CS686: Configuration Space I

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(윤성의)

Course URL:
http://sgvr.kaist.ac.kr/~sungeui/MPA

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Announcements

- **Make a project team of 2 persons for your project**
  - **Each student needs a clear role**
  - **Declare team members at KLMS by Sep-26; you don't need to define the topic by then**
- **Each student**
  - **Present two papers related to the project; 15 min for each talk**
  - **Declare your papers at KLMS by Oct-10**
- **Each team**
  - **Give a mid-term presentation for the project**
  - **Give the final project presentation**

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Tentative schedule

- Oct. 23: no class (reserved)
- Oct. 25: Students Presentation I (2 talks per each class)
- Oct. 30/Nov-1:
- Nov. 6,
- Nov 8, 13: Mid-term project presentation
- Nov. 15 : Students Presentation II
- Nov. 20, 22
- Nov. 27
- Nov. 29: no class (no class due to undergraduate interview)
- Dec. 4/6: Final project presentation
- Dec. 11, 13 Reserved (final exam week; no exam for us)

Deadlines

- **Declare project team members**
  - **By 9/26 at KLMS**
  - **Confirm schedules of paper talks and project talks at 9/27**

- **Declare two papers for student presentations**
  - **First come, first served**
  - **Paper title, conf. name, publication year**
  - **by 10/10 at KLMS**
  - **Discuss them at the class of 10/11**
  - **Choose papers from 2019 ~ now, published on top-tier conf./journals**

Class Objectives (Ch. 3)

- Configuration space
  - Definitions and examples
  - Obstacles
  - Paths
  - Metrics

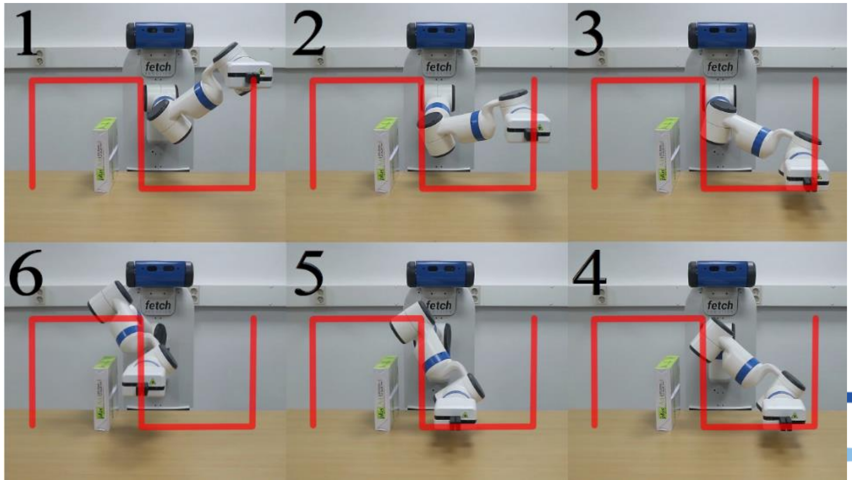
- Last time:
  - Classic motion planning approaches including roadmap, cell decomposition and potential field

Questions

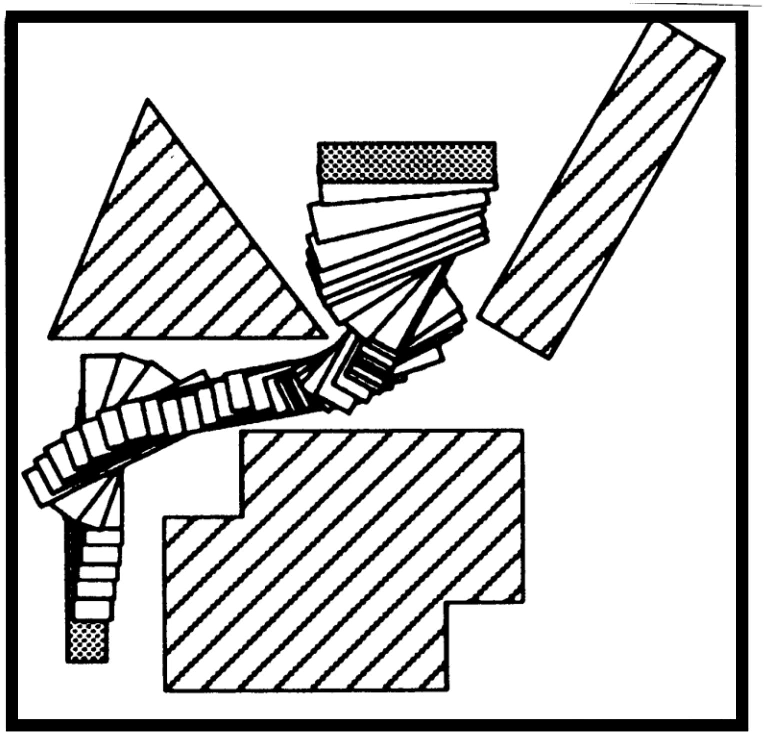
- **Are all path planning problems solved by graph navigation problems?**
  - **Trajectory optimization is also useful**

  TORM: Fast and Accurate Trajectory Optimization of Redundant Manipulator given an End-Effector Path, by Mincheul Kang, Heechan Shin, Donghyuk Kim, and Sung-Eui Yoon

