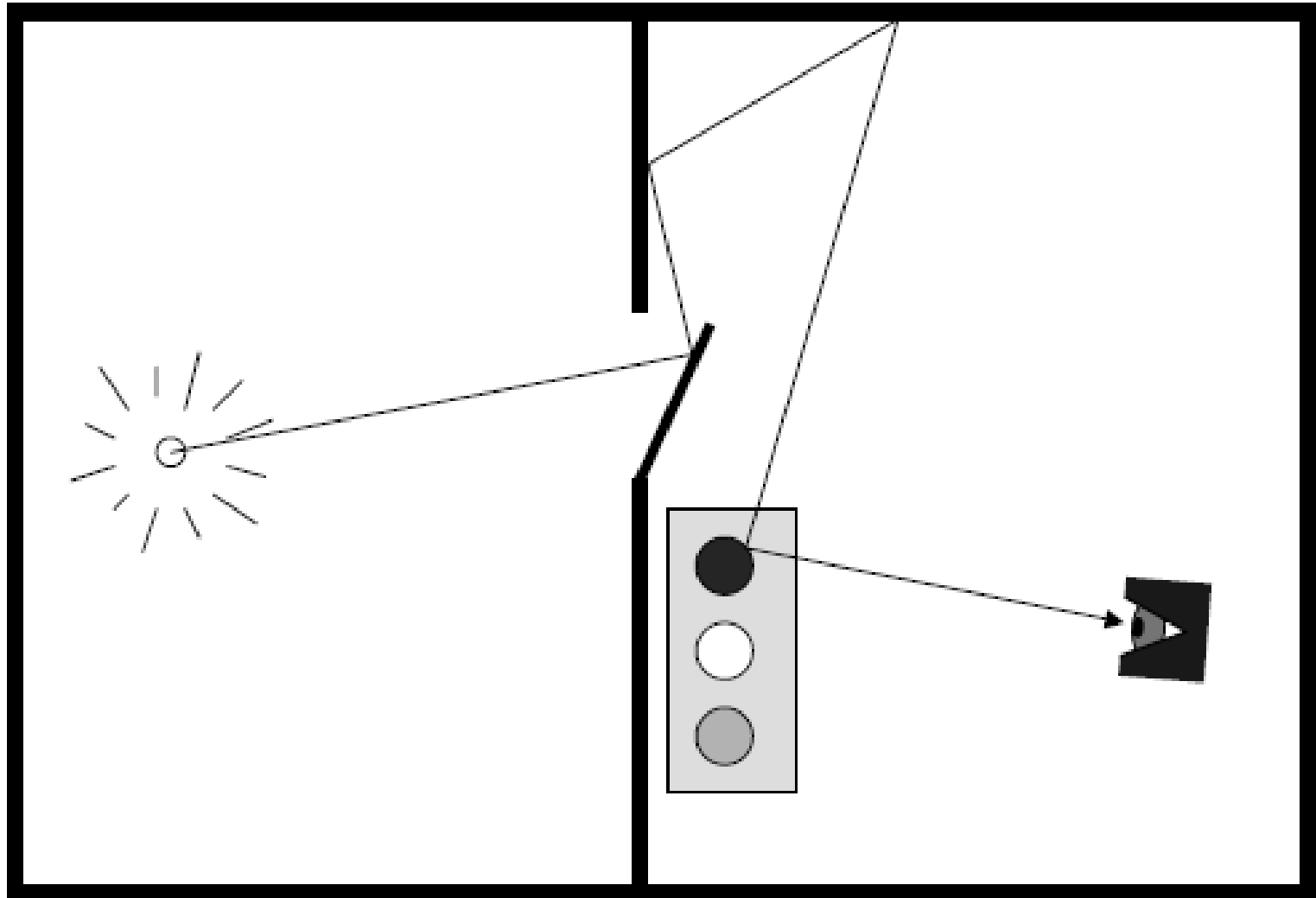
- Or paths generated from both camera and source at the same time ...!



