

---

---

**CS688/WST665: Web-Scale Image Retrieval**  
**Keypoint Localization**

---

---

**Sung-Eui Yoon**  
(윤성익)

**Course URL:**  
**<http://sglab.kaist.ac.kr/~sungeui/IR>**

**KAIST**



# Homework for Every Class

---

---

- **Go over the next lecture slides**
- **Come up with one question on what we have discussed today**
  - 1 for typical questions (that were answered in the class)
  - 2 for questions with thoughts or that surprised me
- **Write questions at least 4 times before the mid-term**
  - Multiple questions in one time will be counted as one time
- **Common questions are addressed at my draft**
  - Some of questions will be discussed in the class
- **If you want to know the answer of your question, ask me or TA **on person****

# Homework for Every Class

---

---

- Go over recent papers on image search
  - High quality papers: Papers published at the top-tier conf. or close it can be presented; e.g., CVPR, ICCV, ECCV, MM, SIGGRAPH
  - Recent publication : papers published since 2011
  - Find and browse two papers, and submit your summary before every beginning of the Thur. class; **submit two summaries**
  - **Online submission is possible**
- Think about possible team members
- Too late if you think them later..

# What we will learn today?

- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector
  - Hessian detector

# What we will learn today?

- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector
  - Hessian detector

# Content-Based Image Retrieval (CBIR)

---

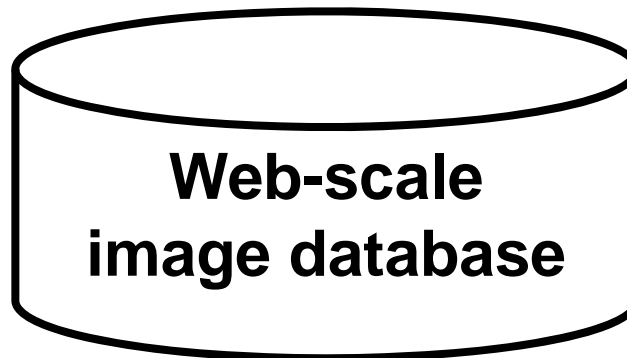
---

- Identify similar images given a user-specified image or other types of inputs

Extract image descriptors (e.g., SIFT)



Input



Output

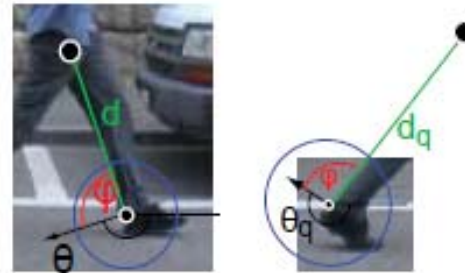
# Motivation

- Global representations have major limitations
- Instead, describe and match only local regions
- Increased robustness to

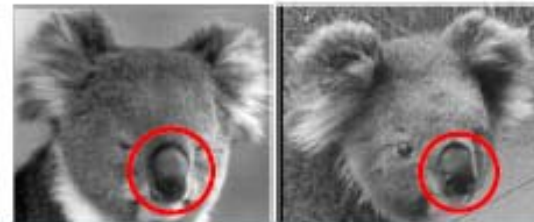
- Occlusions



- Articulation



- Intra-category variations



# Application: Image Matching



by [Diva Sian](#)



by [swashford](#)

Slide credit: Steve Seitz



# Harder Case



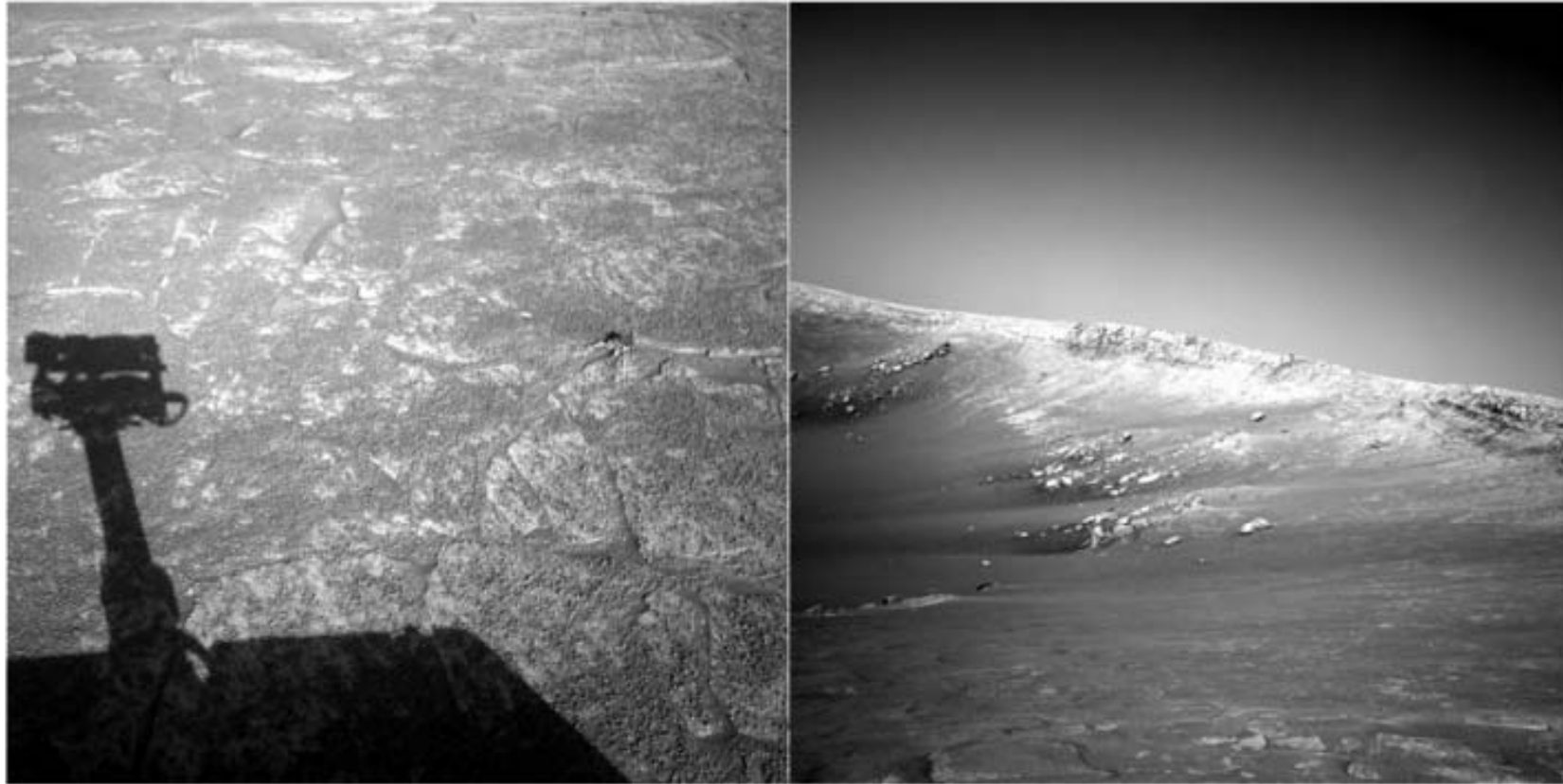
by [Diva Sian](#)



by [scgbt](#)

Slide credit: Steve Seitz

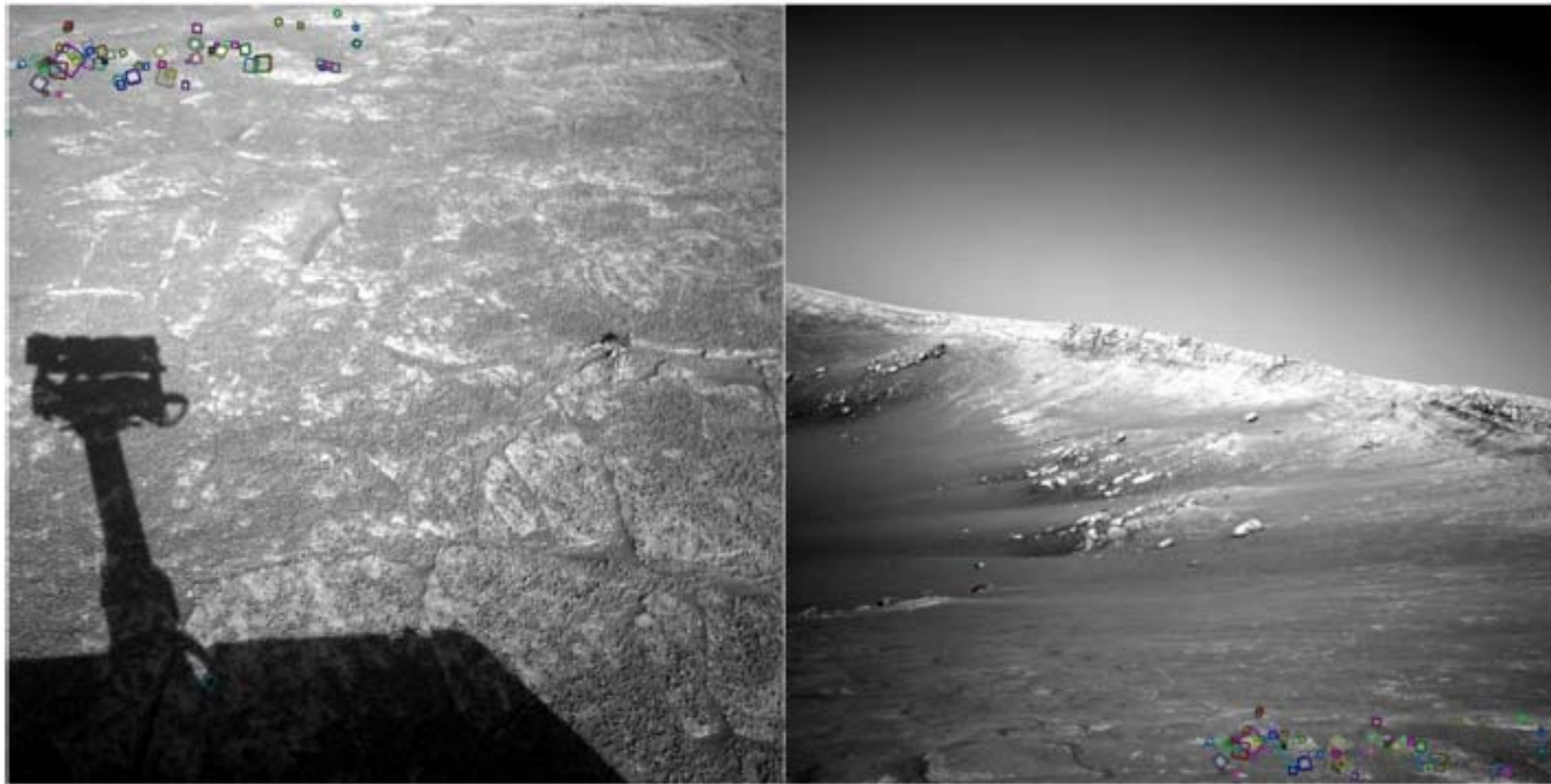
# Harder Still?



