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# CS686: RRT

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Course URL:  
<http://sgvr.kaist.ac.kr/~sungeui/MPA>

# Class Objectives

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- **Understand the RRT technique and its recent advancements**
  - **RRT\***
  - **Kinodynamic planning**
- **Last time**
  - **Probabilistic roadmap techniques**
  - **Sampling and re-sampling techniques**

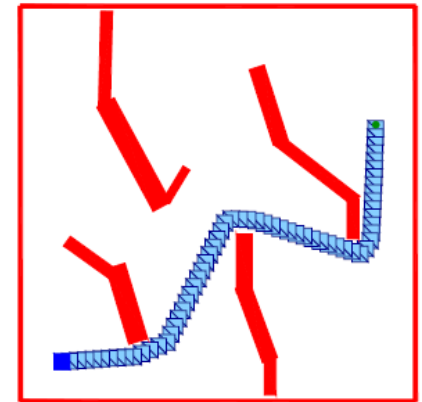
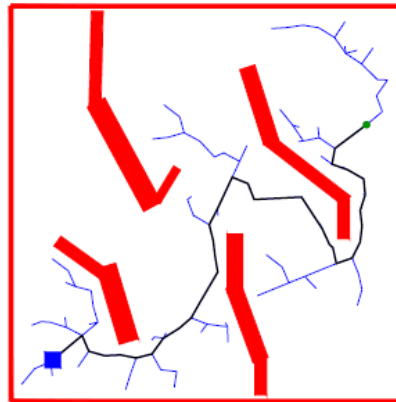
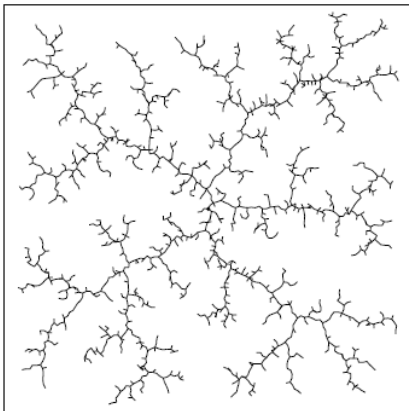
# Question

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- **PRM assumes that we know the global map, but how can we handle the case where we have only a partial map due to the limited sensor range?**
  - 지난시간에 배운 PRM 기법들은 글로벌 맵을 알고 있어야 문제 해결이 가능한데, 전체 맵의 일부분(센서 탐지거리 제약 등으로)만을 알고 있는 상황에서 PRM알고리즘을 적용하려면 어떤 방식으로 해야 하는지요?

# Rapidly-exploring Random Trees (RRT) [LaValle 98]

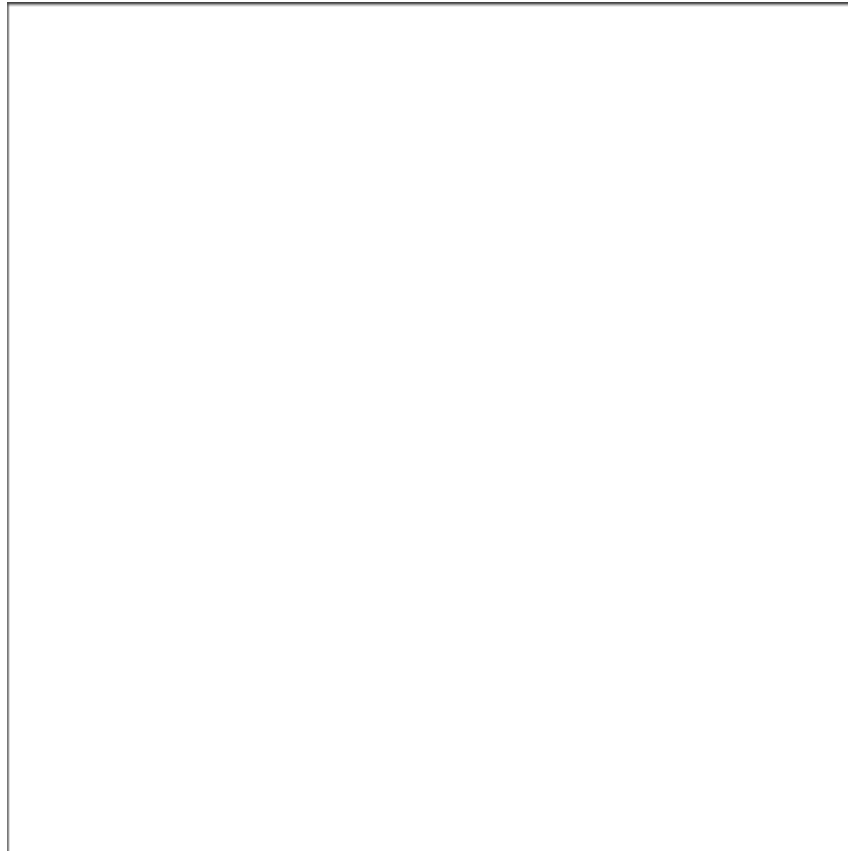
- Present an efficient randomized path planning algorithm for single-query problems
  - Converges quickly
  - Probabilistically complete
  - Works well in high-dimensional C-space