- Connect endpoints to compute final contribution

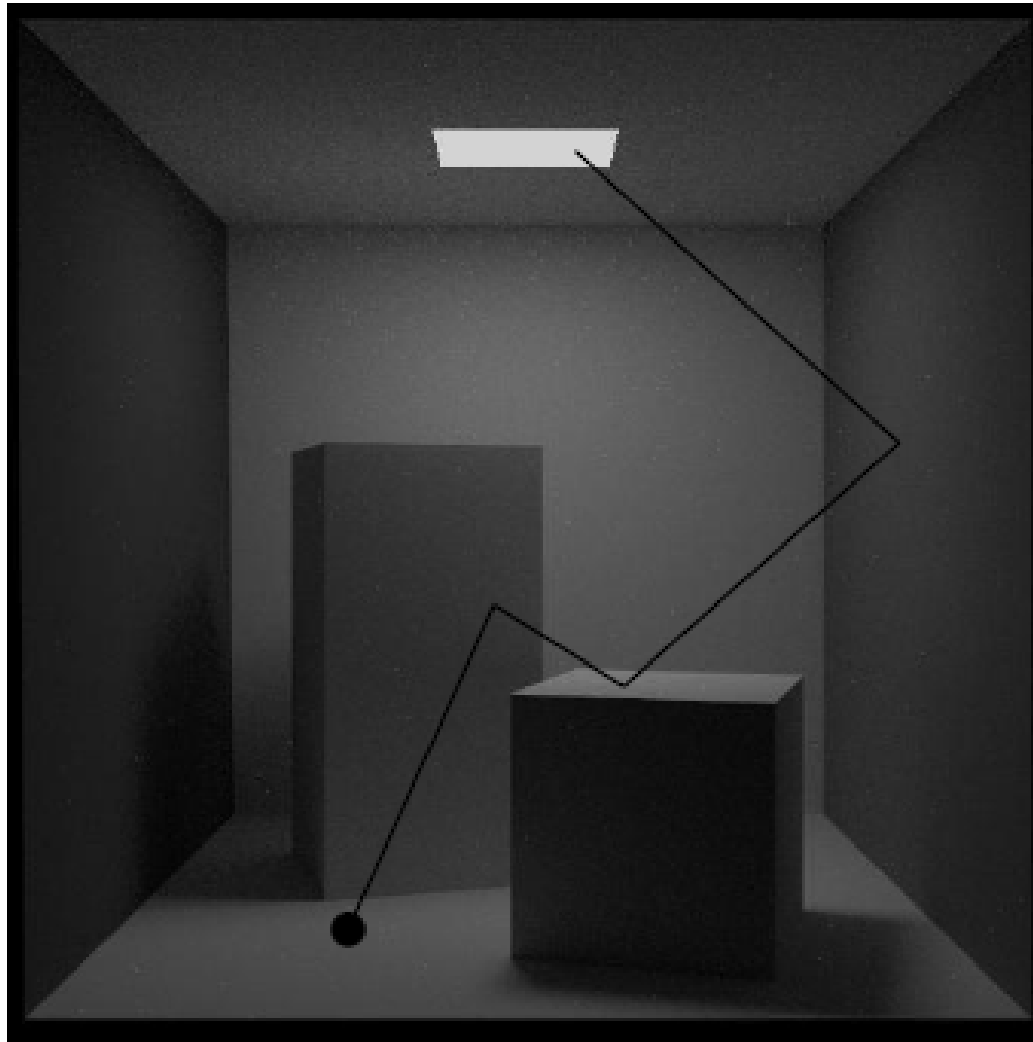
# Metropolis

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# Metropolis

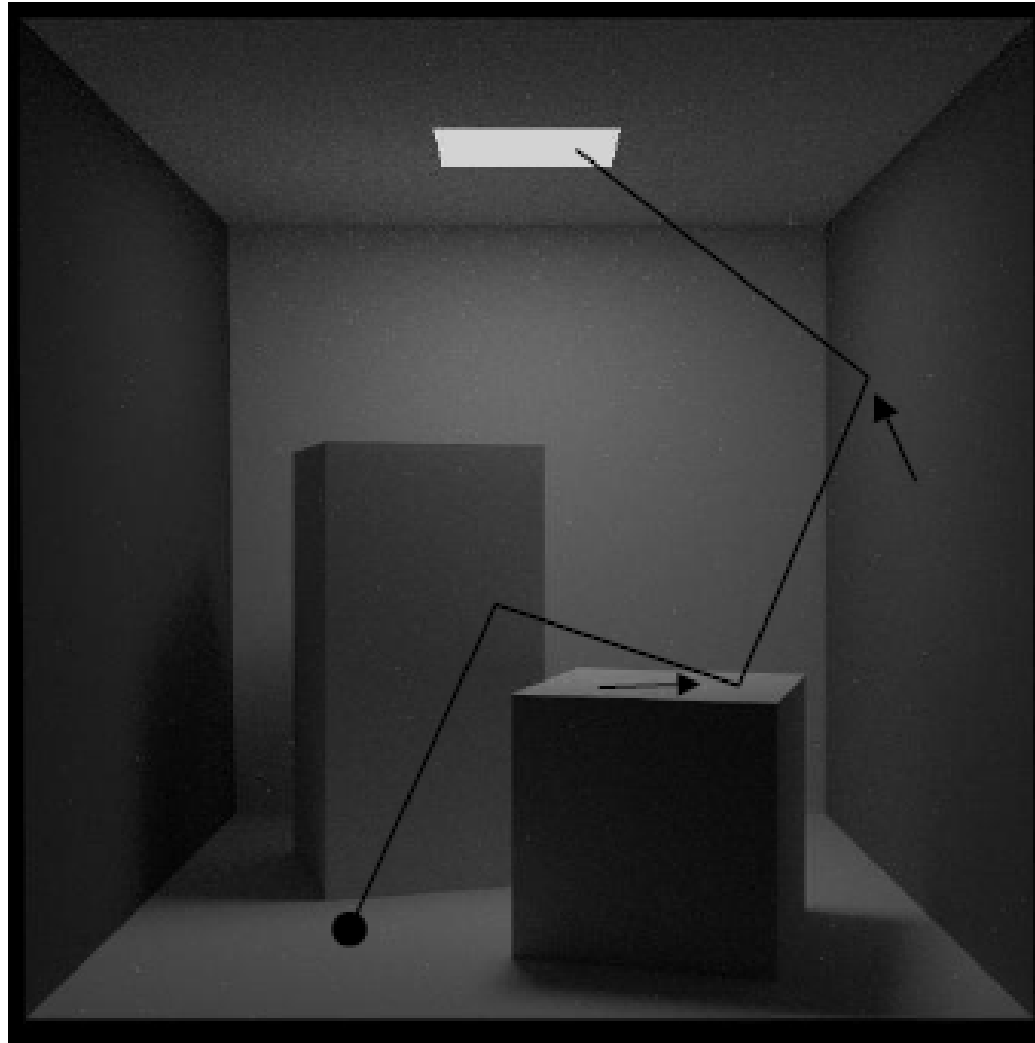
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valid path