NASA Mars Rover images

Slide credit: Steve Seitz

# Answer Below (Look for tiny colored squares)



NASA Mars Rover images with SIFT feature matches  
(Figure by Noah Snavely)

Slide credit: Steve Seitz

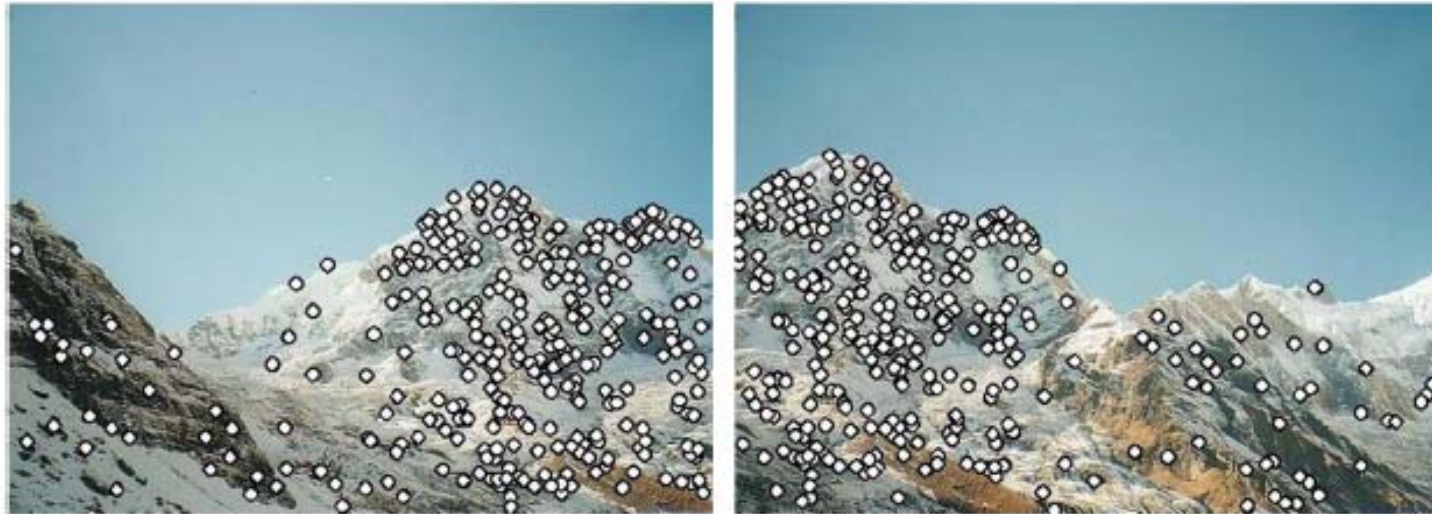


# Application: Image Stitching



Slide credit: Darya Frolova, Denis Simakov

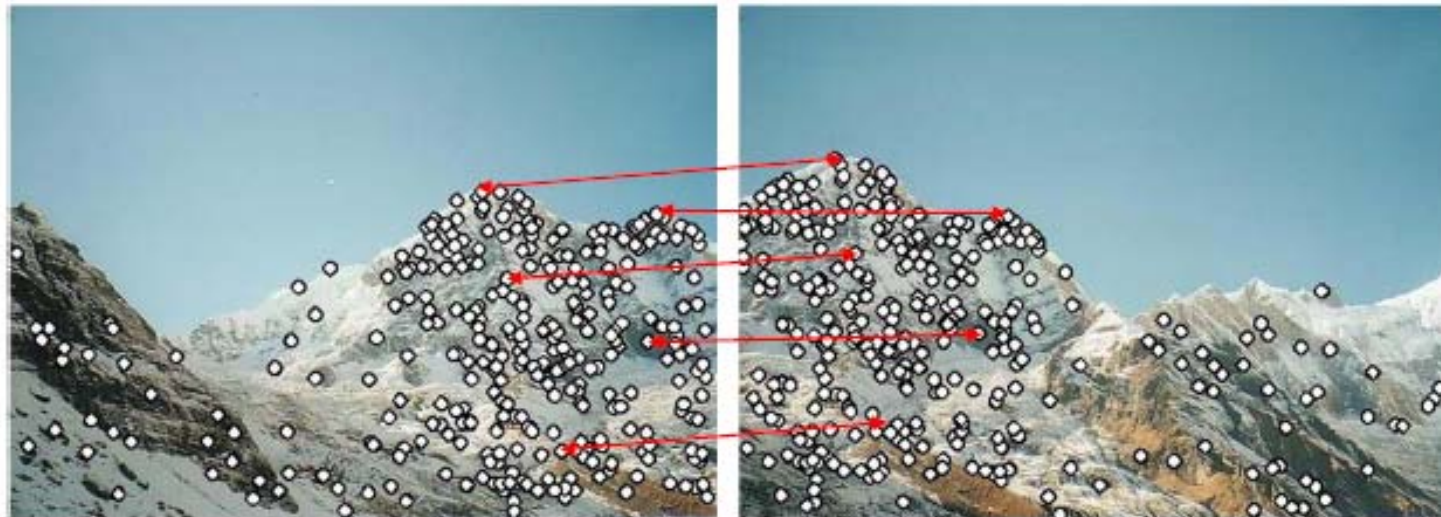
# Application: Image Stitching



- Procedure:
  - Detect feature points in both images

Slide credit: Darya Frolova, Denis Simakov

# Application: Image Stitching



- Procedure:
  - Detect feature points in both images
  - Find corresponding pairs

Slide credit: Darya Frolova, Denis Simakov

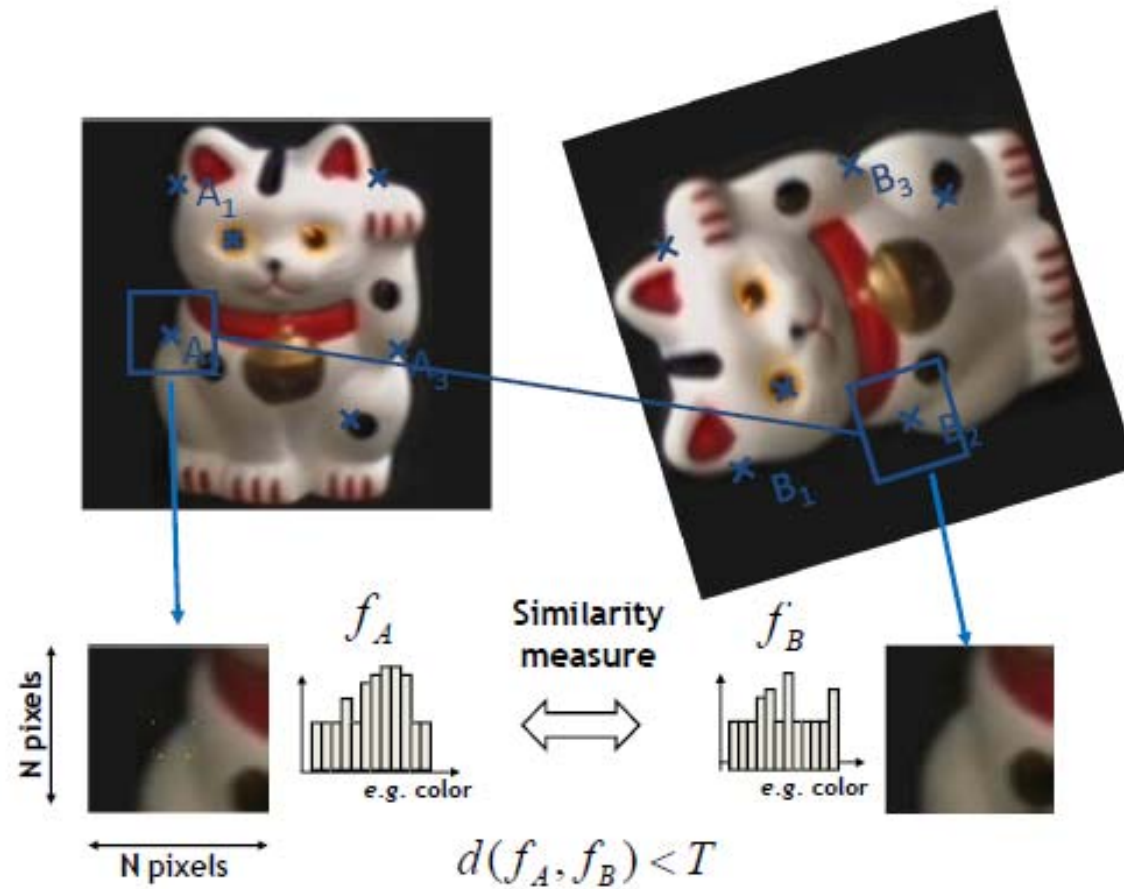


# Application: Image Stitching



- Procedure:
  - Detect feature points in both images
  - Find corresponding pairs
  - Use these pairs to align the images

# General Approach



1. Find a set of distinctive key-points
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

Slide credit: Bastian Leibe



# Common Requirements

- Problem 1:
  - Detect the same point *independently* in both images



No chance to match!

