CS686:
Configuration Space II

Sung-Eui Yoon
(윤성의)

Course URL:
http://sgvr.kaist.ac.kr/~sungeui/MPA

Class Objectives (Ch. 3)

- Configuration space
  - Definitions and examples
  - Obstacles
  - Paths
  - Metrics

- Last time:
  - Degrees-of-freedom (DoFs) of C-space w/ holonomic, non-holonomic and dynamic constraints

Obstacles in the Configuration Space

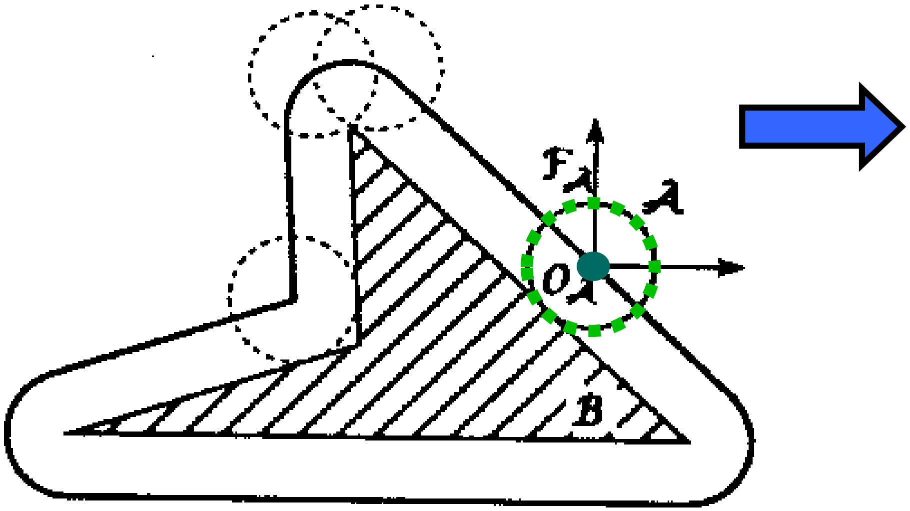
- **A configuration q is collision-free, or free, if a moving object placed at q does not intersect any obstacles in the workspace**

- **The free space F is the set of free configurations**

- **A configuration space obstacle (C-obstacle) is the set of configurations where the moving object collides with workspace obstacles**