# Metropolis

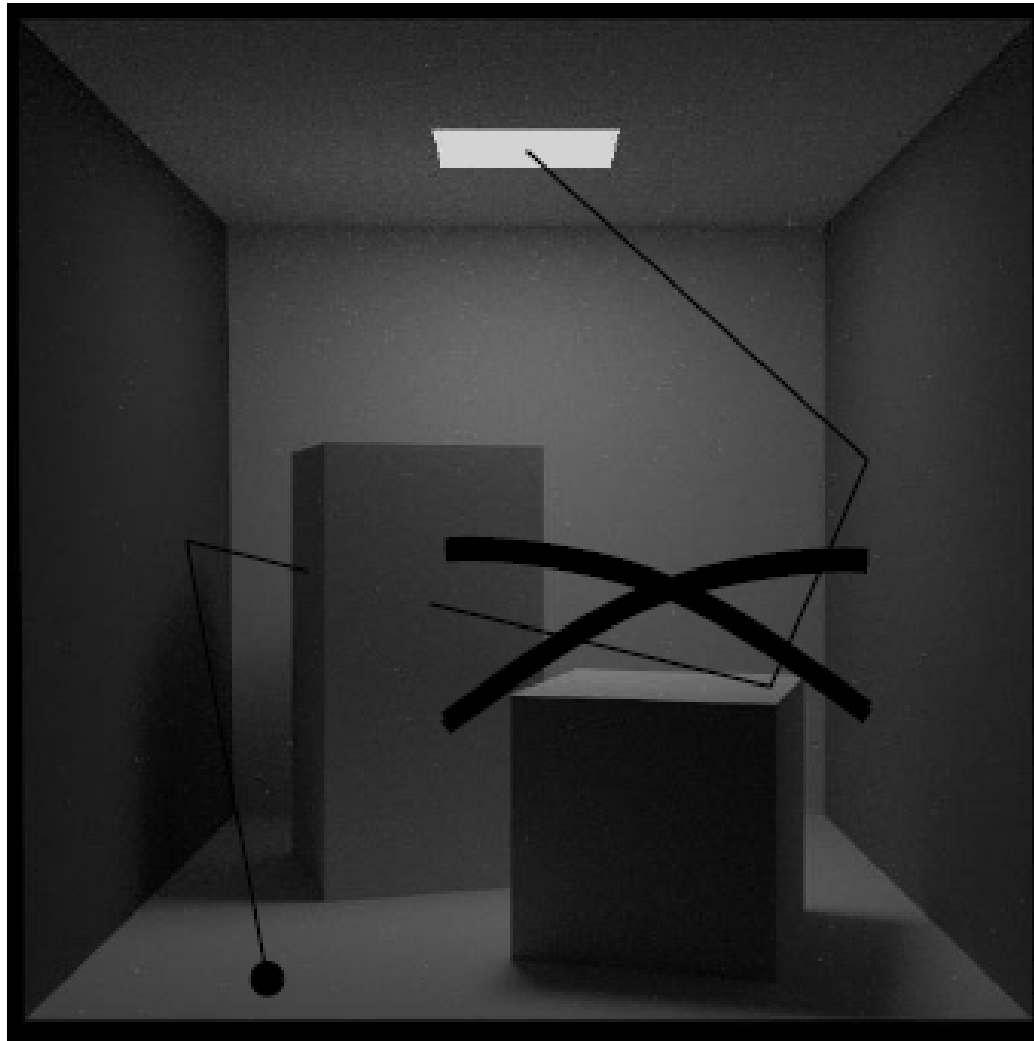
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small  
perturbations

# Metropolis

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Accept  
mutations  
based on  
energy  
transport

# Unbiased vs. Consistent

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- **Unbiased**

- **No systematic error**
- **$E[\mathbf{I}_{\text{estimator}}] = \mathbf{I}$**
- **Better results with larger N**

- **Consistent**

- **Converges to correct results with more samples**
- **$E[\mathbf{I}_{\text{estimator}}] = \mathbf{I} + \boldsymbol{\varepsilon}$ , where  $\lim_{n \rightarrow \infty} \boldsymbol{\varepsilon} = \mathbf{0}$**