**This lecture**

**We need a repeatable detector!**

# Common Requirements

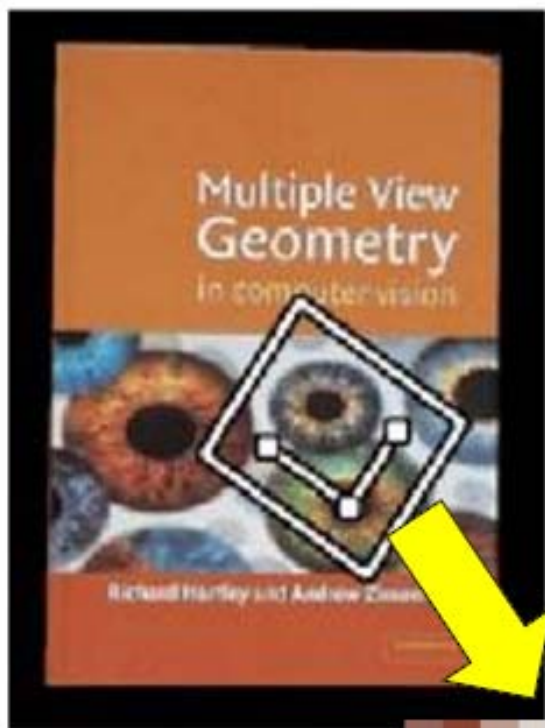
- Problem 1:
  - Detect the same point *independently* in both images
- Problem 2:
  - For each point correctly recognize the corresponding one



**Next lecture**

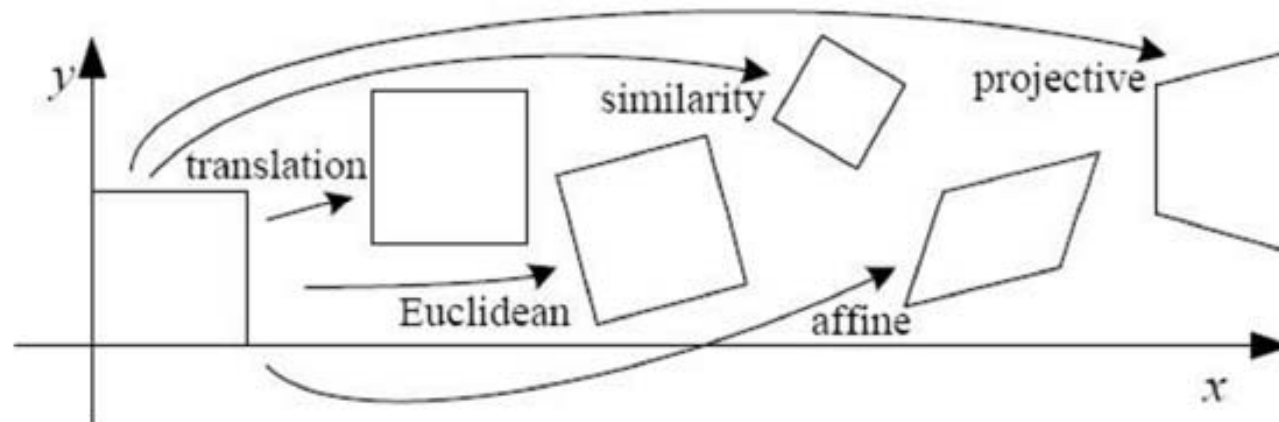
**We need a reliable and distinctive descriptor!**

# Invariance: Geometric Transformations



Slide credit: Steve Seitz

# Levels of Geometric Invariance



Slide credit: Bastian Leibe

# Invariance: Photometric Transformations



- Often modeled as a linear transformation:
  - Scaling + Offset

Slide credit: Tinne Tuytelaars



# Requirements

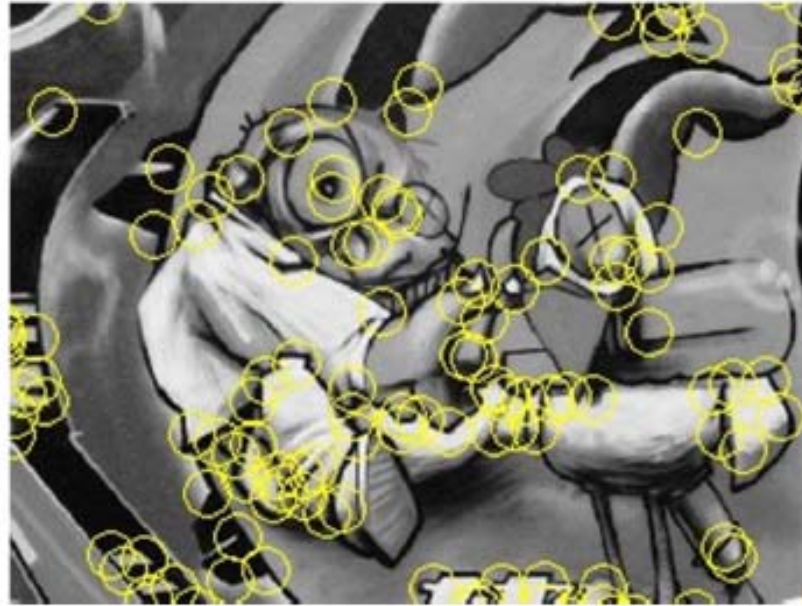
- Region extraction needs to be **repeatable** and **accurate**
  - **Invariant** to translation, rotation, scale changes
  - **Robust** or **covariant** to out-of-plane ( $\approx$ affine) transformations
  - **Robust** to lighting variations, noise, blur, quantization
- **Locality**: Features are local, therefore robust to occlusion and clutter.
- **Quantity**: We need a sufficient number of regions to cover the object.
- **Distinctiveness** : The regions should contain “interesting” structure.
- **Efficiency**: Close to real-time performance.

Slide credit: Bastian Leibe

# Many Existing Detectors Available

- Hessian & Harris [Beaudet '78], [Harris '88]
  - Laplacian, DoG [Lindeberg '98], [Lowe '99]
  - Harris-/Hessian-Laplace [Mikolajczyk & Schmid '01]
  - Harris-/Hessian-Affine [Mikolajczyk & Schmid '04]
  - EBR and IBR [Tuytelaars & Van Gool '04]
  - MSER [Matas '02]
  - Salient Regions [Kadir & Brady '01]
  - Others...
- *Those detectors have become a basic building block for many recent applications in Computer Vision.*

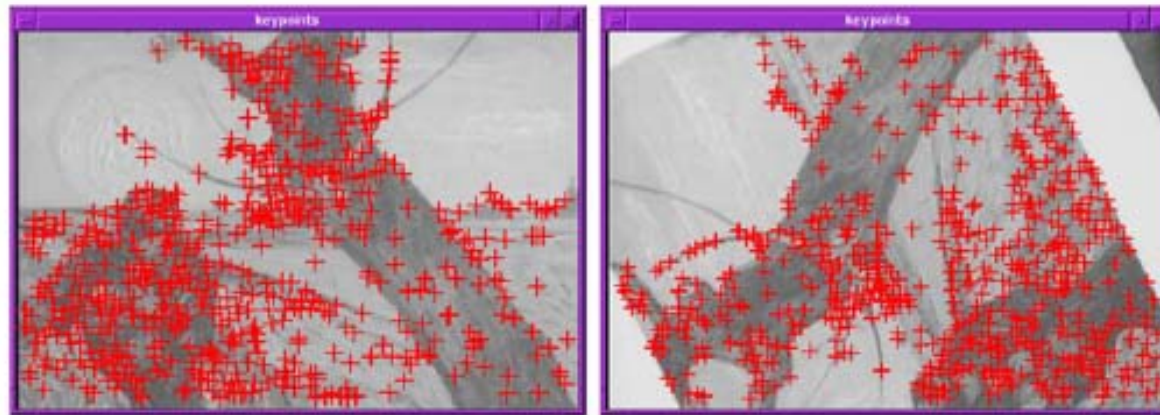
# Keypoint Localization



- Goals:
    - Repeatable detection
    - Precise localization
    - Interesting content
- ⇒ *Look for two-dimensional signal changes*



# Finding Corners

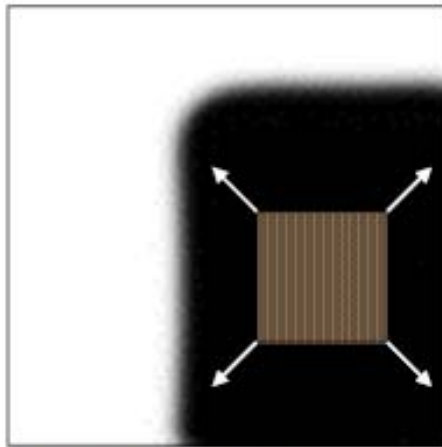


- Key property:
  - In the region around a corner, image gradient has two or more dominant directions
- Corners are *repeatable* and *distinctive*

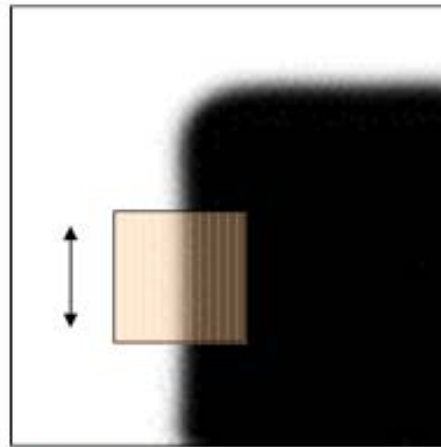
C.Harris and M.Stephens. "[A Combined Corner and Edge Detector.](#)"  
*Proceedings of the 4th Alvey Vision Conference, 1988.*