# Rapidly-Exploring Random Tree

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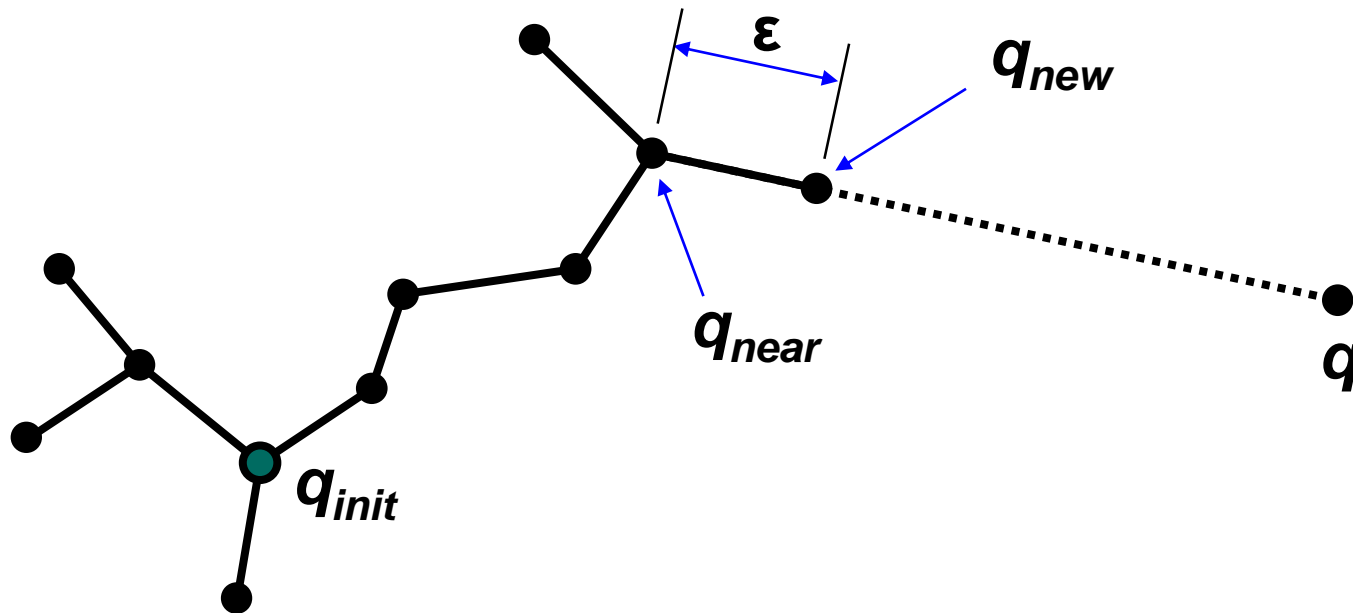
- A growing tree from an initial state