Disc in 2-D Workspace

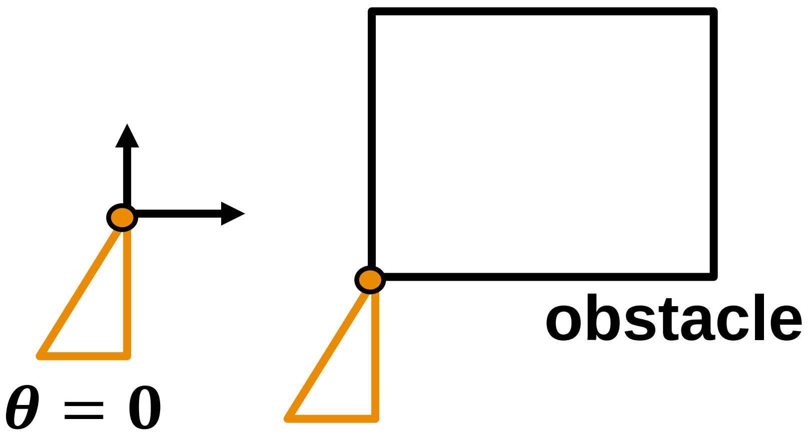
workspace



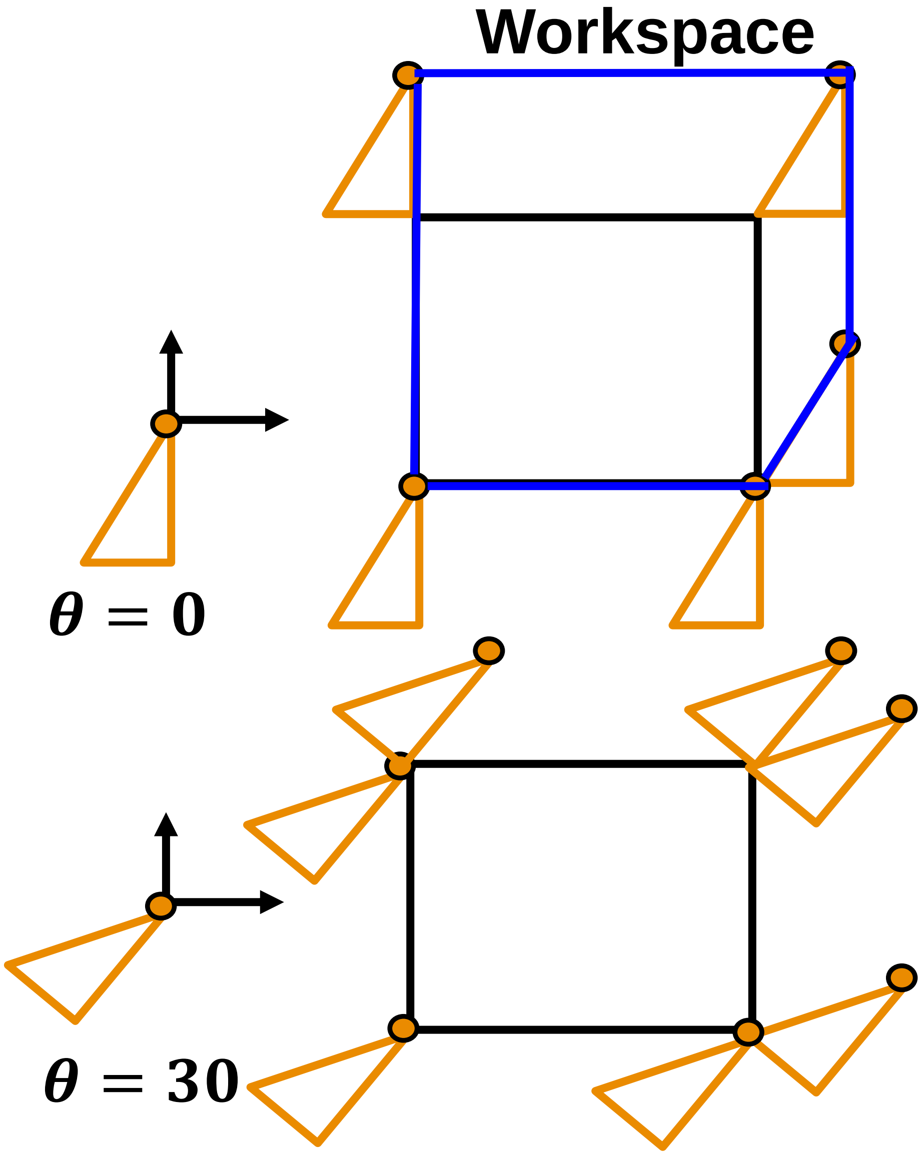
configuration space

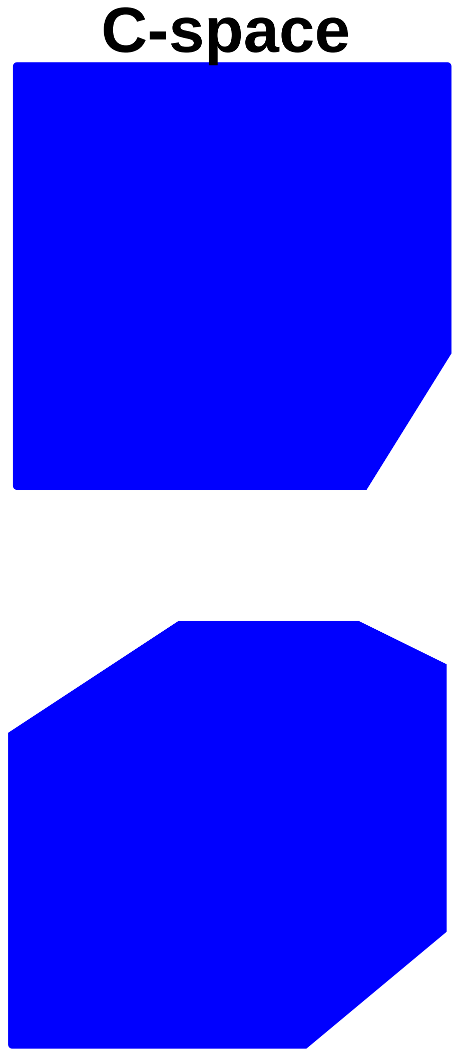
Polygonal Robot Translating & Rotating in 2-D Workspace

Workspace **C-space**

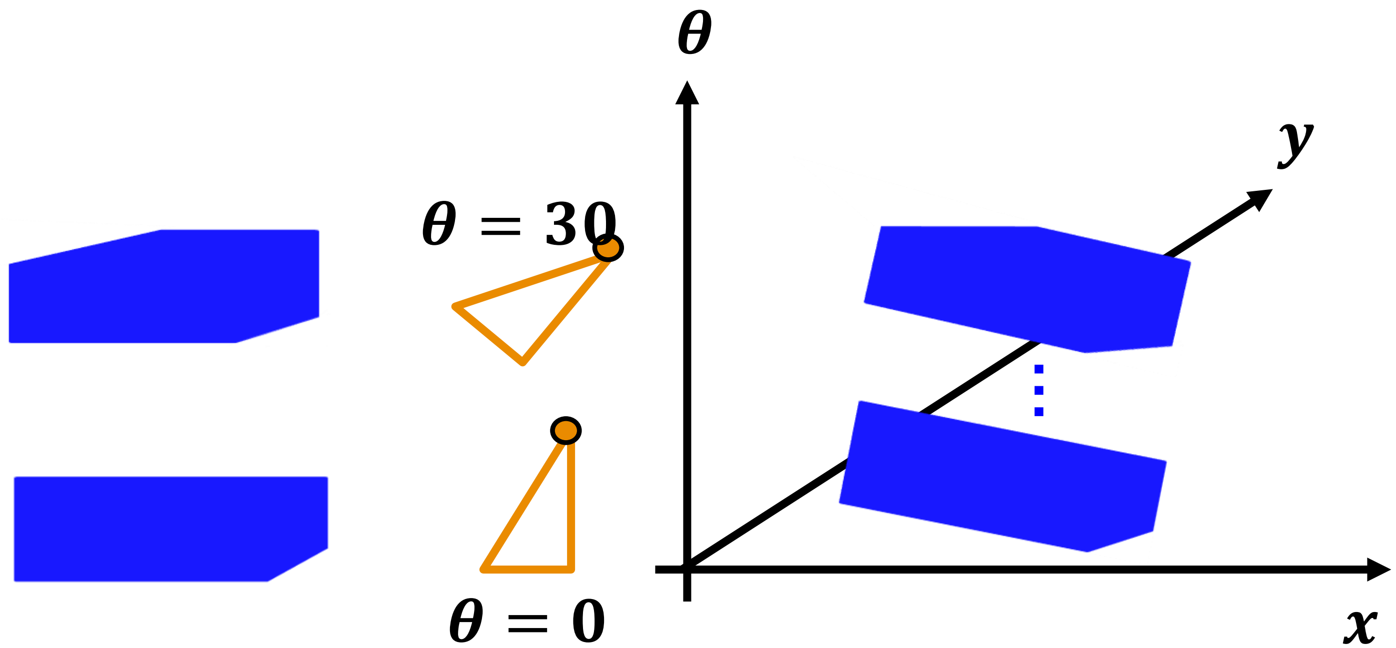


Polygonal Robot Translating & Rotating in 2-D Workspace





Polygonal Robot Translating & Rotating in 2-D Workspace



3D C-space

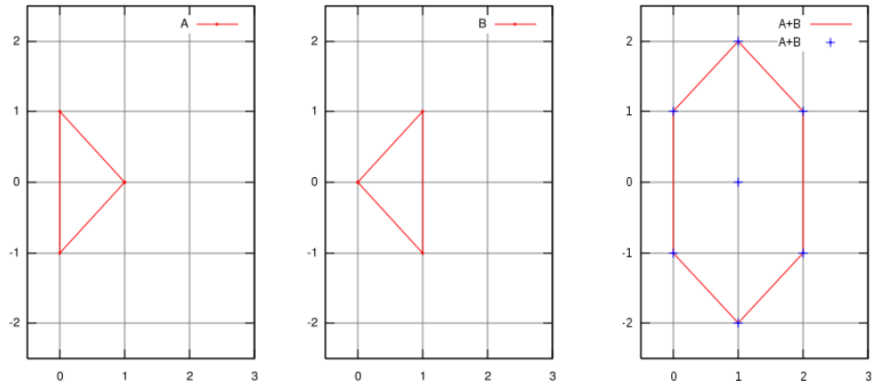
C-Obstacle Construction

- **Input:**
  - **Polygonal moving object translating in 2-D workspace**
  - **Polygonal obstacles**
- **Output:**
  - **Configuration space obstacles represented as polygons**