# Corners as Distinctive Interest Points

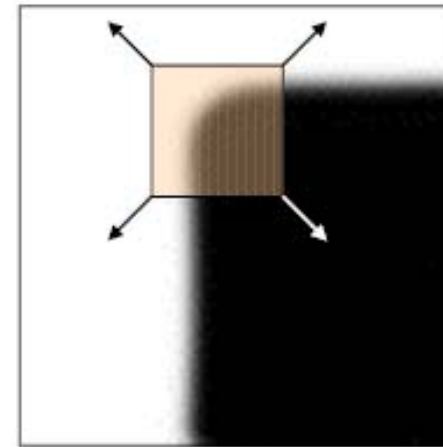
- Design criteria
  - We should easily recognize the point by looking through a small window (*locality*)
  - Shifting the window in *any direction* should give a large change in intensity (*good localization*)



“flat” region:  
no change in all  
directions



“edge”:  
no change along  
the edge direction



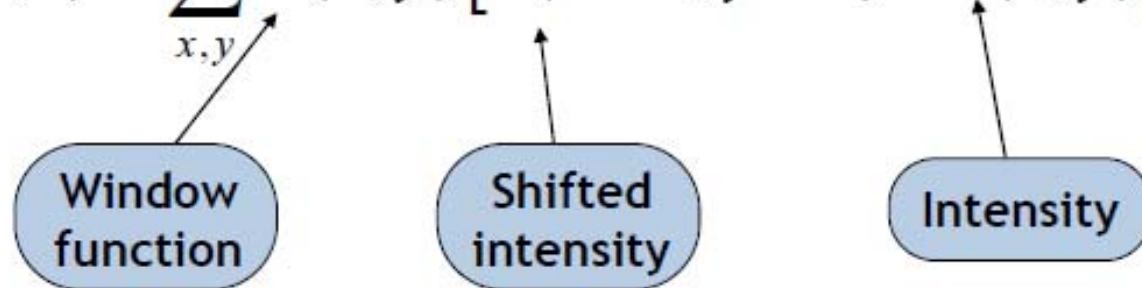
“corner”:  
significant change  
in all directions

Slide credit: Alyosha Efros

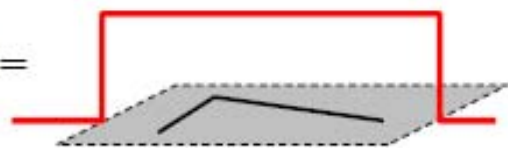
# Harris Detector Formulation

- Change of intensity for the shift  $[u,v]$ :

$$E(u, v) = \sum_{x,y} w(x, y) [I(x+u, y+v) - I(x, y)]^2$$

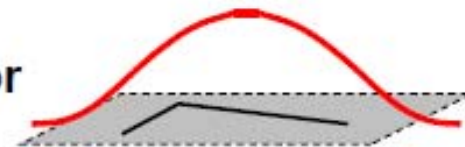


Window function  $w(x,y) =$



1 in window, 0 outside

or



Gaussian

Slide credit: Rick Szeliski

# Harris Detector Formulation

- This measure of change can be approximated by:

$$E(u, v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

where  $M$  is a  $2 \times 2$  matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Sum over image region – the area we are checking for corner

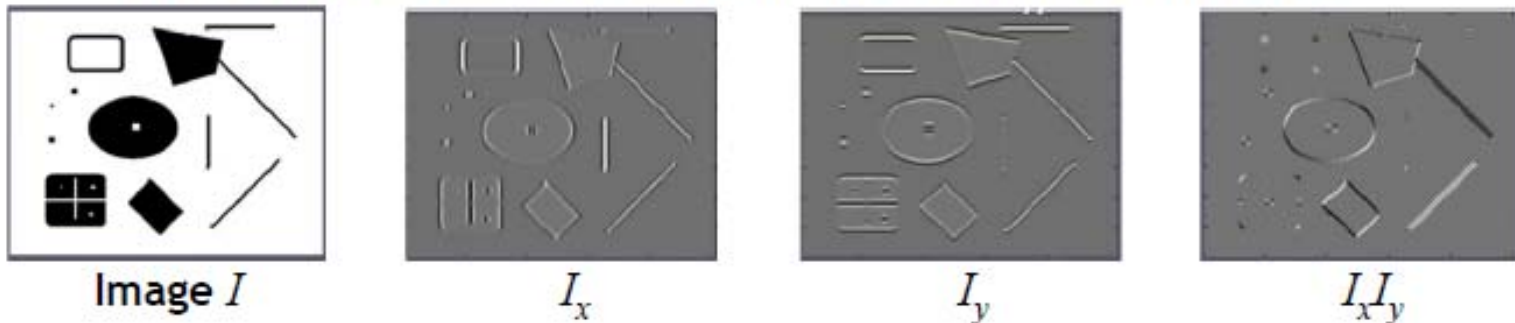
Gradient with respect to  $x$ , times gradient with respect to  $y$

(Second moment matrix)

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y]$$



# Harris Detector Formulation



where  $M$  is a  $2 \times 2$  matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

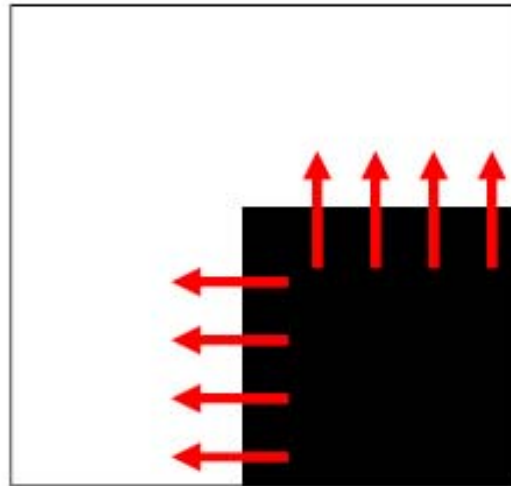
Sum over image region – the area we are checking for corner

Gradient with respect to  $x$ , times gradient with respect to  $y$

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y]$$

# What Does This Matrix Reveal?

- First, let's consider an axis-aligned corner:

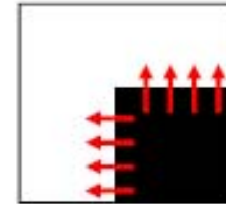


Slide credit: Kristen Grauman

# What Does This Matrix Reveal?

- First, let's consider an axis-aligned corner:

$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$



- This means:
  - Dominant gradient directions align with  $x$  or  $y$  axis
  - If either  $\lambda$  is close to 0, then this is not a corner, so look for locations where both are large.
- What if we have a corner that is not aligned with the image axes?

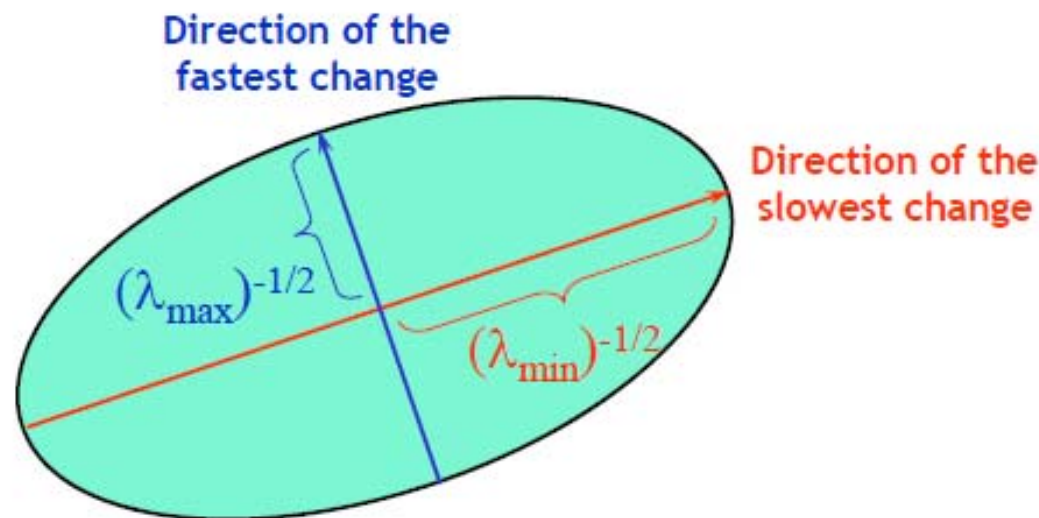
Slide credit: David Jacobs

# General Case

- Since  $M$  is symmetric, we have 
$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

(Eigenvalue decomposition)

- We can visualize  $M$  as an ellipse with axis lengths determined by the eigenvalues and orientation determined by  $R$