# RRT Construction Algorithm

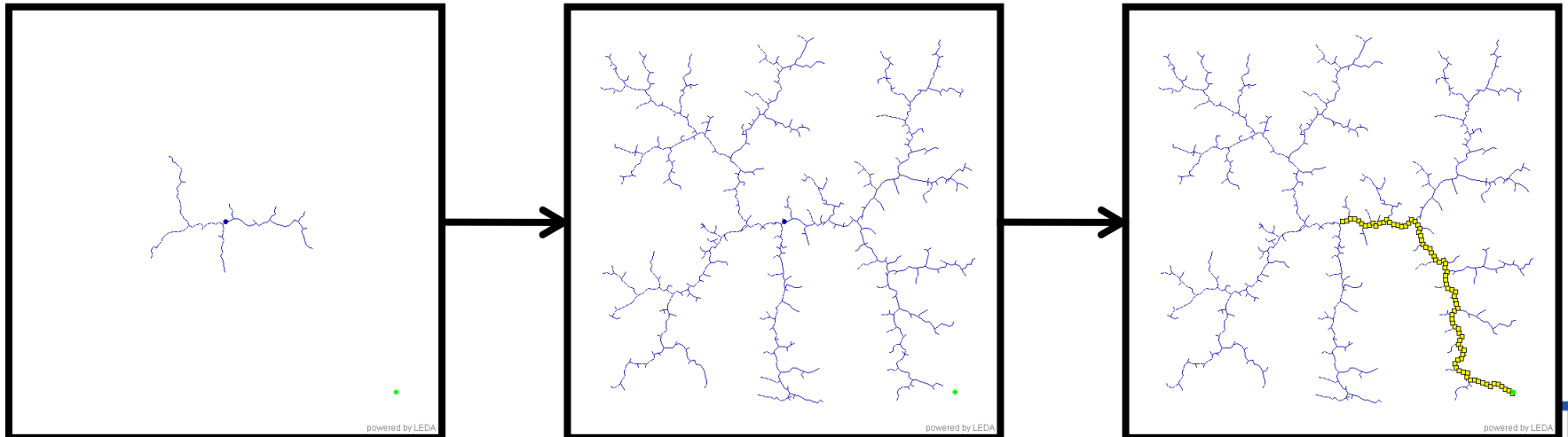
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- **Extend a new vertex in each iteration**
  - **Alternatively, one can simply connect**