Minkowski Sum

- **The Minkowski sum of two sets P and Q, denoted by $P \oplus Q$, is defined as**

$$P \oplus Q = \{\, p+q \mid p \in P,\ q \in Q \,\}$$



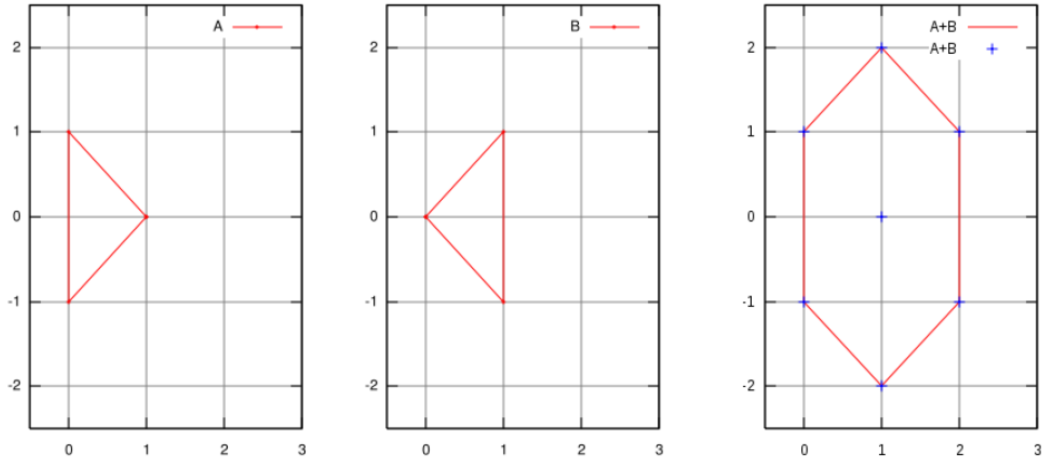
wiki

- **Similarly, the Minkowski difference is defined as**

$$P \ominus Q = \{\, p-q \mid p \in P,\ q \in Q \,\}$$
$$= P \oplus -Q$$

Minkowski Sum of Convex Polygons

- **The Minkowski sum of two convex polygons P and Q of m and n vertices respectively is a convex polygon $P \oplus Q$ of $m + n$ vertices.**
  - **The vertices of $P \oplus Q$ are the "sums" of vertices of P and Q.**