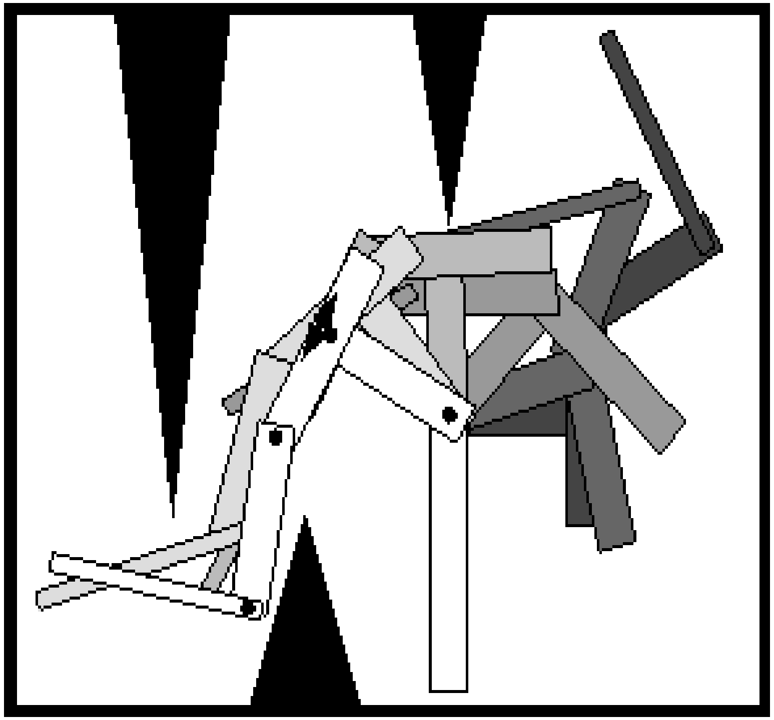
  http://sglab.kaist.ac.kr/TORM/



What is a Path?



A box robot

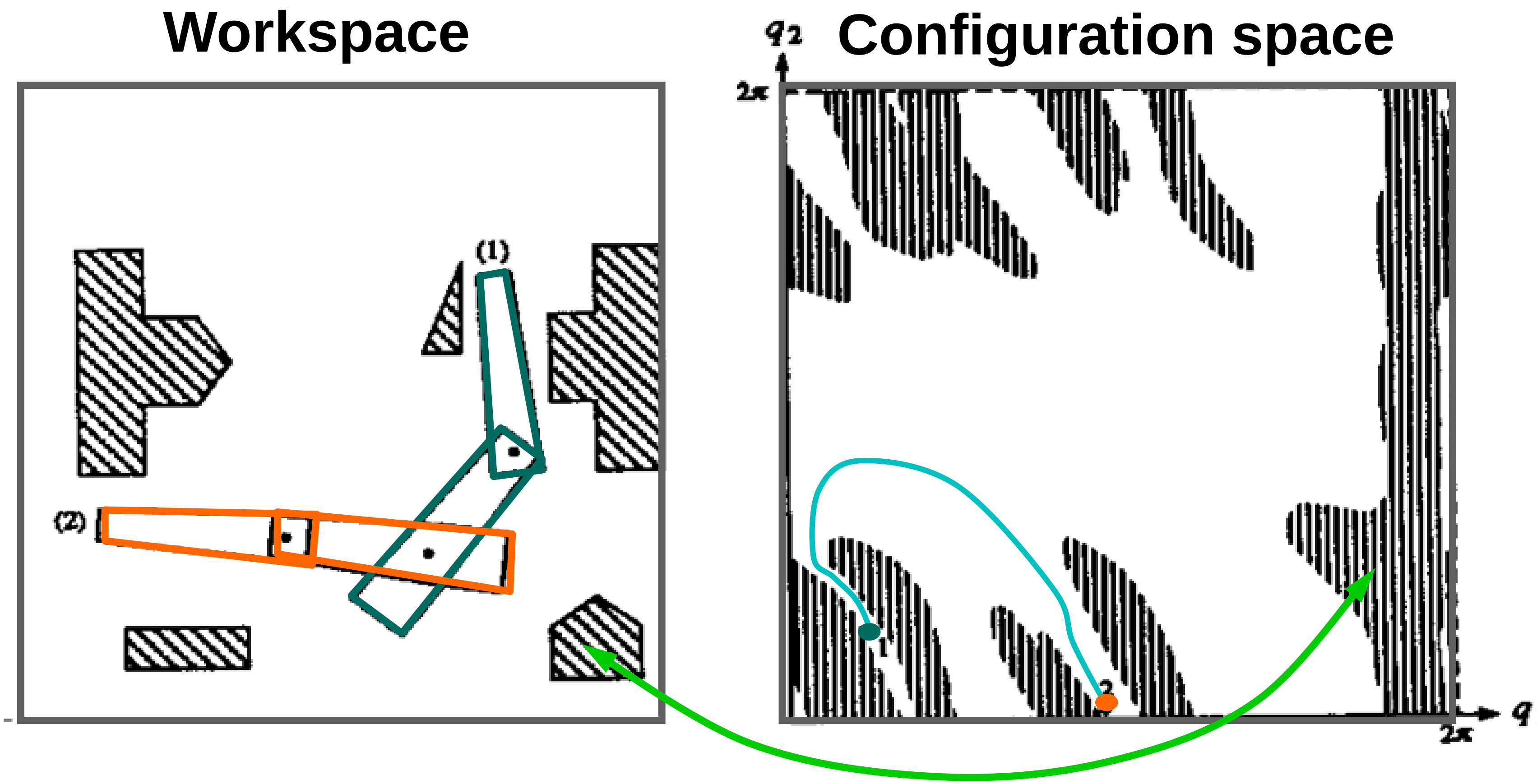


Linked robot

Rough Idea of C-Space

- **Represent degrees-of-freedom (DoFs) of rigid robots, articulated robots, *etc.* into points**
- **Apply algorithms in that space, in addition to the workspace**