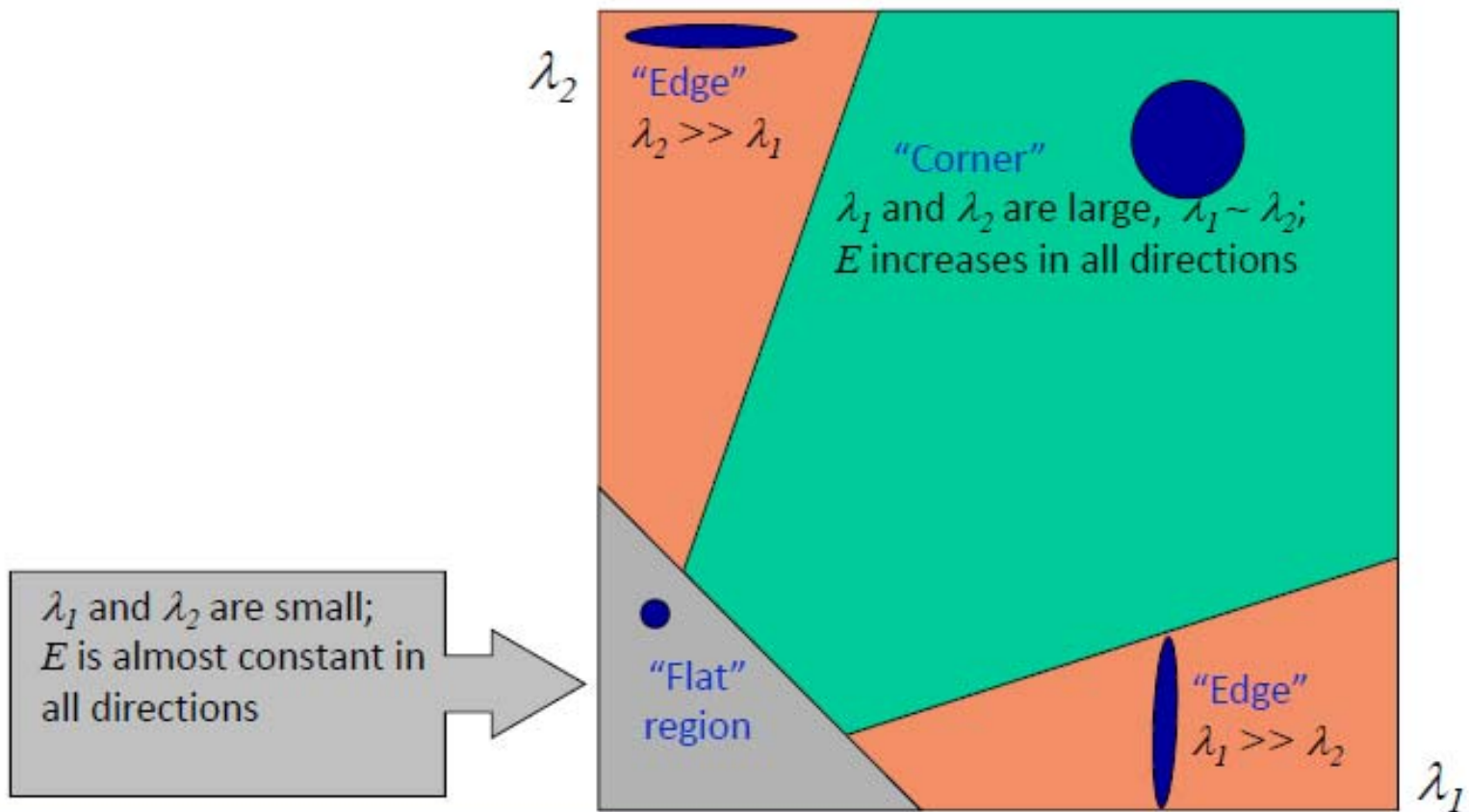
adapted from Darya Frolova, Denis Simakov



# Interpreting the Eigenvalues

- Classification of image points using eigenvalues of  $M$ :

Slide credit: Kristen Grauman



# Corner Response Function

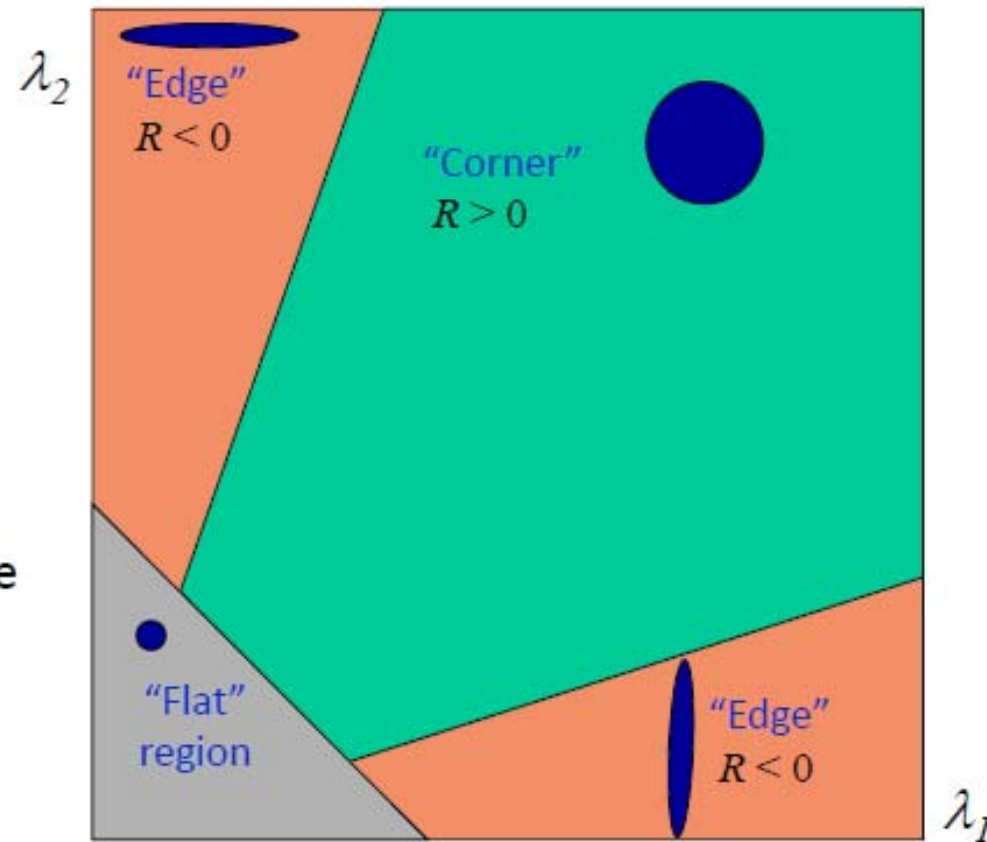
$$R = \det(M) - \alpha \text{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

$$M = \begin{bmatrix} A & B \\ B & C \end{bmatrix},$$

$$\text{trace}(M) = A + C,$$

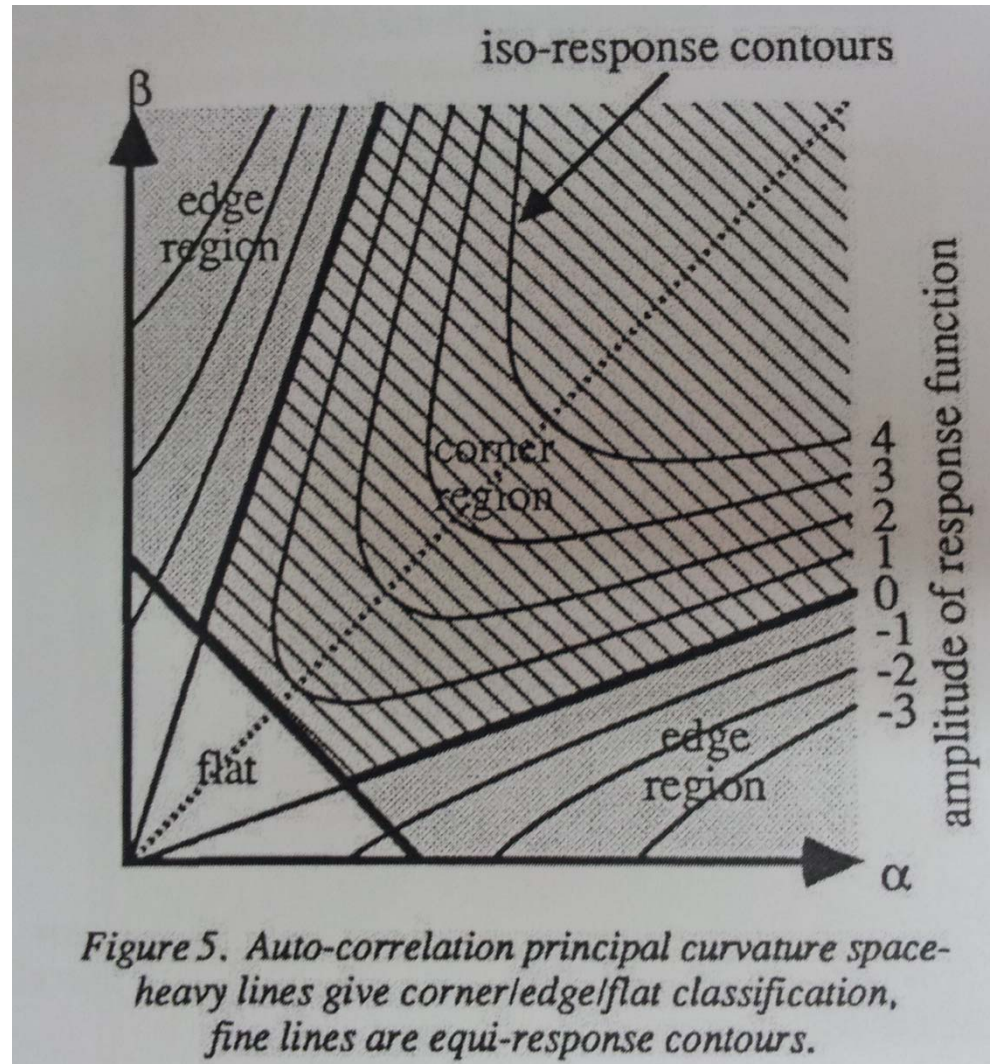
$$\det(M) = AC - B^2$$

- Fast approximation
  - Avoid computing the eigenvalues
  - $\alpha$ : constant (0.04 to 0.06)



Slide credit: Kristen Grauman

# Corner Response Function



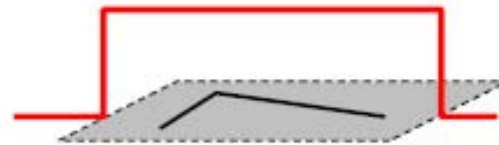
# Window Function $w(x,y)$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Option 1: uniform window
  - Sum over square window