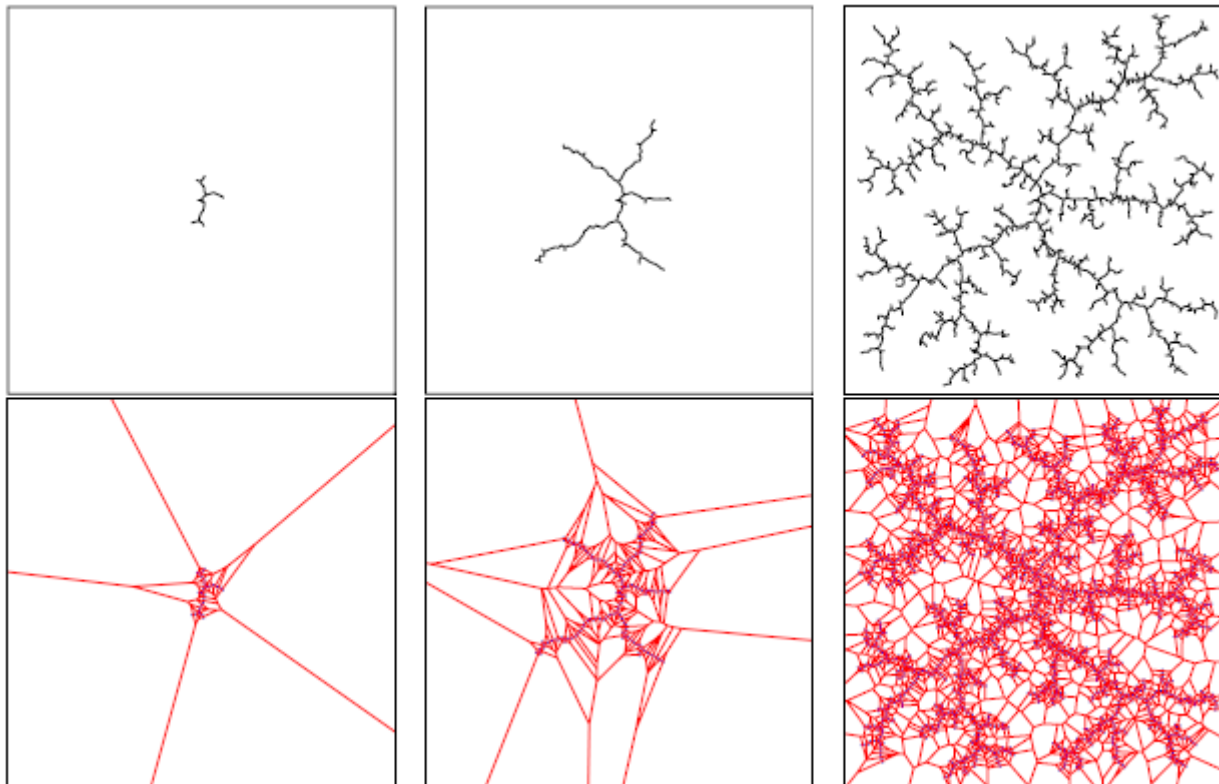
# Overview – Planning with RRT

- **Extend RRT until a nearest vertex is close enough to the goal state**
  - **Can handle nonholonomic constraints and high degrees of freedom**
- **Probabilistically complete, but does not converge to the optimal one**



# Voronoi Region

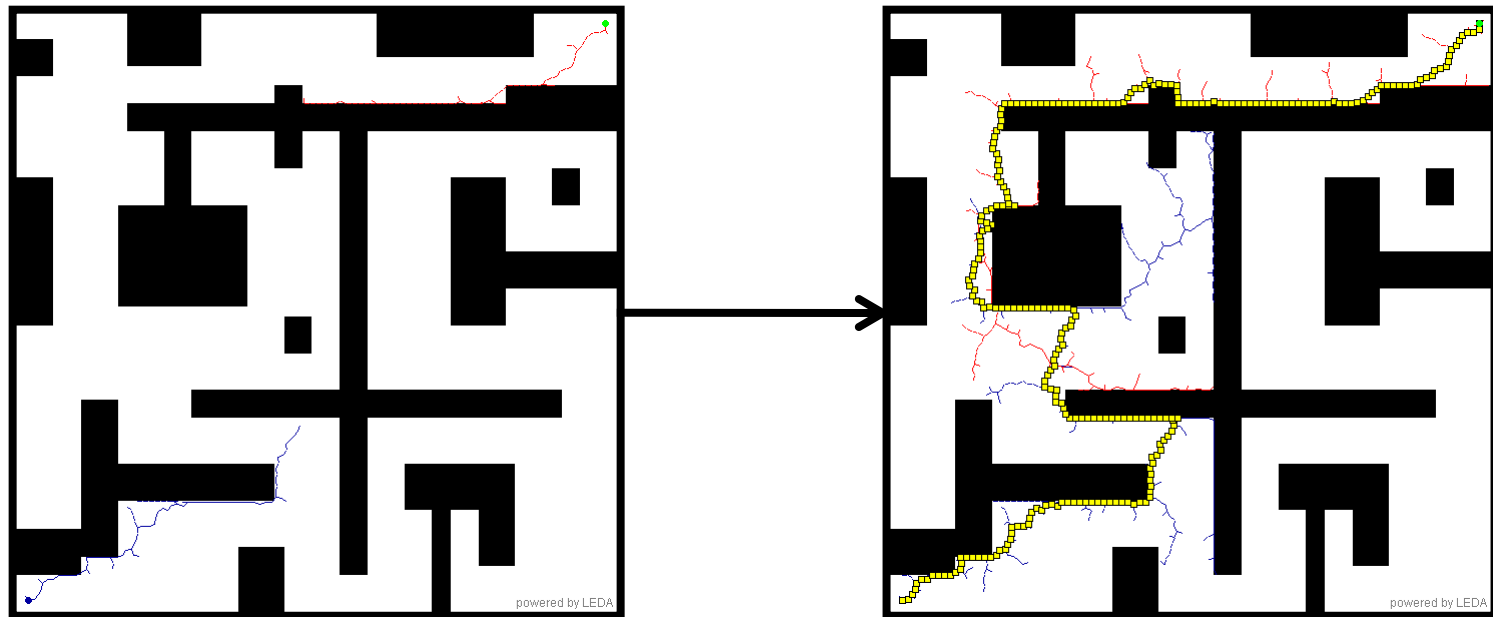
- An RRT is biased by large Voronoi regions to rapidly explore, before uniformly covering the space





# Overview – With Dual RRT

- **Extend RRTs from both initial and goal states**
- **Find path much more quickly**