Mapping from the Workspace to the Configuration Space



Configuration Space

- **Definitions and examples**
- **Obstacles**
- **Paths**
- **Metrics**

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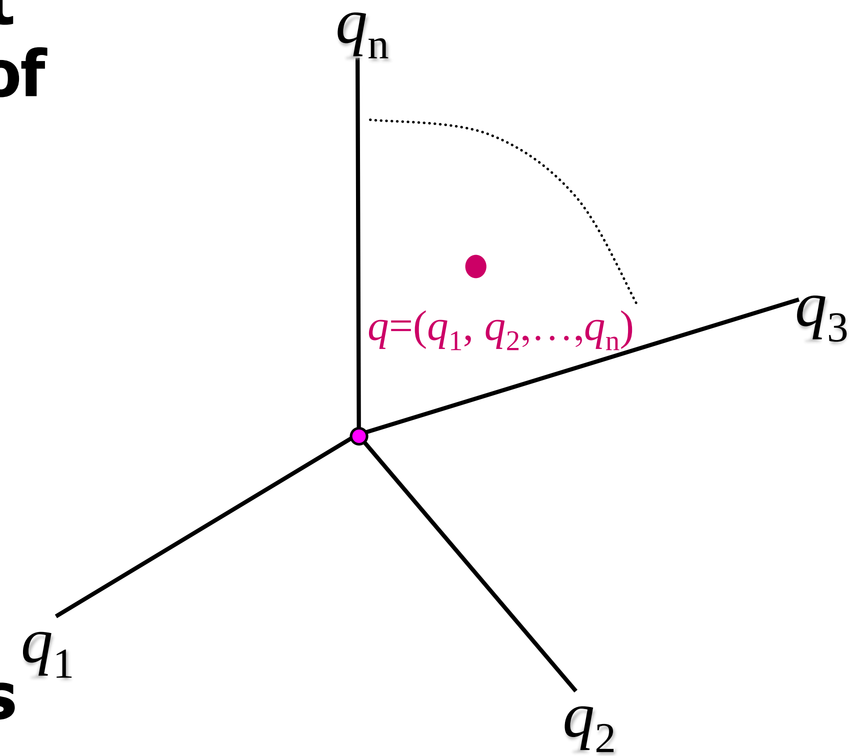
Configuration Space (C-space)

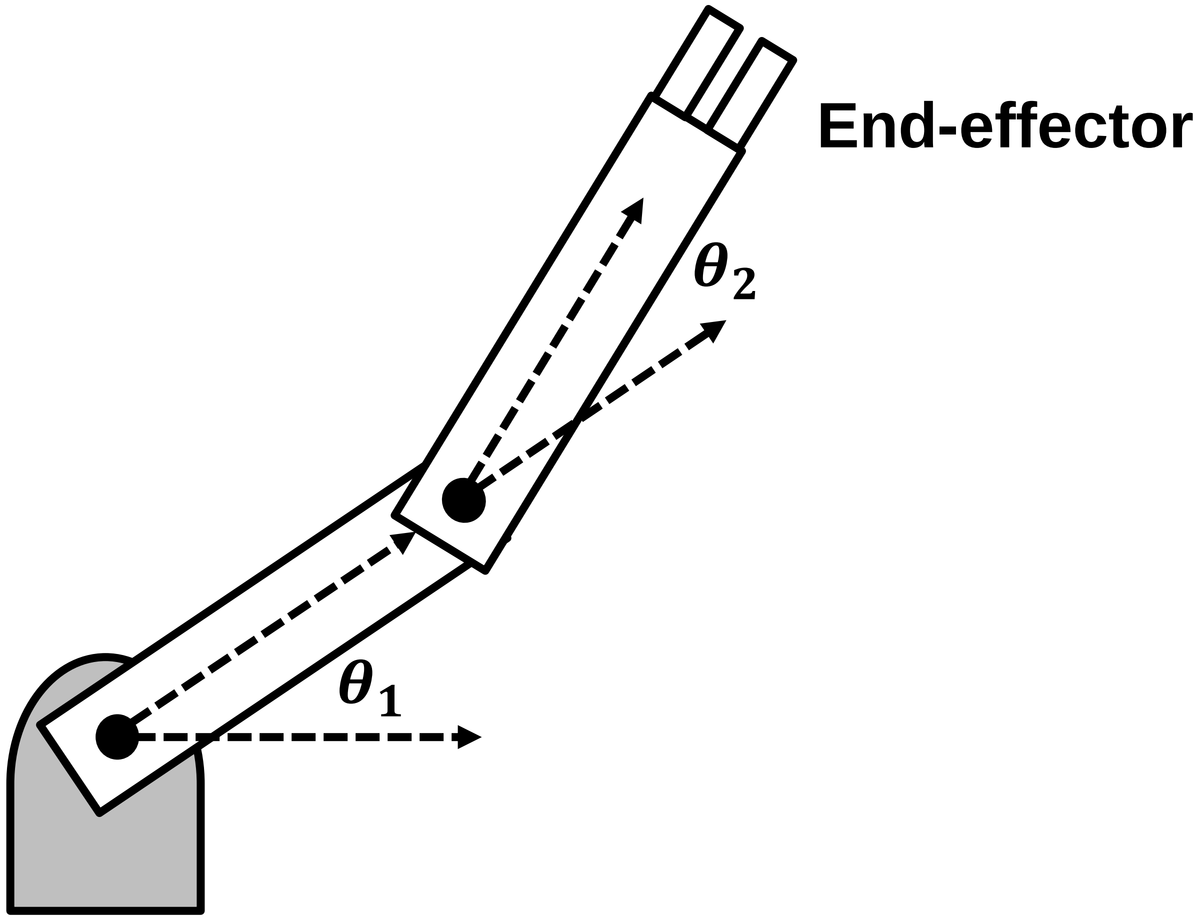
- **The configuration of a robot is a complete specification of the position of every point on the robot**
  - **Usually a configuration is expressed as a vector of position & orientation parameters:** $q = (q_1, q_2,\ldots,q_n)$