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Problem: not rotation invariant



1 in window, 0 outside



# Window Function $w(x,y)$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Option 1: uniform window
  - Sum over square window

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Problem: not rotation invariant

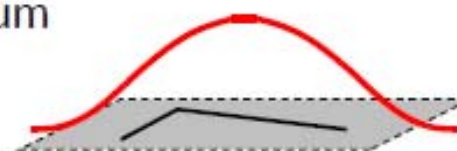


1 in window, 0 outside

- Option 2: Smooth with Gaussian
  - Gaussian already performs weighted sum

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Result is rotation invariant



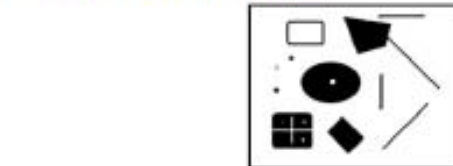
Gaussian

# Summary: Harris Detector [Harris88]

- Compute second moment matrix (autocorrelation matrix)

$$M(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

1. Image derivatives



2. Square of derivatives



3. Gaussian filter  $g(\sigma_I)$



4. Cornerness function - two strong eigenvalues

$$\begin{aligned} R &= \det[M(\sigma_I, \sigma_D)] - \alpha[\text{trace}(M(\sigma_I, \sigma_D))]^2 \\ &= g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2 \end{aligned}$$