# Biased Methods

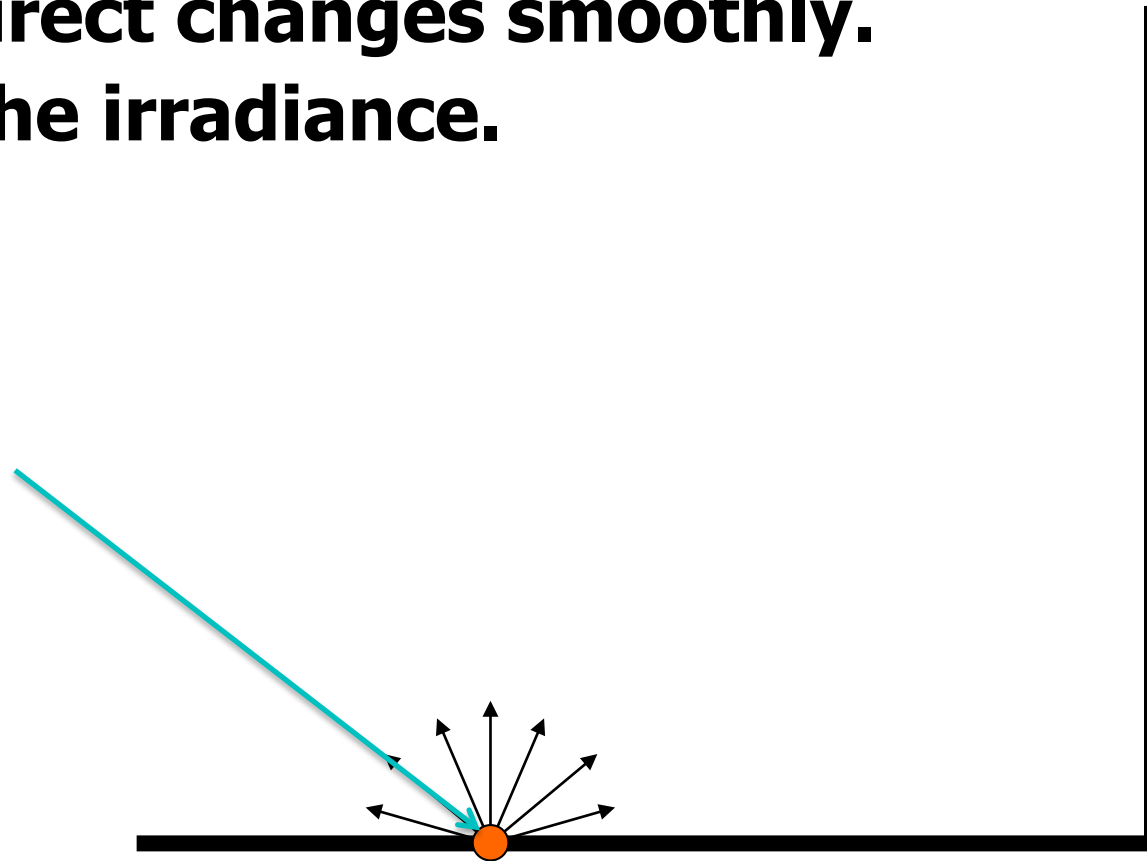
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- **MC methods**
  - **Too noisy and slow**
  - **Noise is objectionable**
- **Biased methods: store information (caching)**
  - **Irradiance caching**
  - **Photon mapping**

# Biased Methods: Irradiance Caching

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- **Indirect changes smoothly.**
- **Cache irradiance.**