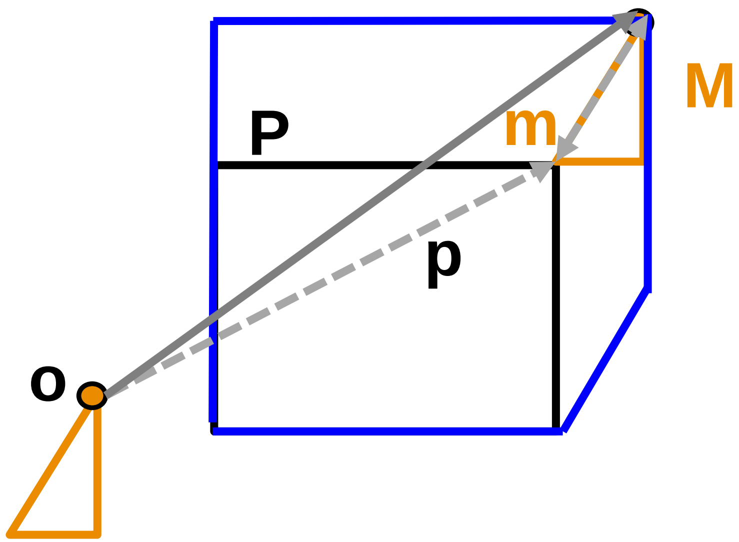


wiki

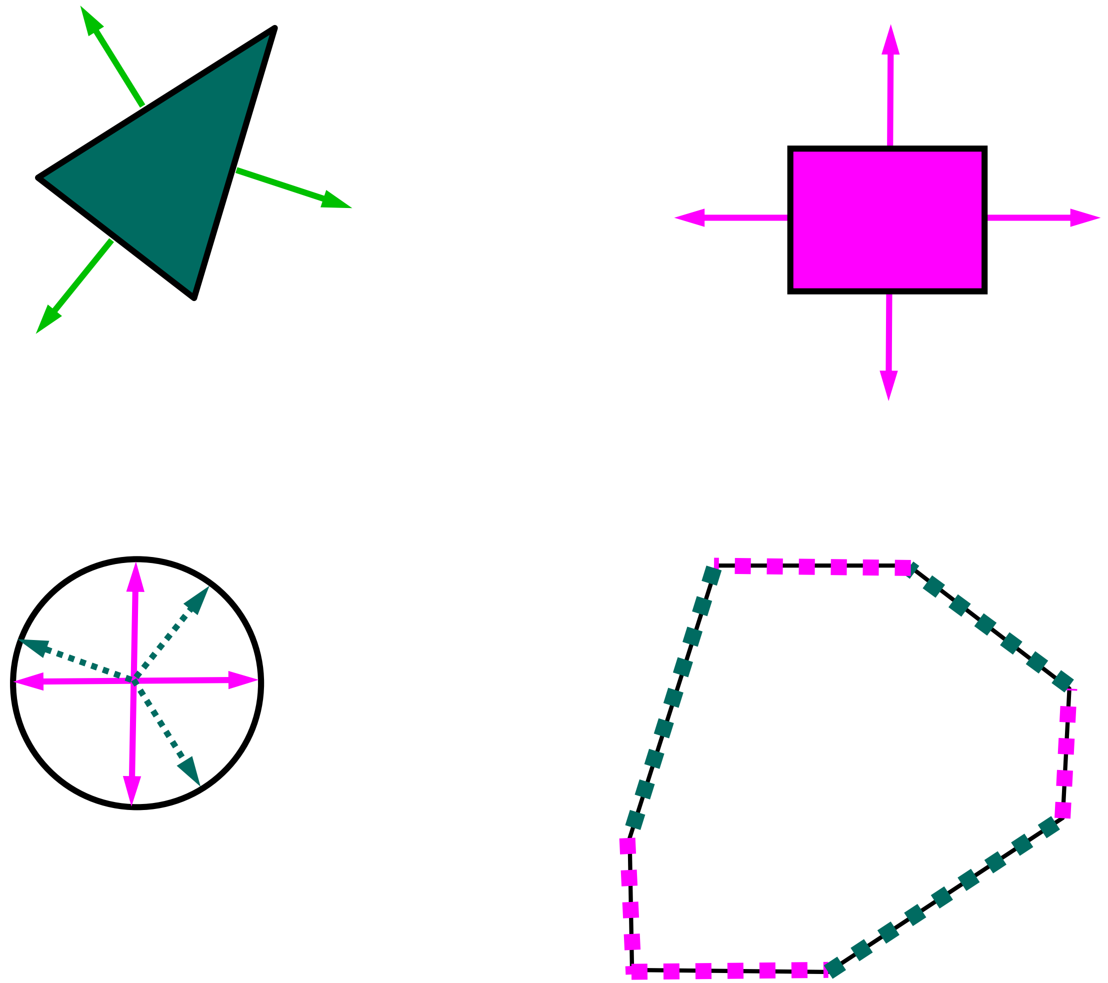
KAIST

Observation

- Suppose P is an obstacle in the workspace and M is a moving object
- Then the C-obstacle is $P \ominus M$



Computing C-obstacles



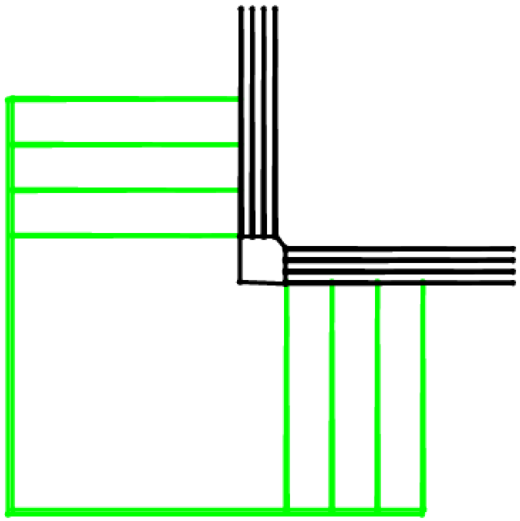
Computational efficiency

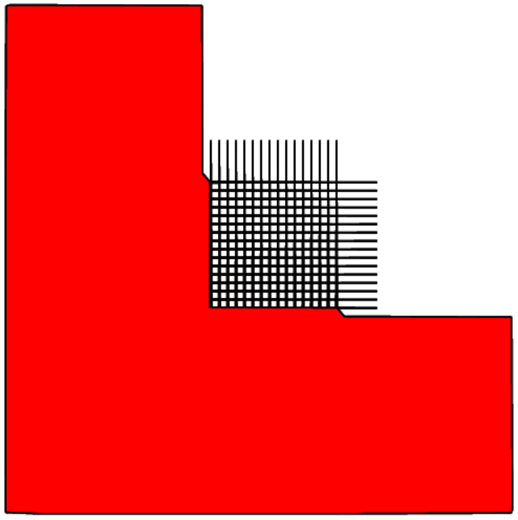
- **Running time $O(n+m)$**
- **Space $O(n+m)$**
- **Non-convex obstacles**
  - **Decompose into convex polygons (*e.g.,* triangles or trapezoids), compute the Minkowski sums, and take the union**
  - **Complexity of Minkowksi sum $O(n^2m^2)$**
- **3-D workspace**
  - **Convex case: $O(nm)$**
  - **Non-convex case: $O(n^3m^3)$**