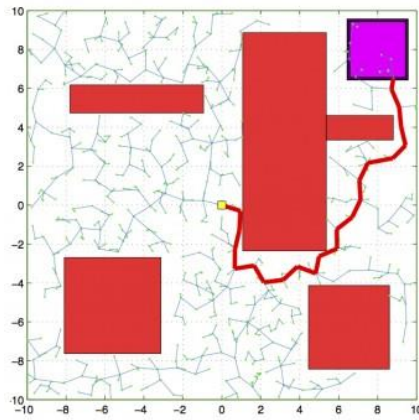


737 nodes are used

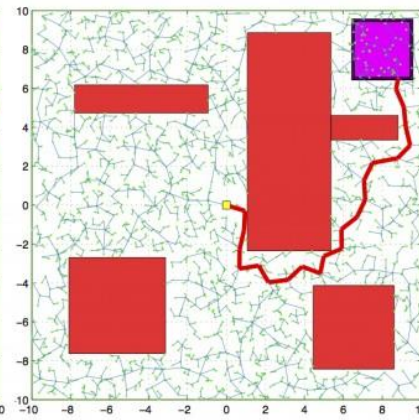
# RRT\*

- RRT does not converge to the optimal solution

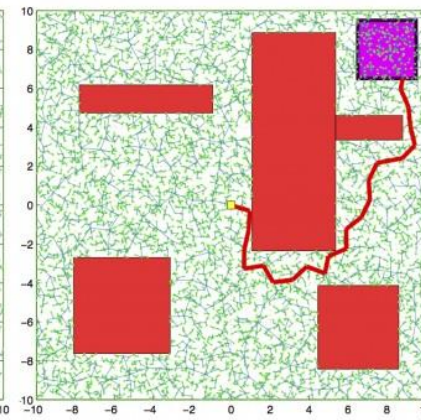
RRT



(a) RRT in iteration 1,000



(b) RRT in iteration 3,000



(c) RRT in iteration 10,000

RRT\*

# RRT\*

- **Asymptotically optimal without a substantial computational overhead**

**Theorem [Karaman & Frazzoli, IJRR 2011]**

(i) The RRT\* algorithm is asymptotically optimal

$$\mathbb{P}\left(\left\{\lim_{n \rightarrow \infty} Y_n^{\text{RRT}^*} = c^*\right\}\right) = 1$$

(ii) RRT\* algorithm has no substantial computational overhead when compared to the RRT:

$$\lim_{n \rightarrow \infty} \mathbb{E} \left[ \frac{M_n^{\text{RRT}^*}}{M_n^{\text{RRT}}} \right] = \text{constant}$$

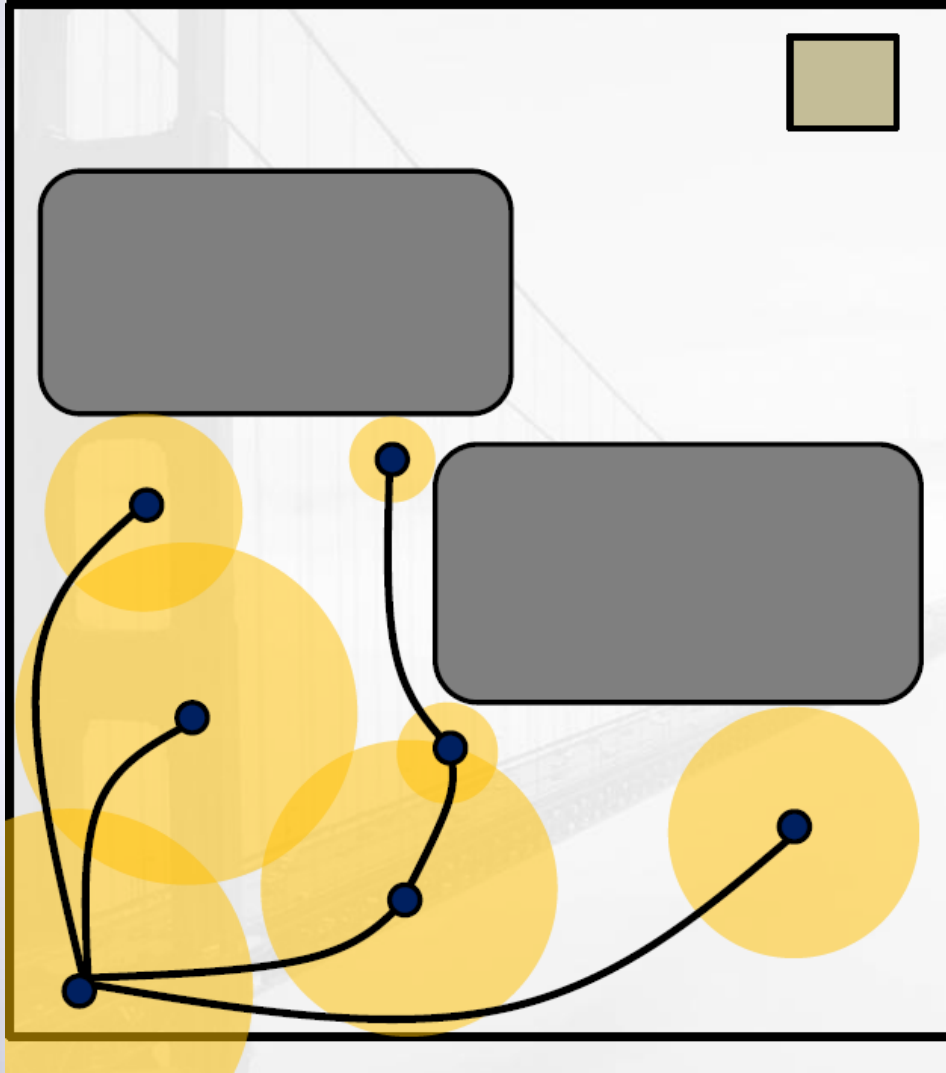
- $Y_n^{\text{RRT}^*}$  : cost of the best path in the RRT\*
- $c^*$  : cost of an optimal solution
- $M_n^{\text{RRT}}$  : # of steps executed by RRT at iteration n
- $M_n^{\text{RRT}^*}$  : # of steps executed by RRT\* at iteration n