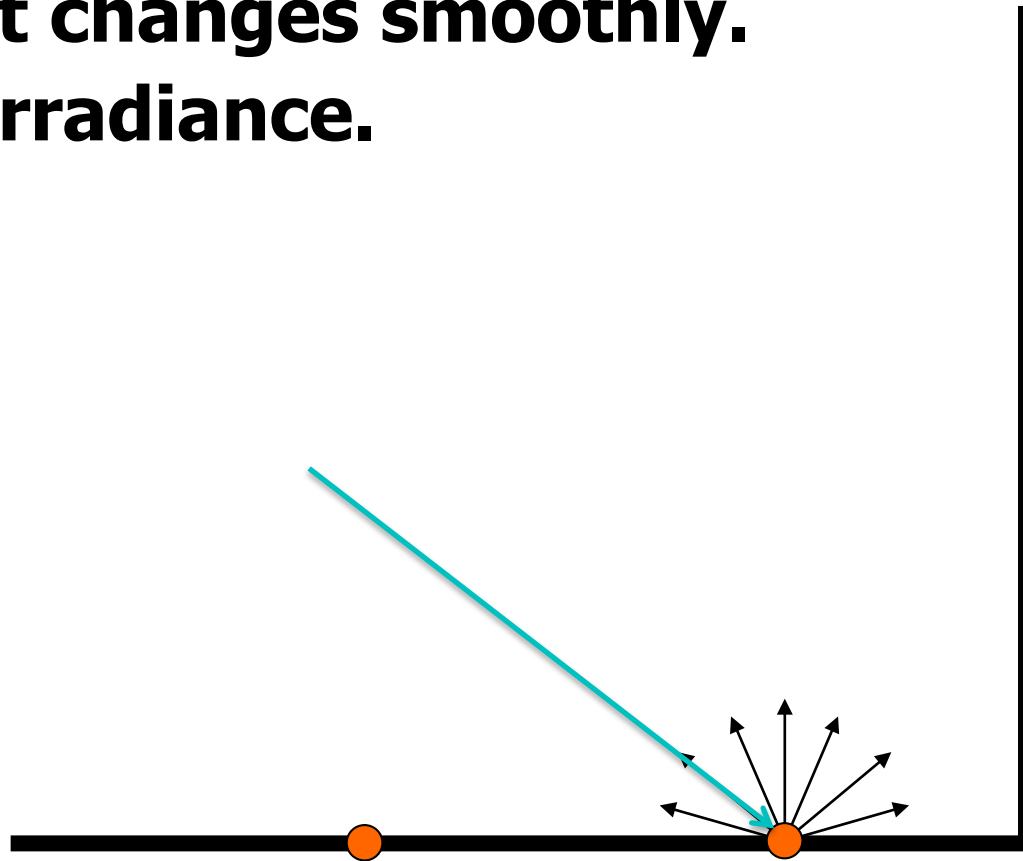




# Irradiance Caching

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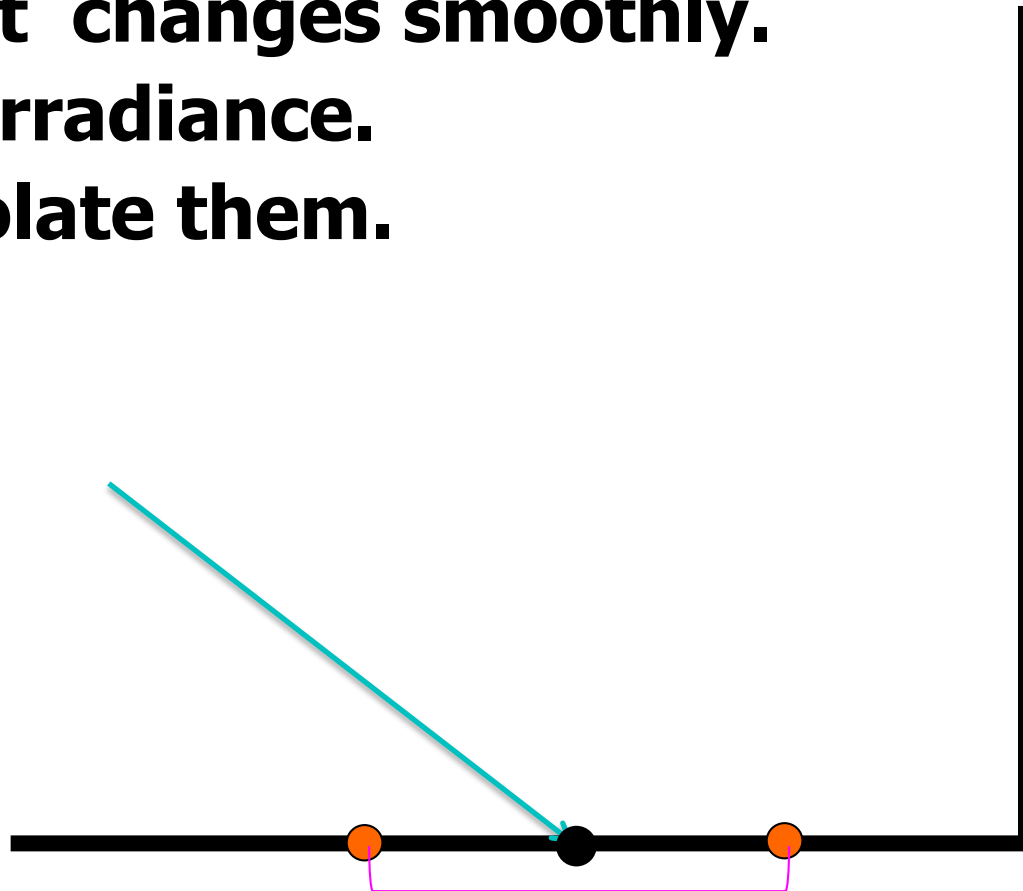
- Indirect changes smoothly.
- Cache irradiance.



# Irradiance Caching

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- **Indirect changes smoothly.**
- **Cache irradiance.**
- **Interpolate them.**



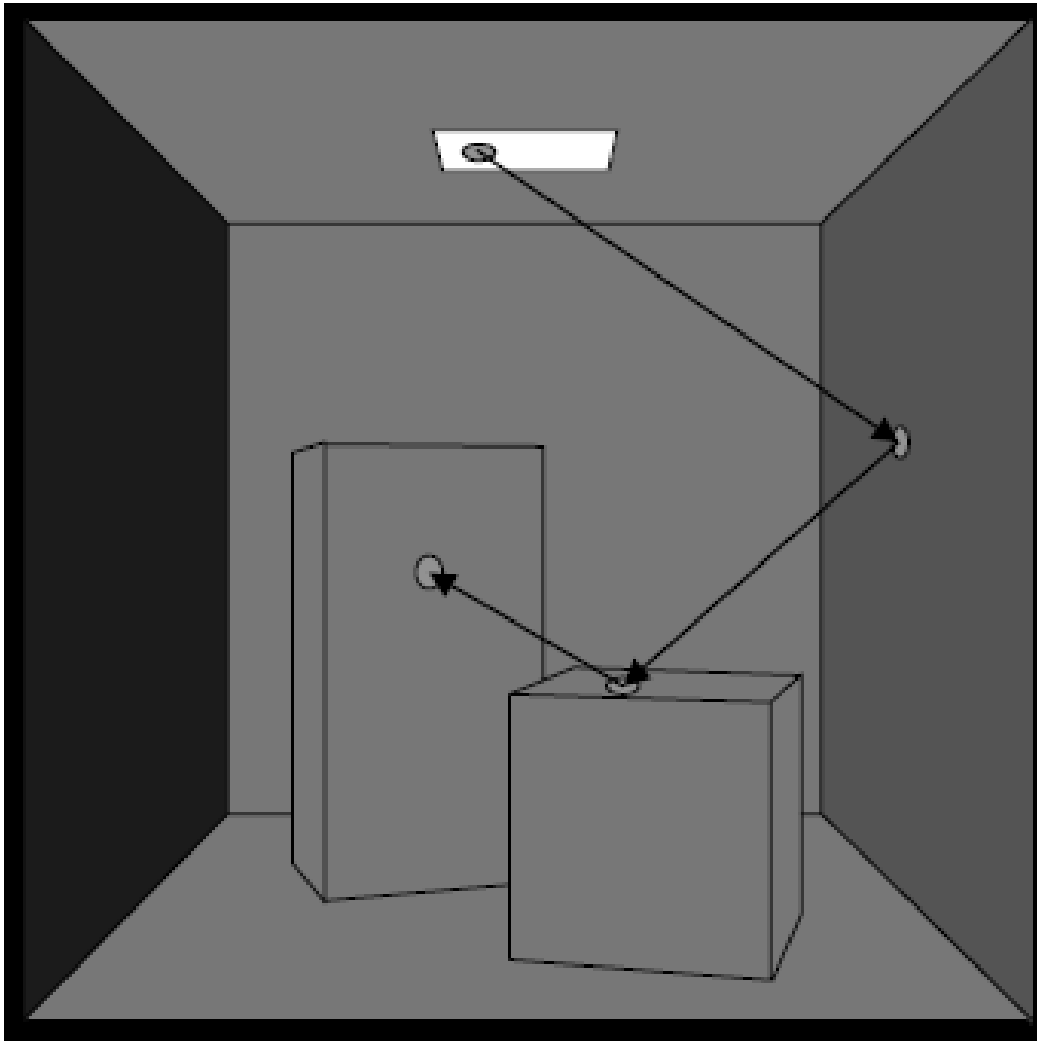
# Biased Method: Photon Mapping

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- **2 passes:**
  - **Shoot “photons” (light-rays) and record any hit-points**
  - **Shoot viewing rays and collect information from stored photons**