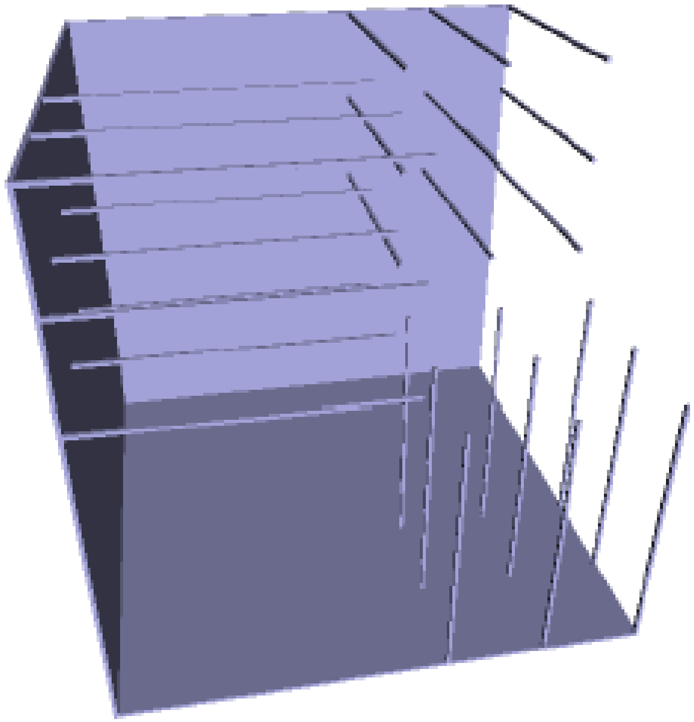
Worst case example

- $O(n^2m^2)$ **complexity**

2D example
Agarwal et al. 02

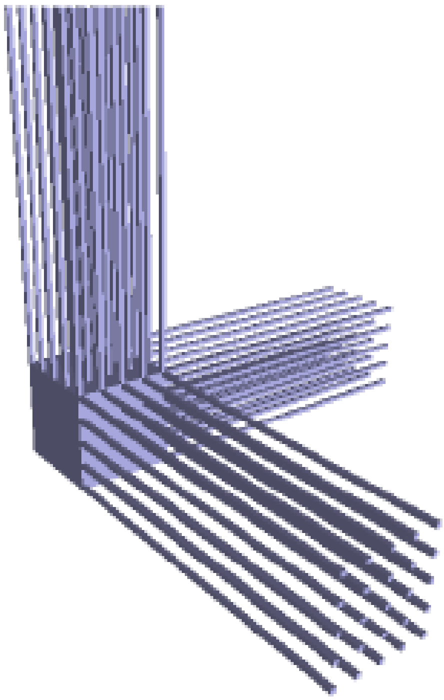




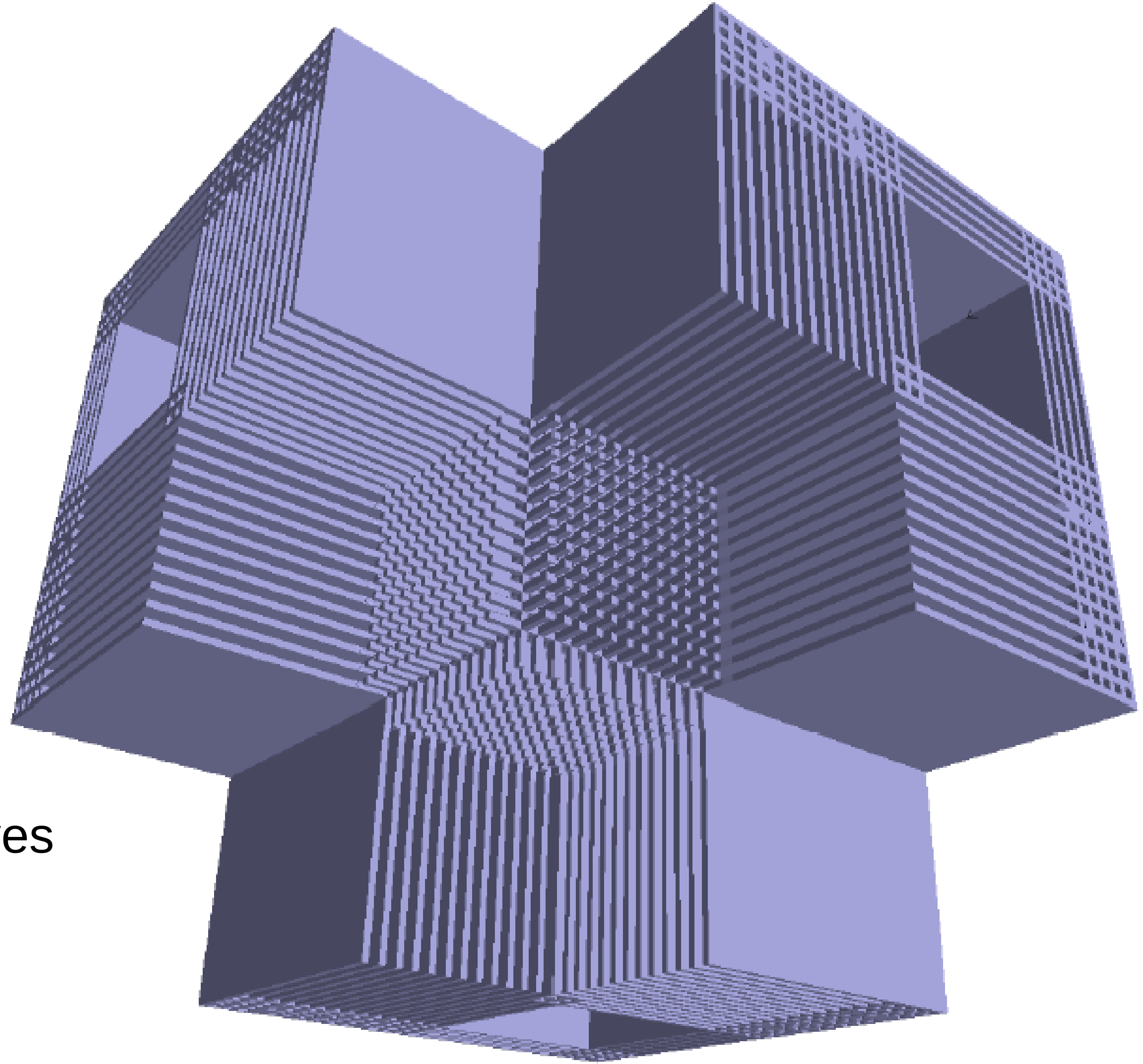


444 tris

$\oplus$



1,134 tris



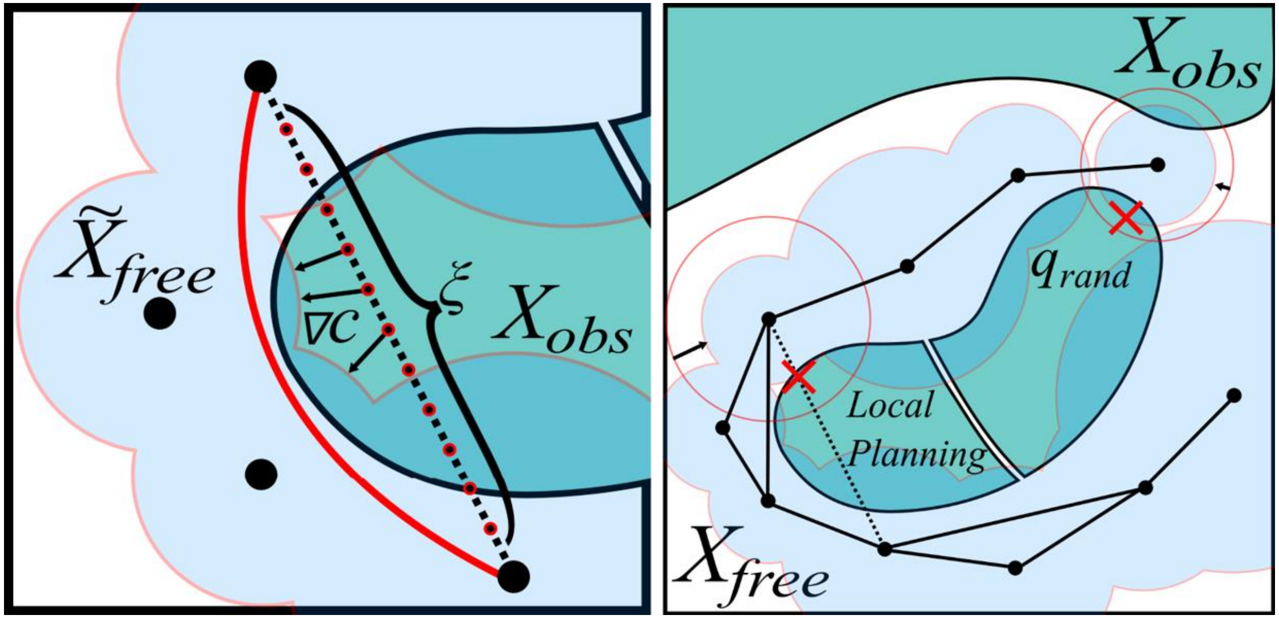
Union of
66,667 primitives

Main Message

- Computing the free or obstacle space in an accurate way is an expensive and non-trivial problem
- Lead to many sampling based methods
  - Locally utilize many geometric concepts developed for designing complete planners

Approximation of Configuration Free Space

- **Dancing PRM* : Simultaneous Planning of Sampling and Optimization with Configuration Free Space Approximation**
  - **Approximate C-Space and perform planning**
  - **Improve the quality in an iterative manner**



Video

Sensors!