5. Perform non-maximum suppression



# Harris Detector: Workflow

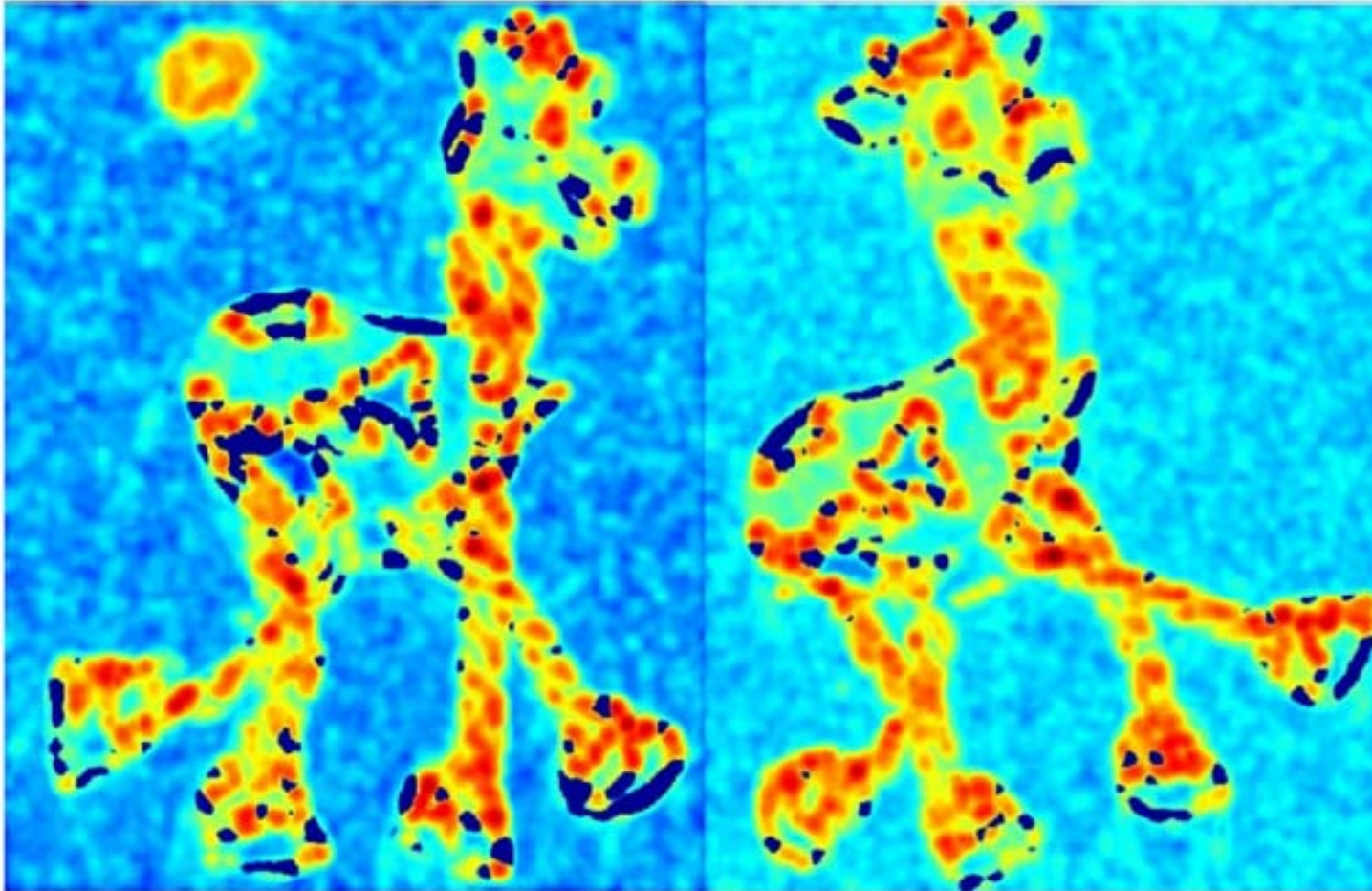


Slide adapted from Darya Frolova, Denis Simakov



# Harris Detector: Workflow

- computer corner responses  $R$

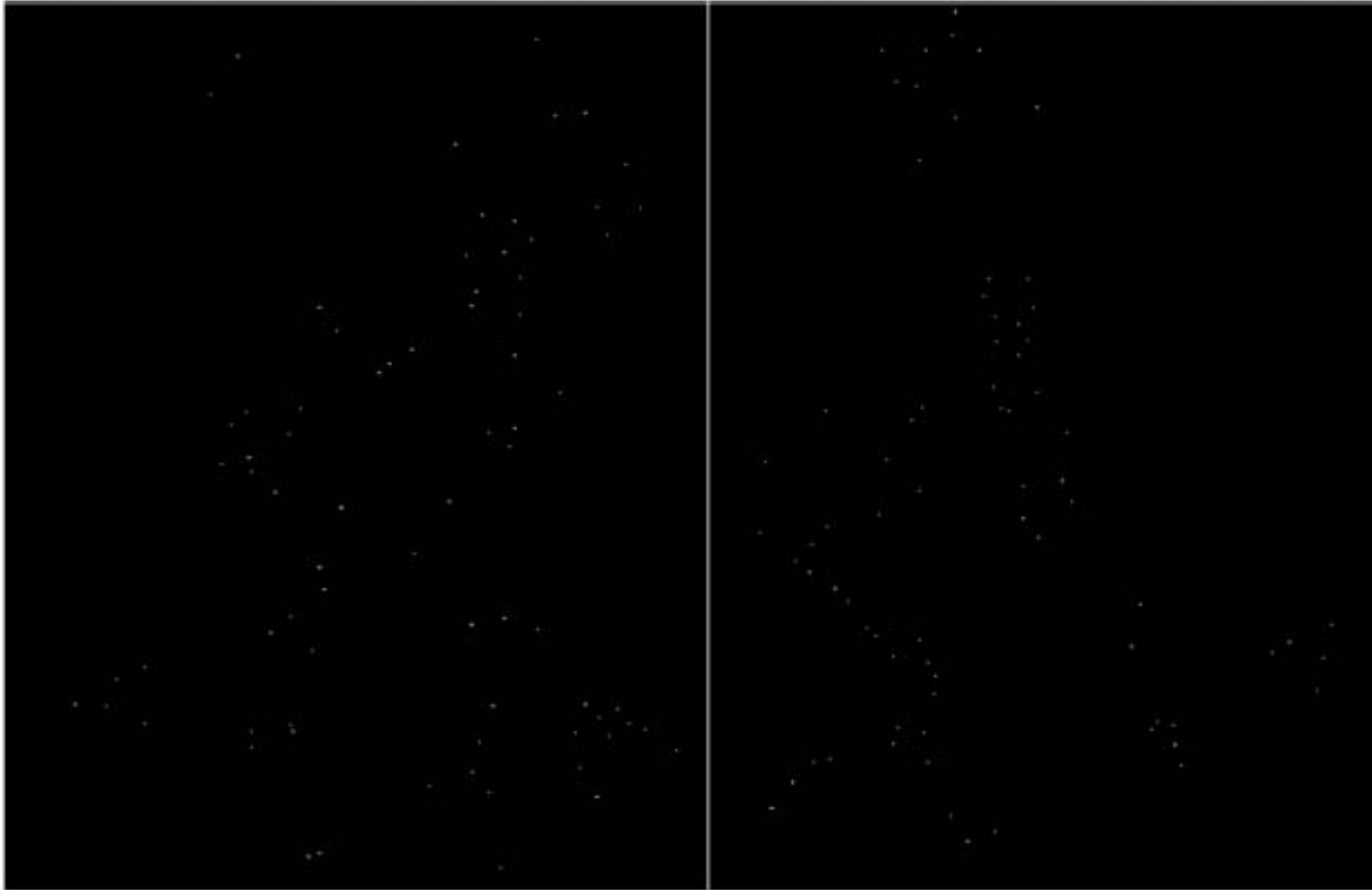


Slide adapted from Darya Frolova, Denis Simakov



# Harris Detector: Workflow

- Take only the local maxima of  $R$ , where  $R > \text{threshold}$



Slide adapted from Darya Frolova, Denis Simakov

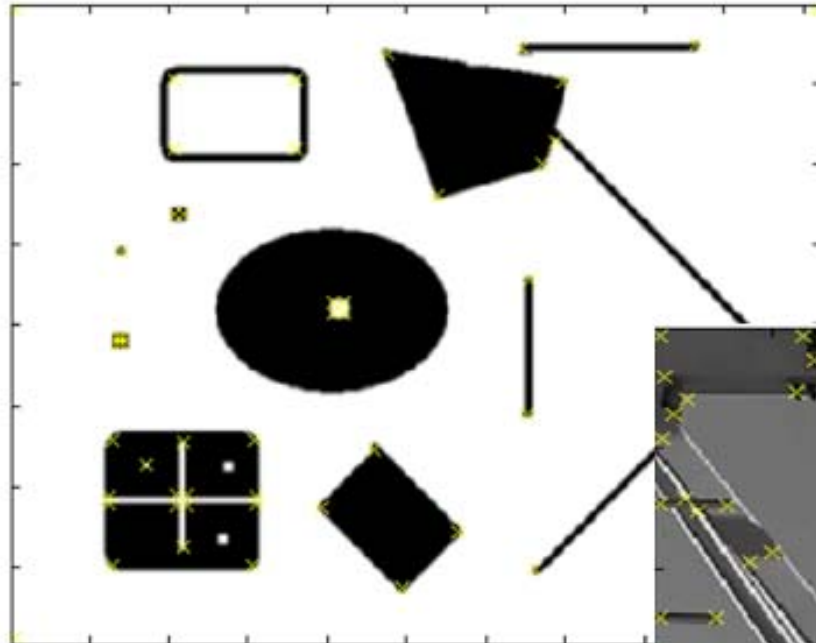
# Harris Detector: Workflow

## - Resulting Harris points

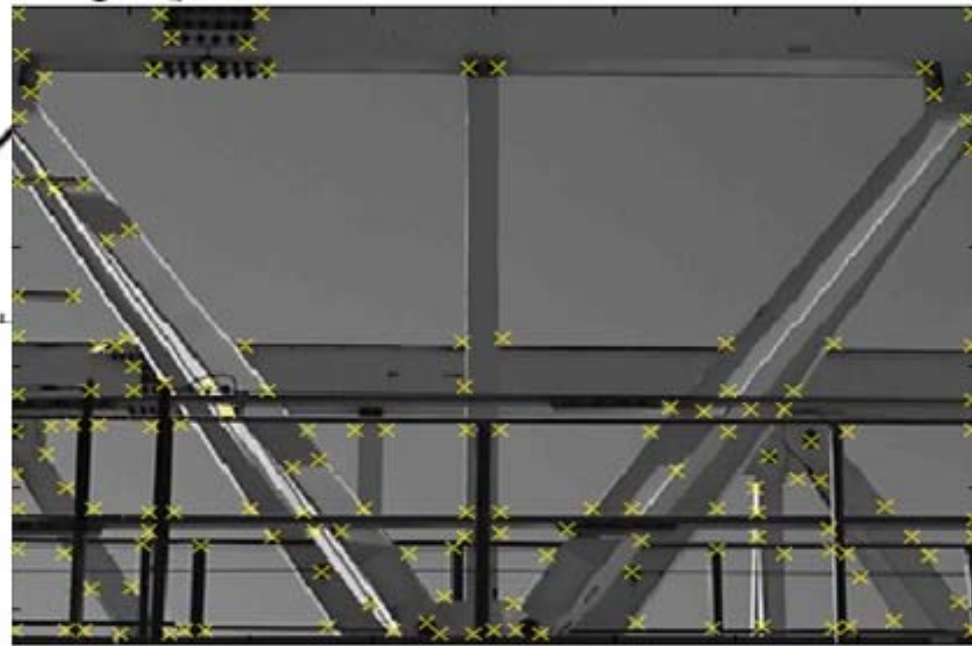


Slide adapted from Darya Frolova, Denis Simakov

# Harris Detector – Responses [Harris88]



*Effect:* A very precise corner detector.



Slide credit: Krystian Mikolajczyk



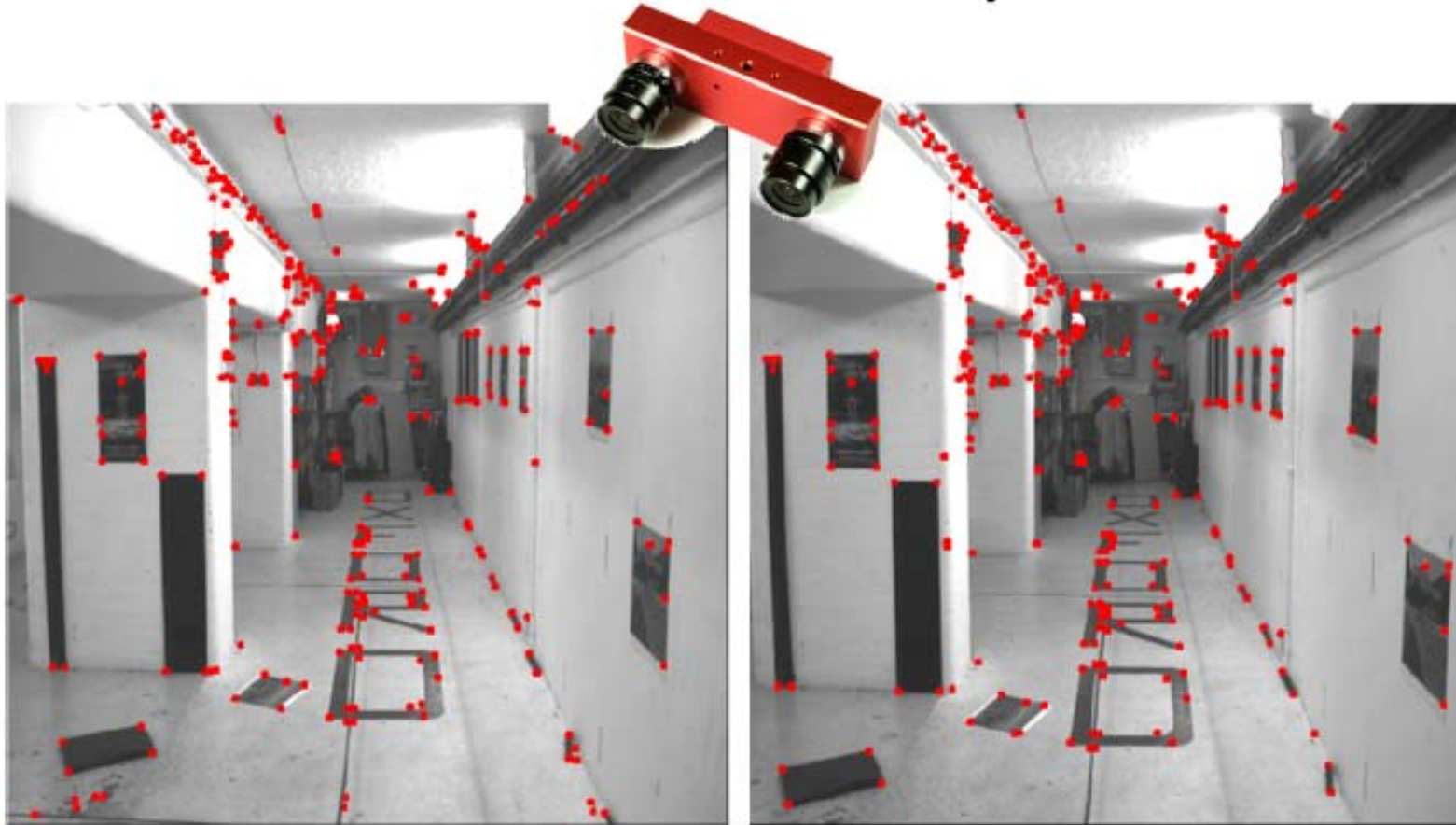
# Harris Detector – Responses [Harris88]



Slide credit: Krystian Mikolajczyk



# Harris Detector – Responses [Harris88]

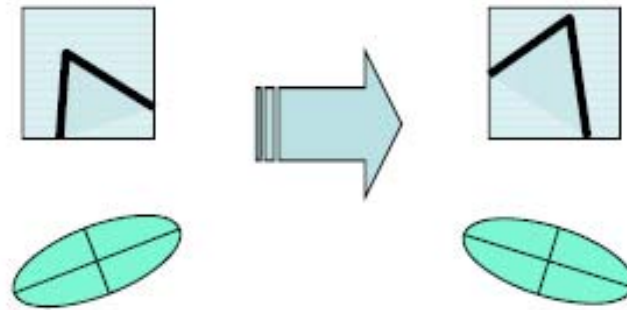


- Results are well suited for finding stereo correspondences

Slide credit: Kristen Grauman

# Harris Detector: Properties

- Rotation invariance?



Ellipse rotates but its shape (i.e. eigenvalues) remains the same

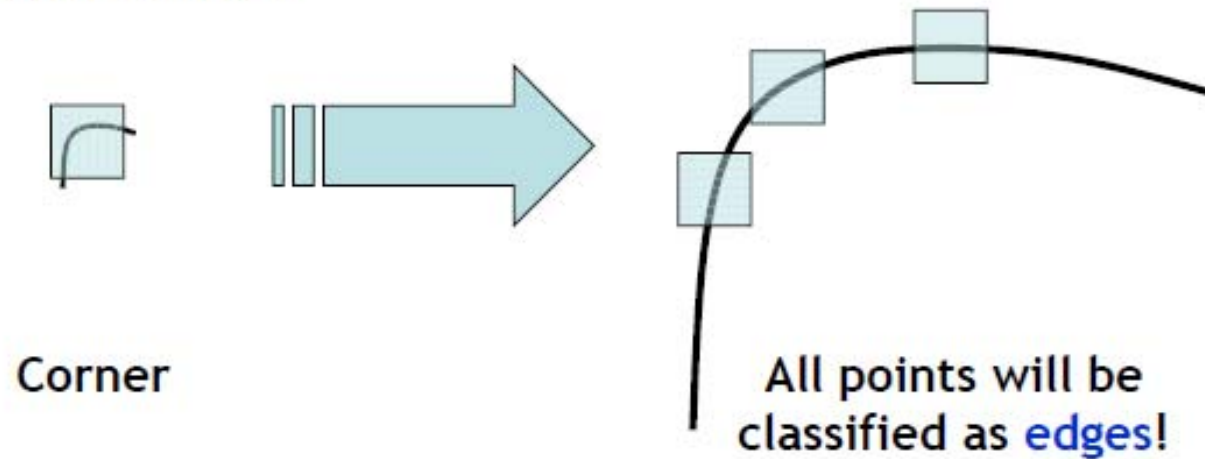
***Corner response  $R$  is invariant to image rotation***

# Harris Detector: Properties

- Rotation invariance
- Scale invariance?