# Pass 1: shoot photons

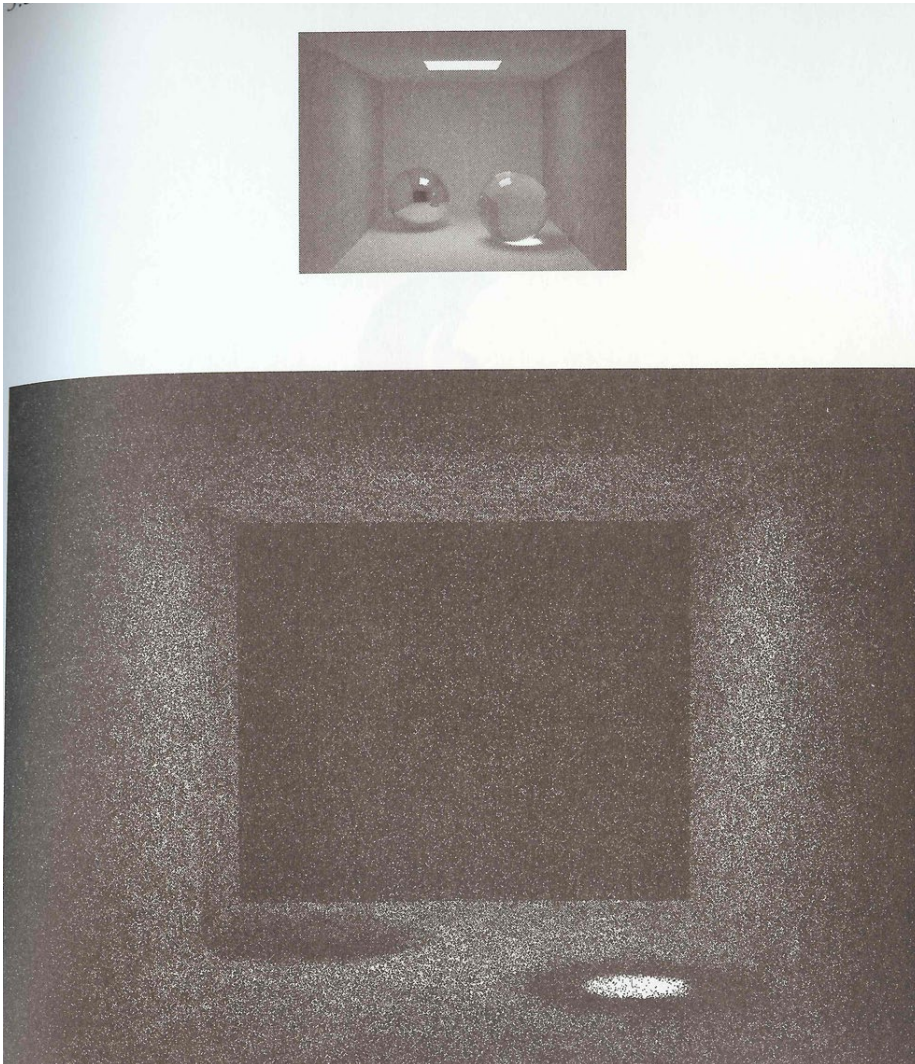
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- Light path generated using MC techniques and Russian Roulette
- Store:
  - position
  - incoming direction
  - color
  - ...

# Stored Photons

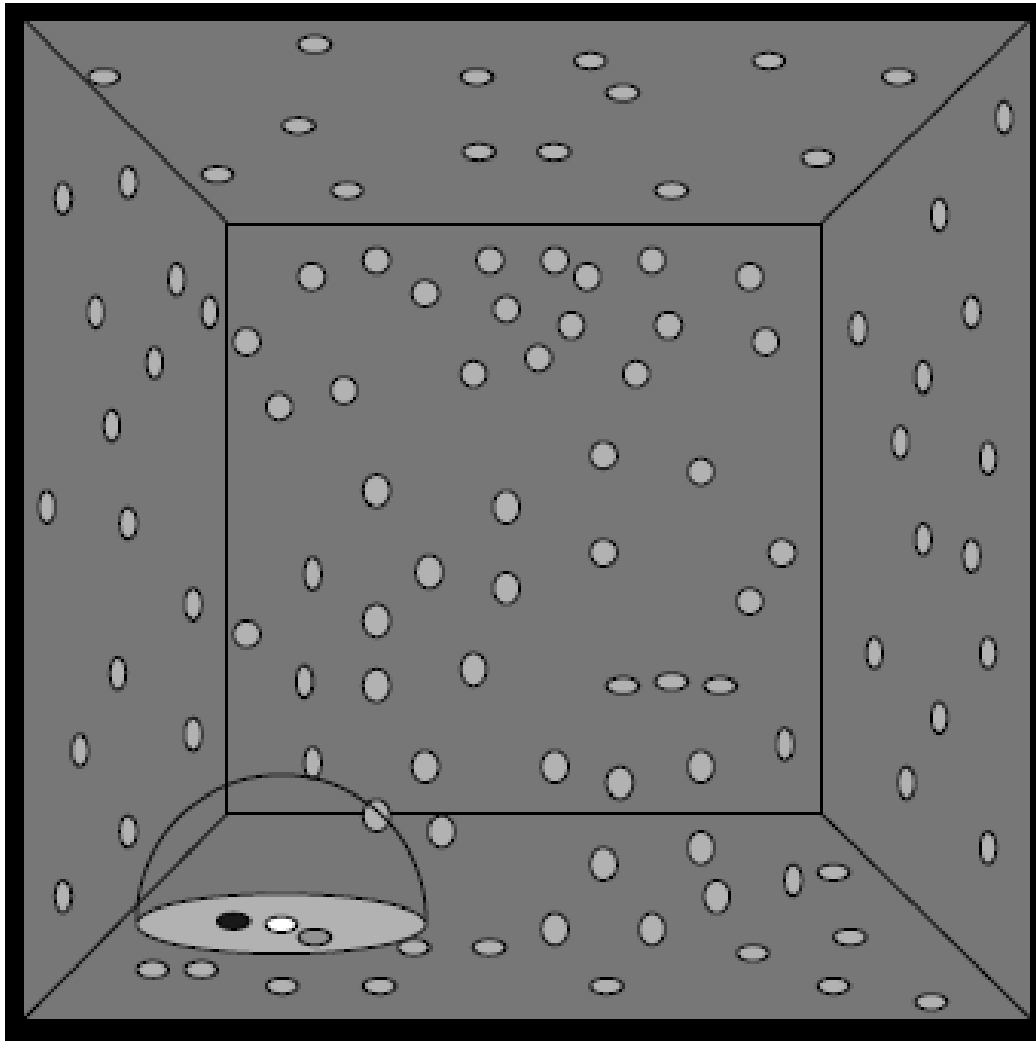
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**Generate a few  
hundreds of  
thousands of  
photons**