Robots' link to the external world...

sonar rangefinder

gyro

Sensors, sensors, sensors!

and tracking what is sensed: world models

compass

IR rangefinder

sonar rangefinder

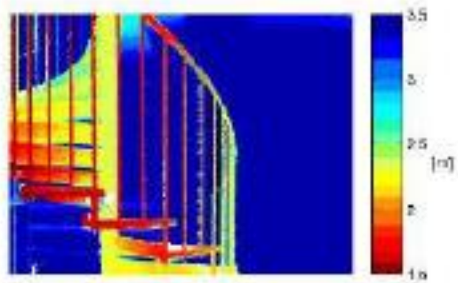
CMU cam with on-board processing

odometry…

Laser Ranging

LIDAR

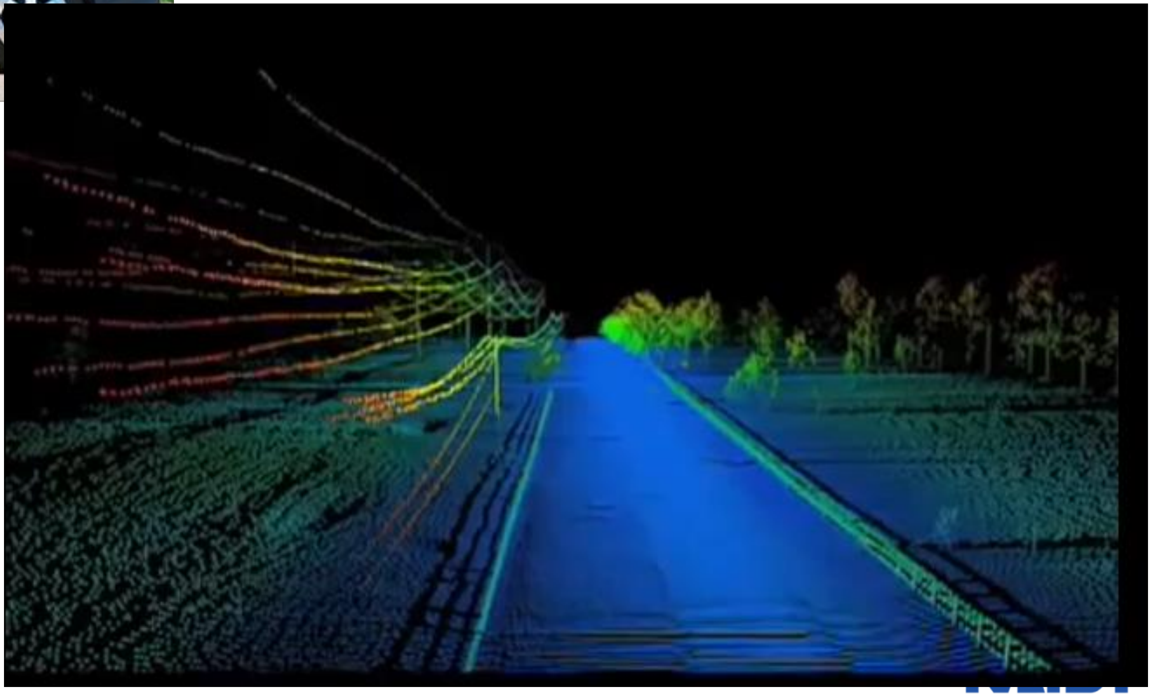
Sick Laser



LIDAR map

Sick laser range finder

Velodyne

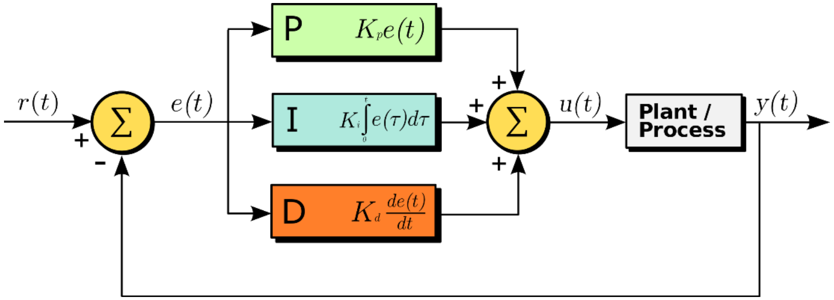


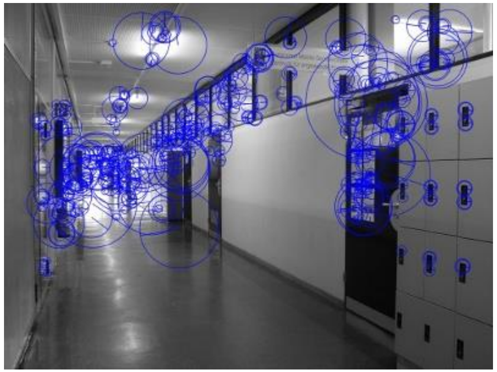
Kinect and Xtion

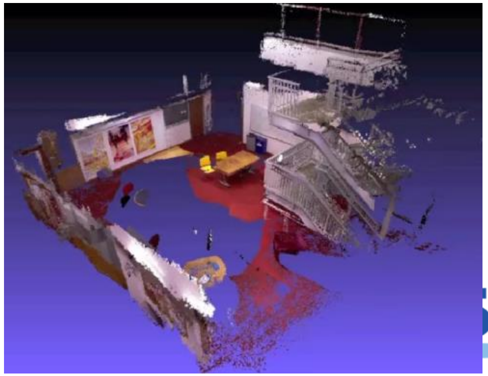
- **Kinect resolution**
  - **640×480 pixels @ 30 Hz (RGB camera)**
  - **640×480 pixels @ 30 Hz (IR depth-finding camera)**