- **The configuration space** C **is the set of all possible configurations**
  - **A configuration is a point in** C

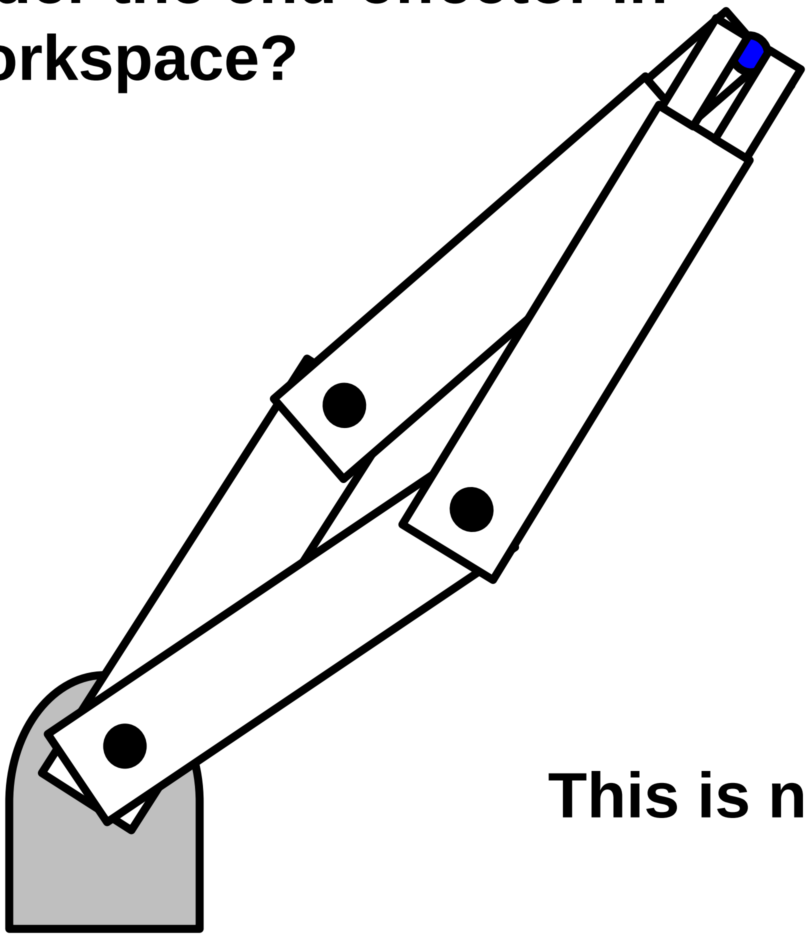
C-space formalism: Lozano-Perez '79

Examples of Configuration Spaces



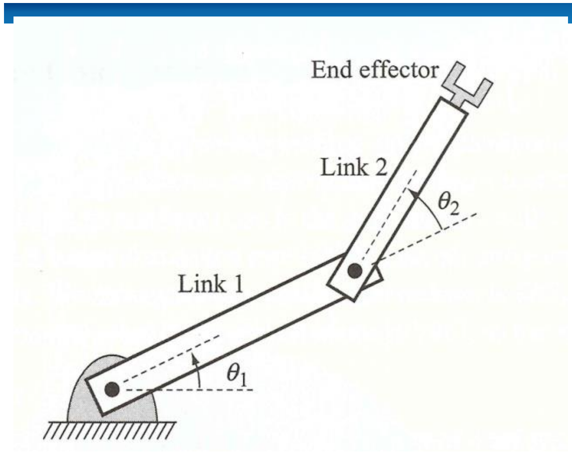
Examples of Configuration Spaces

Consider the end-effector in the workspace?