# Pass 2: viewing ray

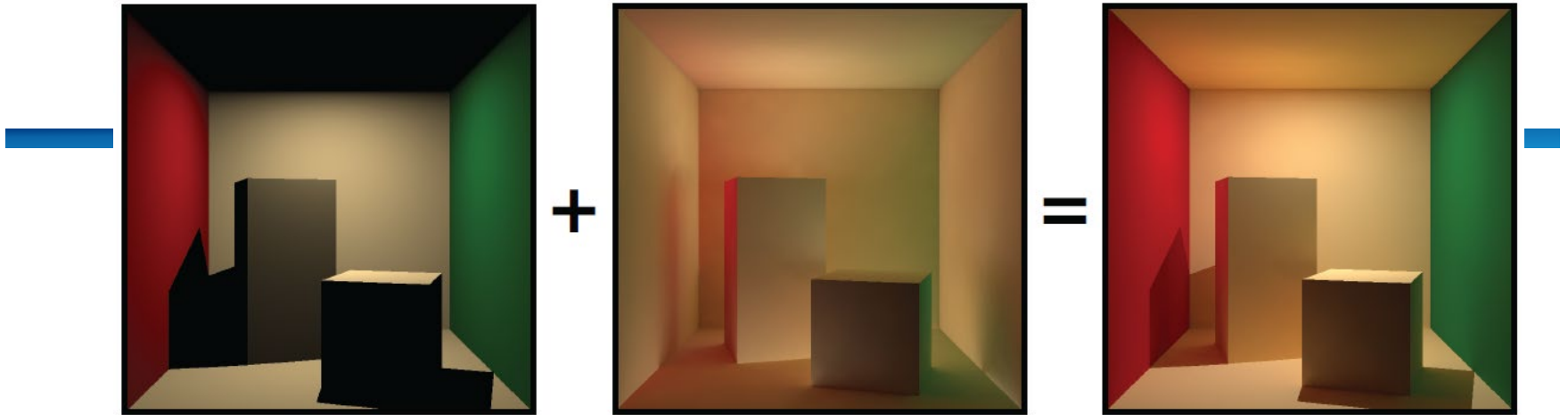
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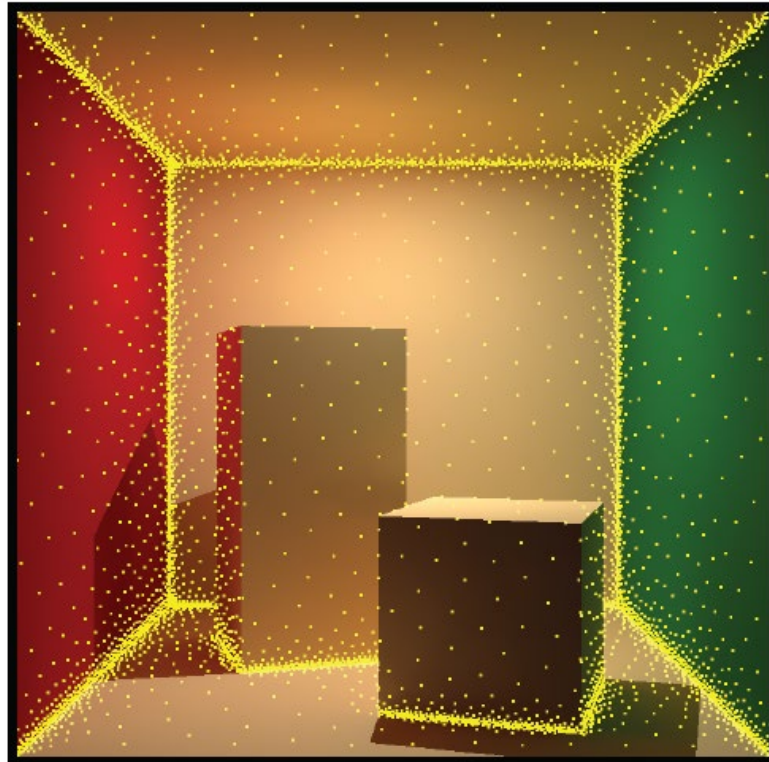
- Search for  $N$  closest photons (+check normal)
- Assume these photons hit the point we're interested in
- Compute average radiance