Whole Picture

- Sensor
  - Point clouds as obstacle map
- SLAM (Simultaneous Localization and Mapping)
- Path/motion planner
- Control
  - Compute force controls given a computed path





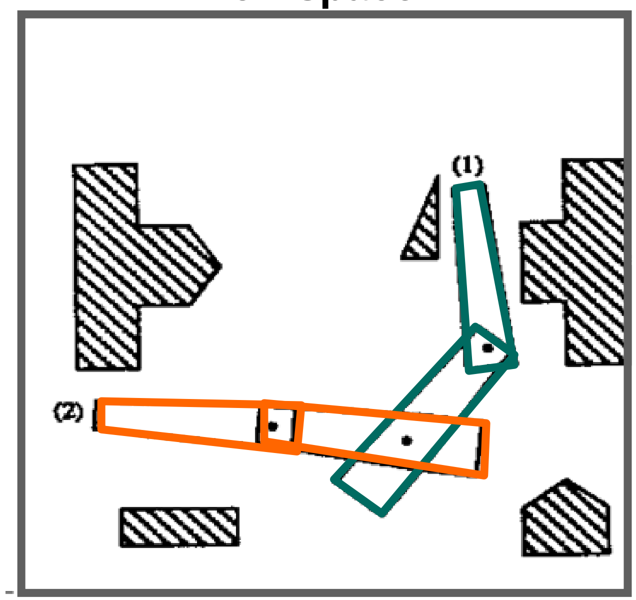


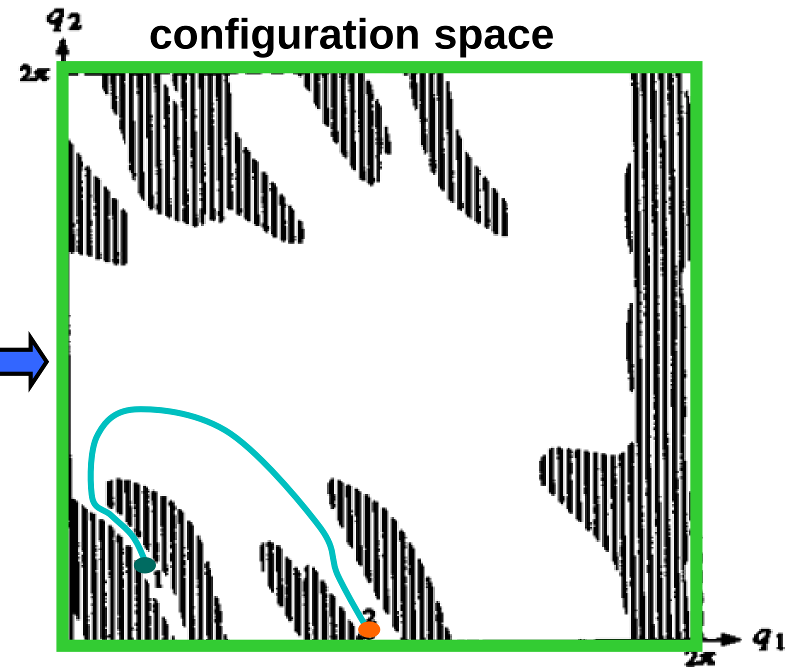
Configuration space

- Definitions and examples
- Obstacles
- Paths
- Metrics

25

Paths in the configuration space





- **A path in C is a continuous curve connecting two configurations q and q':**

$$\tau : s \in [0,1] \rightarrow \tau(s) \in C$$

such that $\tau(0) = q$ and $\tau(1) = q'$.

Constraints on paths

- **A trajectory is a path parameterized by time:**

$$\tau : t \in [0, T] \rightarrow \tau(t) \in C$$

- **Constraints**
  - **Finite length**
  - **Bounded curvature**
  - **Smoothness**
  - **Minimum length**
  - **Minimum time**
  - **Minimum energy**
  - **…**

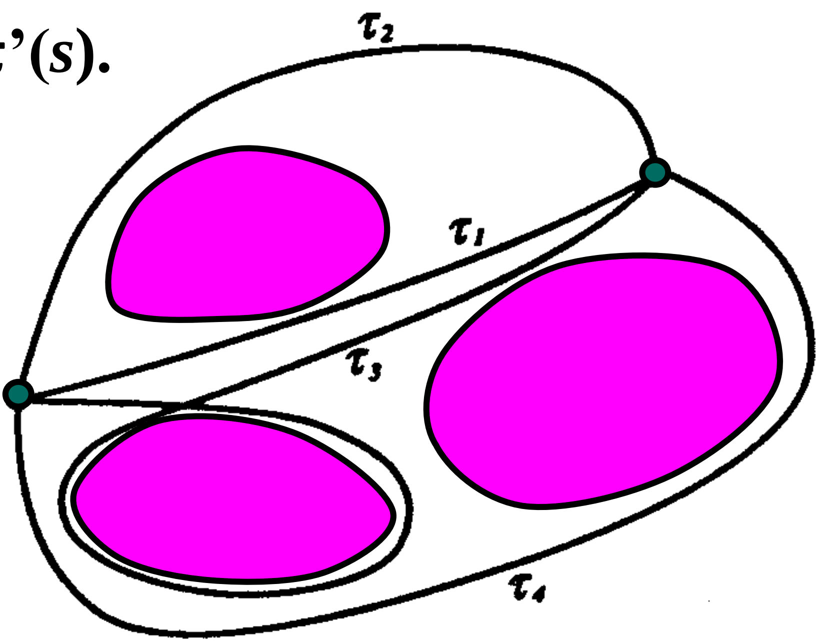
Homotopic Paths

- **Two paths τ and τ' (that map from U to V) with the same endpoints are homotopic if one can be continuously deformed into the other:**

$$h : U \times [0,1] \rightarrow V$$

with $h(s,0) = \tau(s)$ and $h(s,1) = \tau'(s)$.