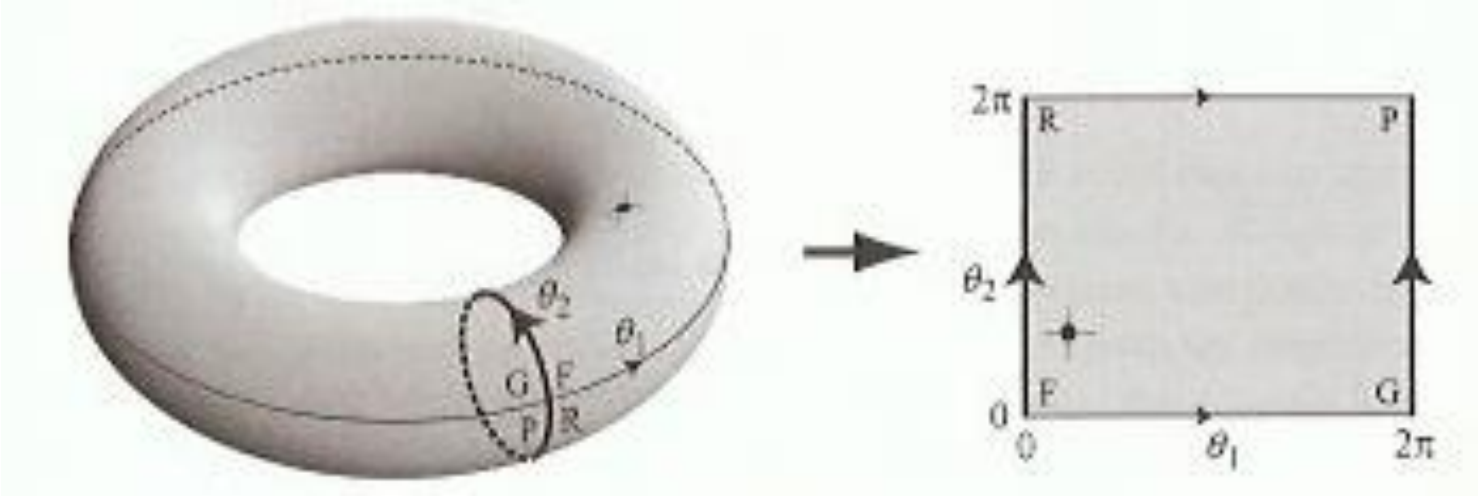


This is not a valid C-space!

Examples of Configuration Spaces

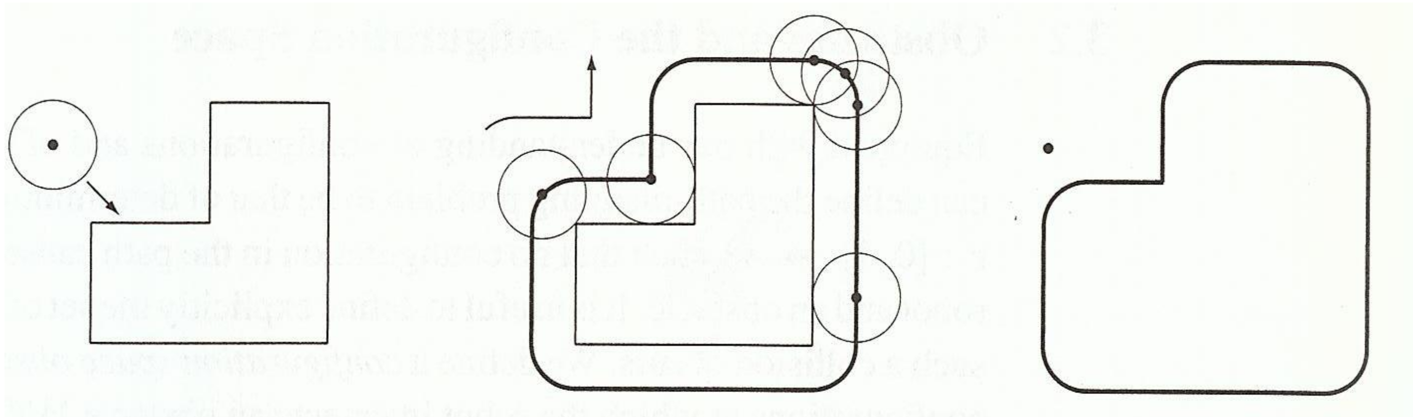


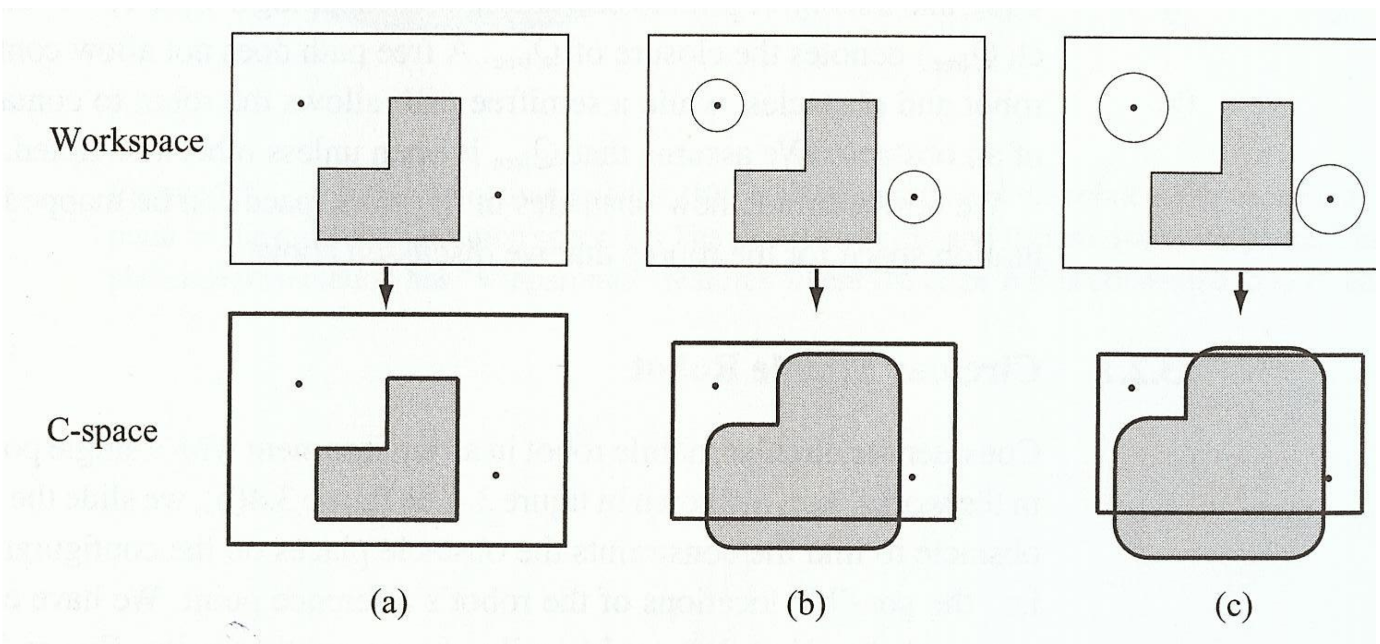
The topology of C is usually **not** that of a Cartesian space R^n.