**Direct**

**Indirect**



**Indirect**  $\approx$



# Result

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**350K photons  
for the caustic  
map**



# Result

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**350K photons  
for the caustic  
map**

# Class Objectives were:

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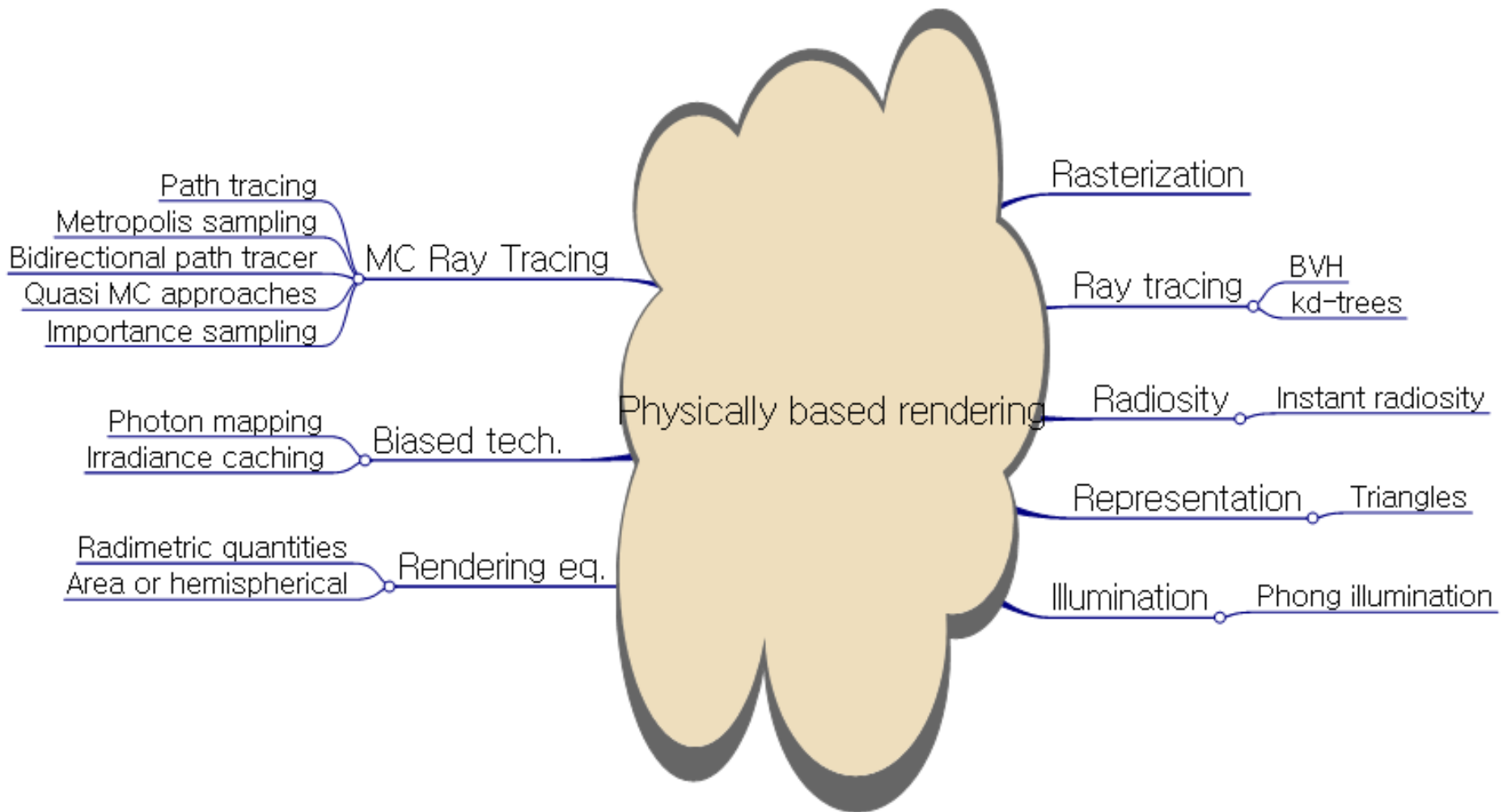
- **Understand a basic structure of Monte Carlo ray tracing**
  - **Russian roulette for its termination**
  - **Path tracing**

# Summary

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- **Two basic building blocks**
- **Radiometry**
- **Rendering equation**
- **MC integration**
- **MC ray tracing**
  - **Unbiased methods**
  - **Biased methods**

# Summary



# Next Time...

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- **Denoising techniques**

# Homework

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- **Go over the next lecture slides before the class**
- **Watch 2 SIG/CVPR/ISMAR videos and submit your summaries every Mon. class**
  - **Just one paragraph for each summary**
  - **Any top-tier conf (e.g., ICRA) is okay**

## Example:

**Title: XXX XXXX XXXX**

**Abstract: this video is about accelerating the performance of ray tracing. To achieve its goal, they design a new technique for reordering rays, since by doing so, they can improve the ray coherence and thus improve the overall performance.**

# Any Questions?

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- **Submit three times before the mid-term exam**
- **Come up with one question on what we have discussed in the class and submit:**
  - **1 for typical or already answered questions**
  - **2 for questions that have some thoughts or surprise me**