# Harris Detector: Properties

- Rotation invariance
- Scale invariance?



**Not invariant to image scale!**

Slide credit: Kristen Grauman



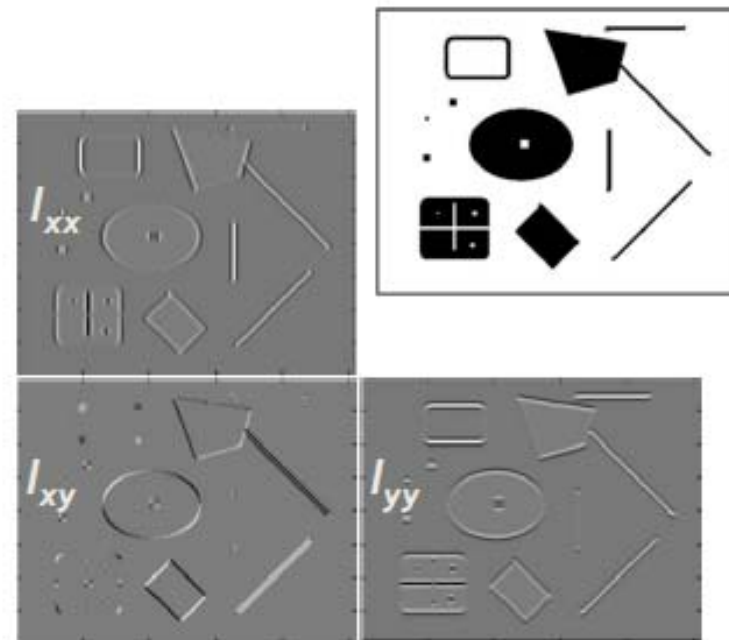
# Hessian Detector [Beaudet78]

- Hessian determinant

$$\text{Hessian}(I) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$

Note: these are 2<sup>nd</sup> derivatives!

*Intuition:* Search for strong derivatives in two orthogonal directions



Slide credit: Krystian Mikołajczyk

# Hessian Detector [Beaudet78]

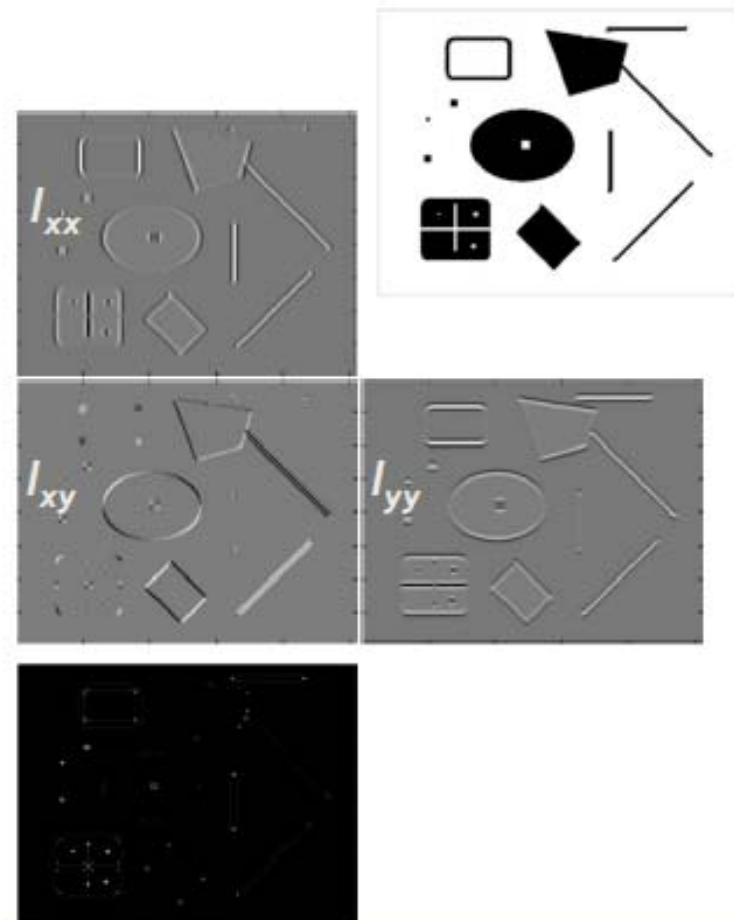
- Hessian determinant

$$\text{Hessian}(I) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$

$$\det(\text{Hessian}(I)) = I_{xx}I_{yy} - I_{xy}^2$$

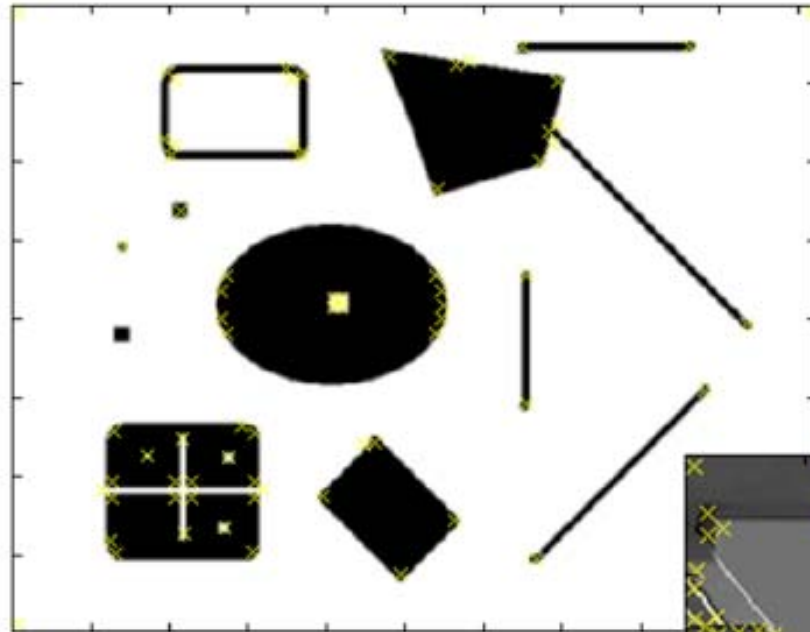
In Matlab:

$$I_{xx} \cdot I_{yy} - (I_{xy})^2$$

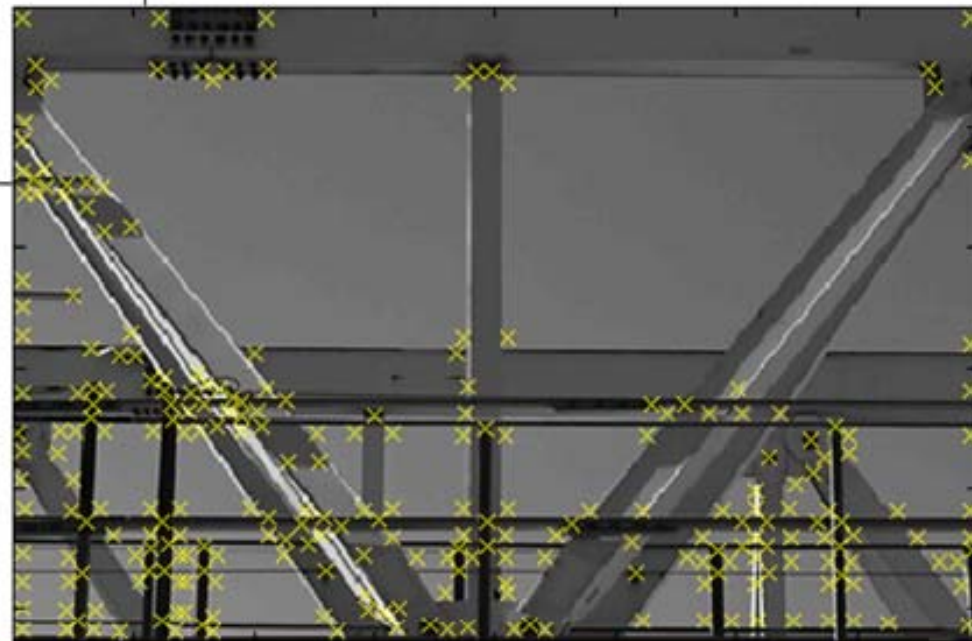


Slide credit: Krystian Mikołajczyk

# Hessian Detector – Responses [Beaudet78]



*Effect:* Responses mainly on corners and strongly textured areas.



Slide credit: Krystian Mikołajczyk

# Hessian Detector – Responses [Beaudet78]



Slide credit: Krystian Mikołajczyk



# Next Time..

---

---

- **Scale invariant region selection**

# Homework for Every Class

---

---

- Go over the next lecture slides
- Come up with one question on what we have discussed today
- Go over recent papers on image search, and submit their summary

# Homework for Every Class

---

---

- **Go over recent papers on image search**
  - **High quality papers: Papers published at the top-tier conf. or close it can be presented; e.g., CVPR, ICCV, ECCV, MM, SIGGRAPH**
  - **Recent publication : papers published since 2011**
  - **Find and browse two papers, and submit your summary before every beginning of the Thur. class**
- **Think about possible team members**
- **Too late if you think them later..**