$S^1 \times S^1 = T^2$

Examples of Circular Robot



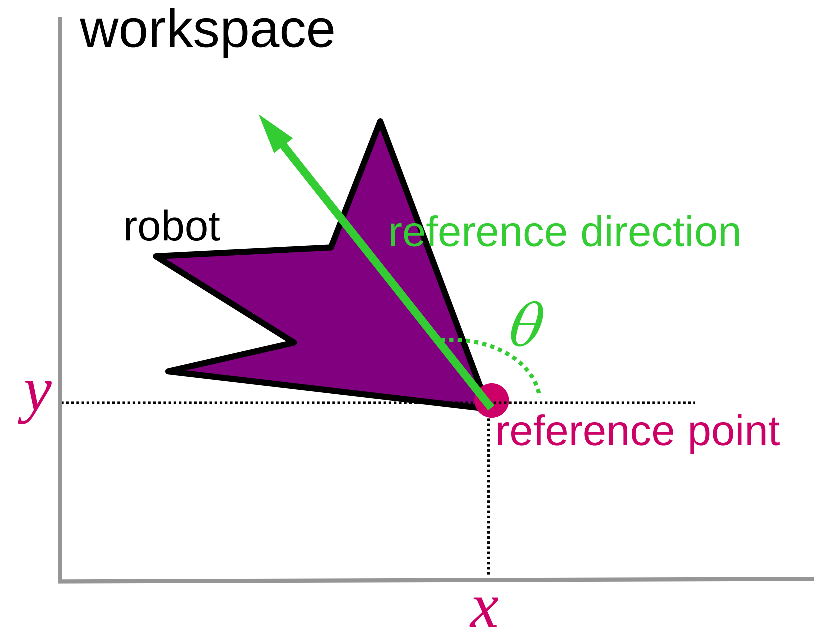


Dimension of Configuration Space

- **The dimension of the configuration space is the minimum number of parameters needed to specify the configuration of the object completely**

- **It is also called the number of degrees of freedom (dofs) of a moving object**

Example: Rigid Robot in 2-D Workspace



- **3-parameter specification:** $q = (x, y, \theta)$ **with** $\theta \in [0, 2\pi)$.
  - **3-D configuration space**

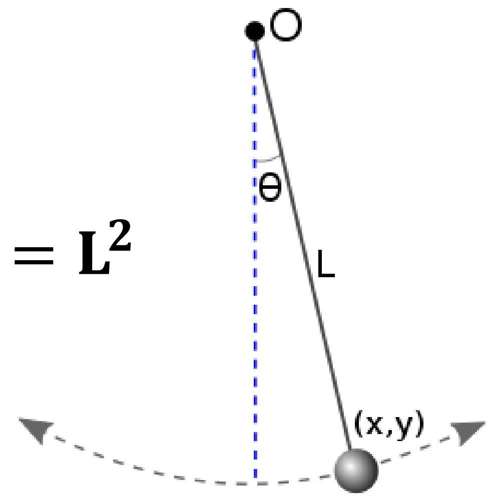
Example: Rigid Robot in 2-D workspace

- **4-parameter specification:** $q = (x, y, u, v)$ **with** $u^2+v^2 = 1$. **Note** $u = \cos\theta$ **and** $v = \sin\theta$

- **dim of configuration space =** **3**
  - **Does the dimension of the configuration space (number of dofs) depend on the parametrization?**

Holonomic and Non-Holonomic Constraints

- **Holonomic constraints**
  - **g (q, t) = 0**
  - **E.g., pendulum motion:** $x^2 + y^2 = L^2$



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- **Non-holonomic constraints**
  - **g (q, q′, t) = 0 (or q′ = f(q, u), where u is an action parameter)**
  - **This is related to the kinematics of robots**
  - **To accommodate this, the C-space is extended to include the position and its velocity**