# Key Operation of RRT\*

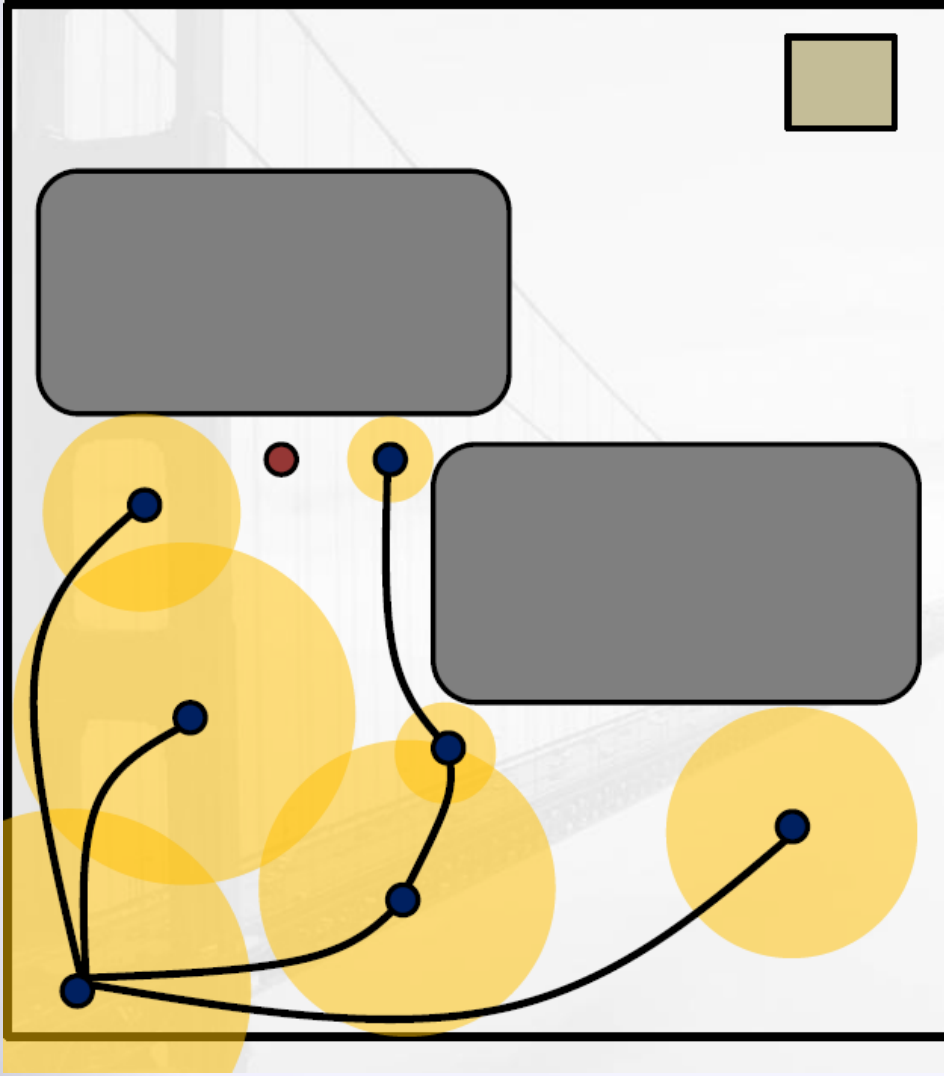
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- **RRT**
  - **Just connect a new node to its nearest neighbor node**
- **RRT\*: refine the connection with re-wiring operation**
  - **Given a ball, identify neighbor nodes to the new node**
  - **Refine the connection to have a lower cost**