- **A homotopic class of paths contains all paths that are homotopic to one another**



KAIST

Configuration space

- **Definitions and examples**
- **Obstacles**
- **Paths**
- **Metrics**

Metric in Configuration Space

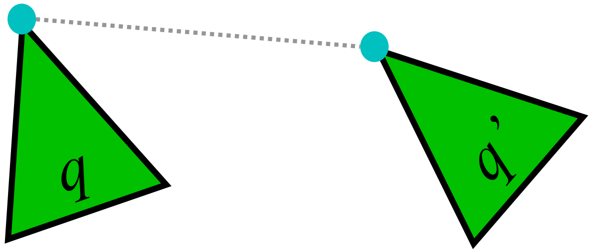
- **A metric or distance function *d* in a configuration space *C* is a function**
$$d:(q,q')\in C^2 \rightarrow d(q,q')\geq 0$$
such that
  - $d(q, q') = 0$ **if and only if** $q = q'$, Identity
  - $d(q, q') = d(q', q)$, Symmetry
  - $d(q,q')\leq d(q,q'')+d(q'',q')$. Triangle inequality

Example

- **Robot** A **and a point** x **on** A
- $x(q)$: **position of** x **in the workspace when** A **is at configuration** q
- **A distance** d **in** C **is defined by**

$$d(q, q') = \max_{x \in A} \| x(q) - x(q') \|,$$

where $||x - y||$ **denotes the Euclidean distance between points** x **and** y **in the workspace.**



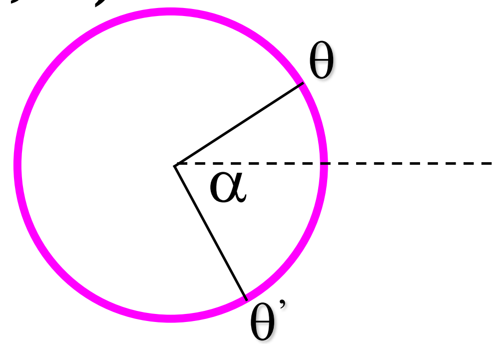
L_p Metrics

$$d(x, x') = \left(\sum_{i=1}^{n} | x_i - x_i' |^p \right)^{1/p}$$

- L_2: Euclidean metric
- L_1: Manhattan metric
- L_∞: Max ($| x_i - x_i' |$)

Examples in $R^2 \times S^1$

- **Consider** $R^2 \times S^1$
  - $q = (x, y, \theta)$, $q' = (x', y', \theta')$ **with** $\theta, \theta' \in [0, 2\pi)$
  - $\alpha = \min\{\, |\theta - \theta'|\,,\ 2\pi - |\theta - \theta'|\,\}$

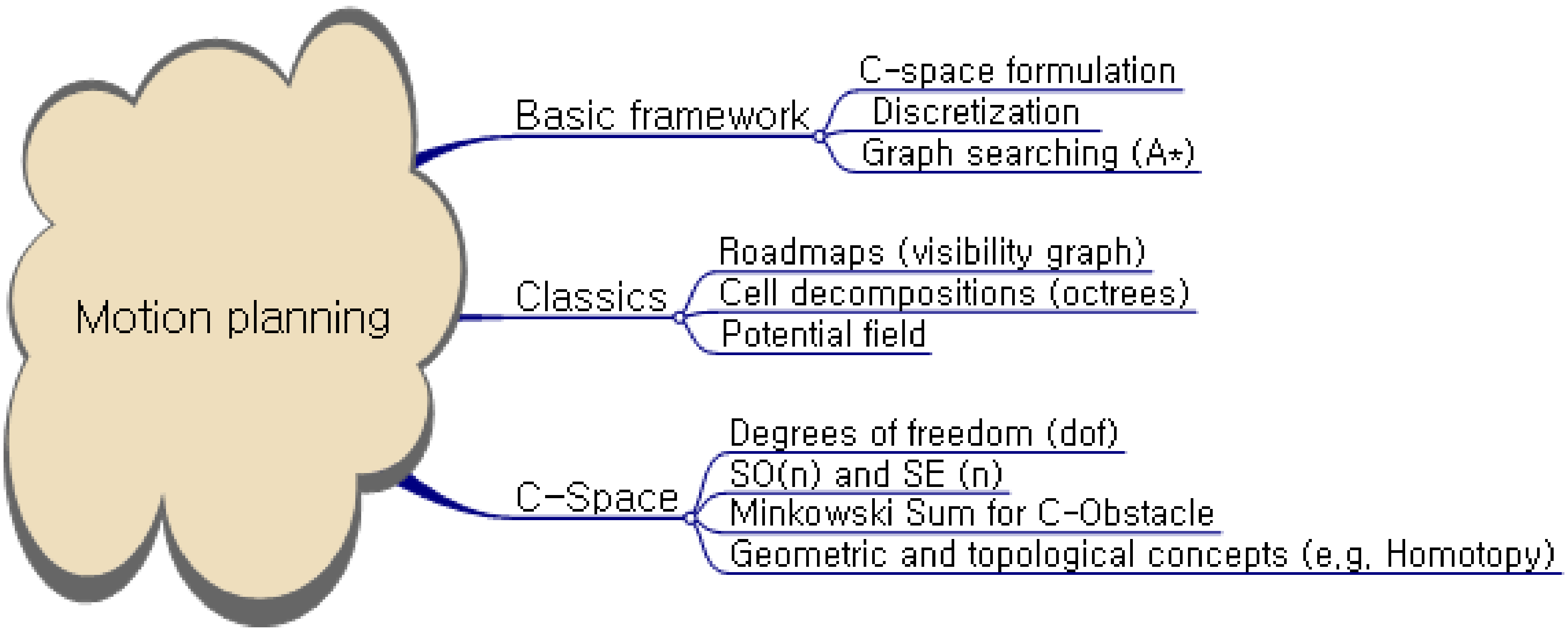


- $d(q, q') = \mathrm{sqrt}(\,(x-x')^2 + (y-y')^2 + \alpha^2\,)$

Class Objectives were:

- **Configuration space**
  - **Definitions and examples**
  - **Obstacles**
  - **Paths**
  - **Metrics**

Summary



Motion planning
Basic framework
C-space formulation
Discretization
Graph searching (A*)
Classics
Roadmaps (visibility graph)
Cell decompositions (octrees)
Potential field
C-Space
Degrees of freedom (dof)
SO(n) and SE (n)
Minkowski Sum for C-Obstacle
Geometric and topological concepts (e.g. Homotopy)

Next Time….

- **Collision detection and distance computation**

Homework

- **Submit summaries of 2 ICRA/IROS/RSS/CoRL/TRO/IJRR papers**
- **Go over the next lecture slides**
- **Come up with two questions before the mid-term exam**