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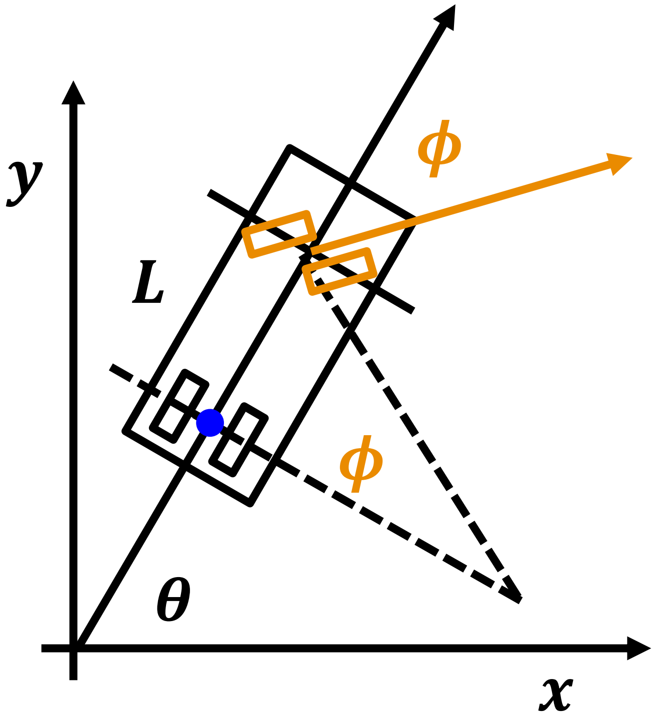
Example of Non-Holonomic Constraints

See Kinematic Car Model of my draft



$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{dy}{dx}$$

$$\sin(\theta)dx - \cos(\theta)dy = 0$$

$$\frac{dx}{dt} = v \cdot \cos(\theta), \quad \frac{dy}{dt} = v \cdot \sin(\theta),$$

$$\frac{d\theta}{dt} = \frac{v}{L}\tan(\phi)$$

Note that v, ϕ are action parameters

Holonomic and Non-Holonomic Constraints

- **Dynamic constraints**
  - **Dynamic equations are represented as $G(q, q', q'') = 0$**
  - **These constraints are reduced to non-holonomic ones when we use the extended C-space such as the state space:**

$$S = (X, X'), \text{ where } X = (q, q')$$

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Computation of Dimension of C-Space

- **Suppose that we have a rigid body that can translate and rotate in 2D workspace**
  - **Start with three points: A, B, C (6D space)**

- **We have the following (holonomic) constraints**
  - **Given A, we know the dist to B: d(A,B) = |A-B|**
  - **Given A and B, we have similar equations: d(A,C) = |A-C|, d(B,C) = |B-C|**

- **Each holonomic constraint reduces one dim.**
  - **Not for non-holonomic constraint**

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Example: Rigid Robot in 3-D Workspace

- **We can represent the positions and orientations of such robots with matrices (i.e., SO (3) and SE (3))**

SO (n) and SE (n)

- **Special orthogonal group, SO(n), of n x n matrices R,**

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

that satisfy:

$$r_{1i}^2 + r_{2i}^2 + r_{3i}^2 = 1 \text{ for all } i,$$

$$r_{1i} r_{1j} + r_{2i} r_{2j} + r_{3i} r_{3j} = 0 \text{ for all } i \neq j,$$

$$\det(R) = +1$$

Refer to the 3D Transformation at the undergraduate computer graphics.
http://sgvr.kaist.ac.kr/~sungeui/render/raster/transformation.pdf

- **Given the orientation matrix R of SO (n) and the position vector p, special Euclidean group, SE (n), is defined as:**

$$\begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

Example: Rigid Robot in 3-D Workspace

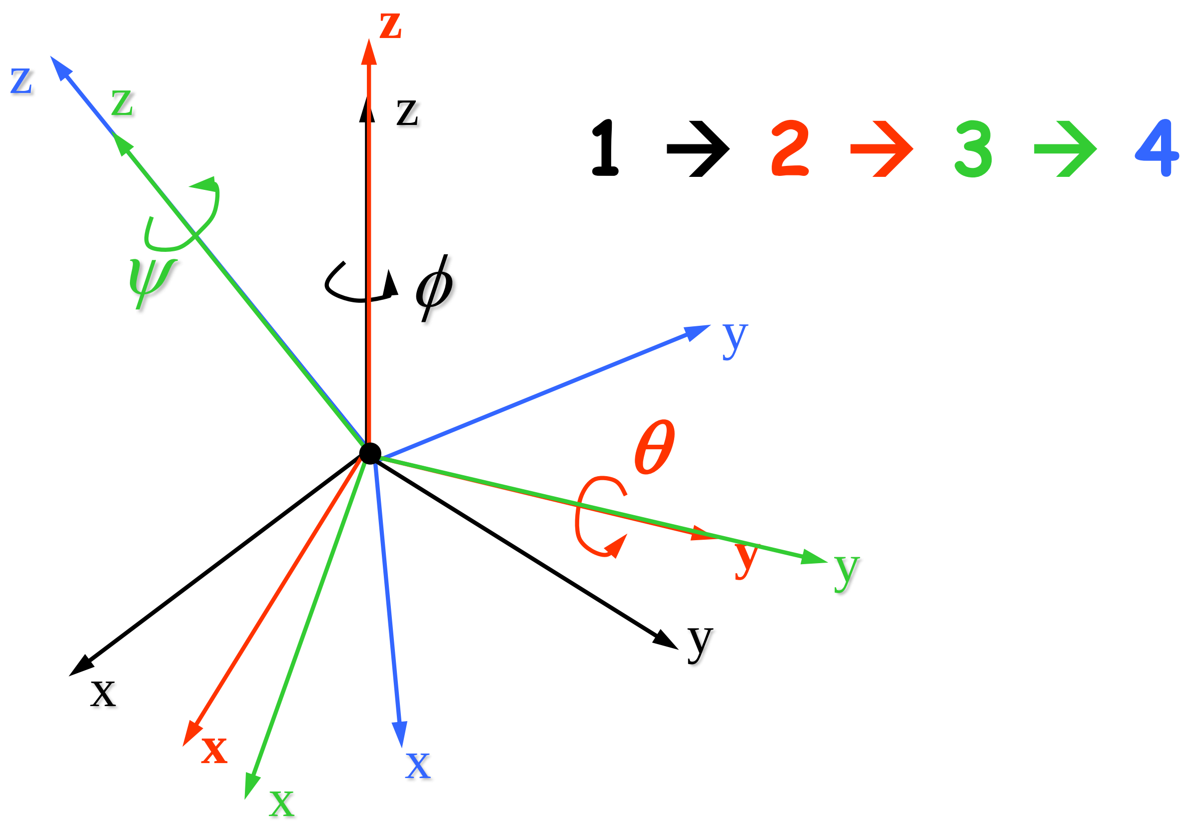
- q = (**position, orientation**) = ($x, y, z,$???)

- **Parametrization of orientations by matrix:**
  $q = (r_{11}, r_{12}, \ldots, r_{33}, r_{33})$ **where** $r_{11}, r_{12}, \ldots, r_{33}$ **are the elements of rotation matrix**

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \in SO(3)$$

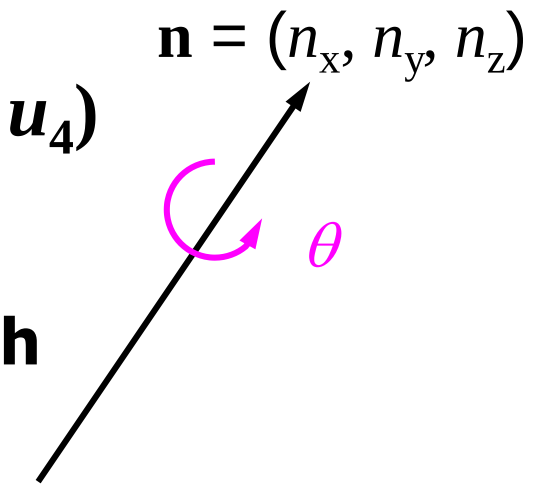
Example: Rigid Robot in 3-D Workspace

- **Parametrization of orientations by Euler angles:** (ϕ, θ, ψ)



Example: Rigid Robot in 3-D Workspace

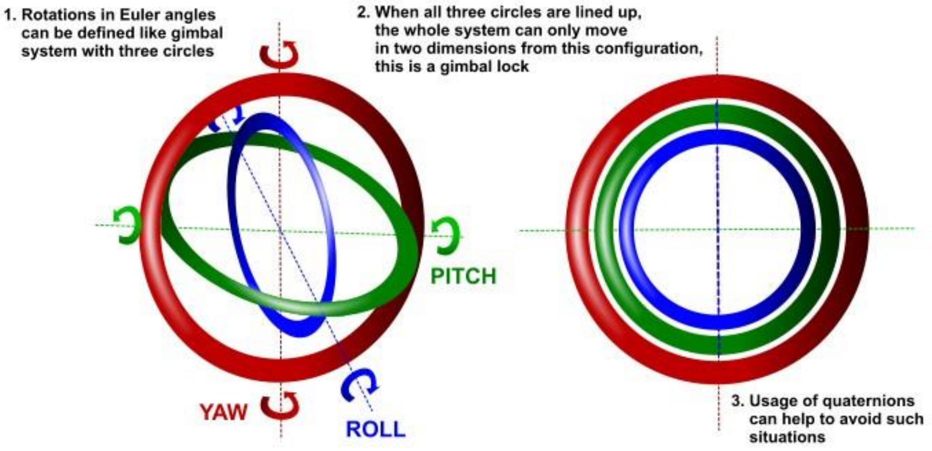
- **Parametrization of orientations by unit quaternion:** $u = (u_1, u_2, u_3, u_4)$ **with** $u_1^2 + u_2^2 + u_3^2 + u_4^2 = 1.$
  - **Note** $(u_1, u_2, u_3, u_4) = (\cos\theta/2, n_x\sin\theta/2, n_y\sin\theta/2, n_z\sin\theta/2)$ **with** $n_x^2 + n_y^2 + n_z^2 = 1$



  - **Compare with representation of orientation in 2-D:** $(u_1,u_2) = (\cos\theta, \sin\theta)$

Example: Rigid Robot in 3-D Workspace

- **Advantage of unit quaternion representation**
  - **Compact**
  - **No singularity (no gimbal lock indicating two axes are aligned)**
  - **Naturally reflect the topology of the space of orientations**



Cyrille Fauvel

- **Number of dofs = 6**
- **Topology:** R^3 **x** $\mathrm{SO}(3)$

Class Objectives were:

- Configuration space
  - Definitions and examples
  - Obstacles
  - Paths
  - Metrics

Next Time….

- Configuration space
  - Definitions and examples
  - Obstacles
  - Paths
  - Metrics

Homework

- **Come up with one question on what we have discussed today**
  - **Write a question two times before the mid-term exam**

- **Browse two papers**
  - **Submit their summaries online before the Mon. Class**