# Example: Re-Wiring Operation

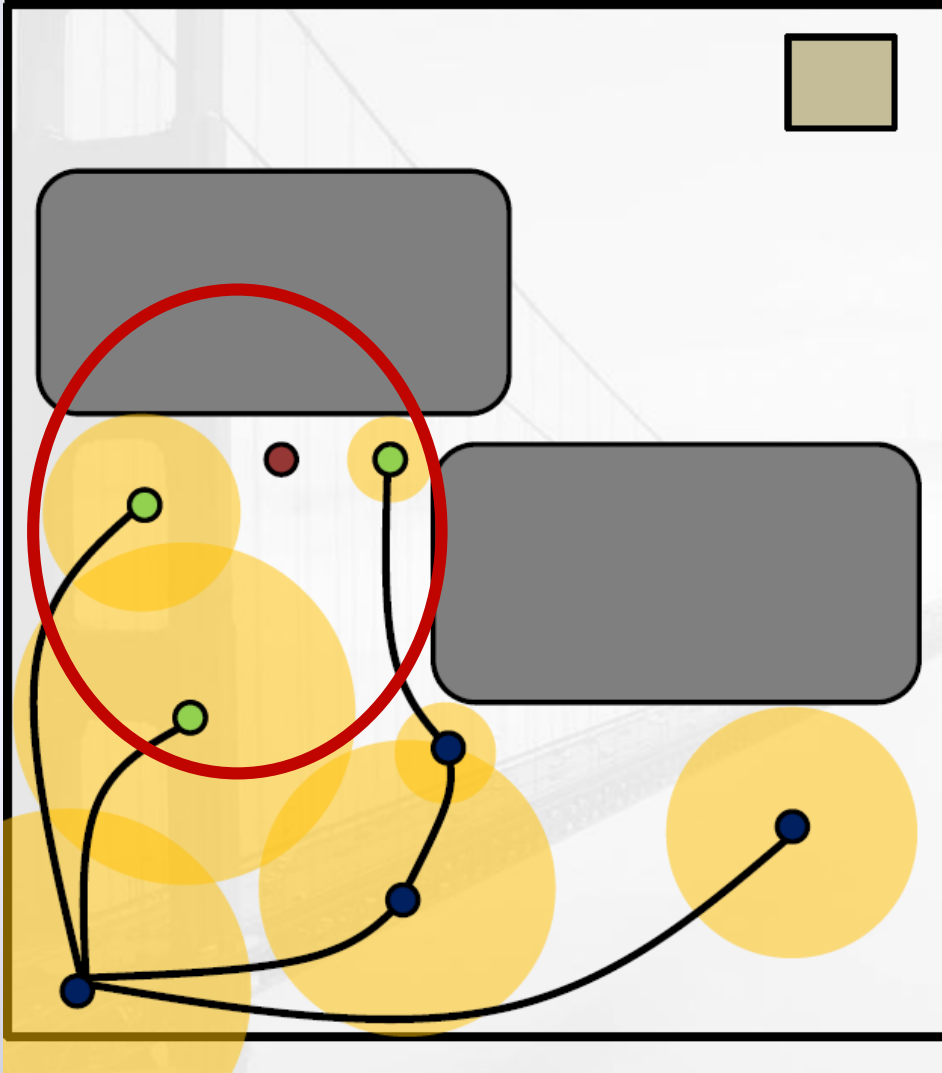


# Example: Re-Wiring Operation



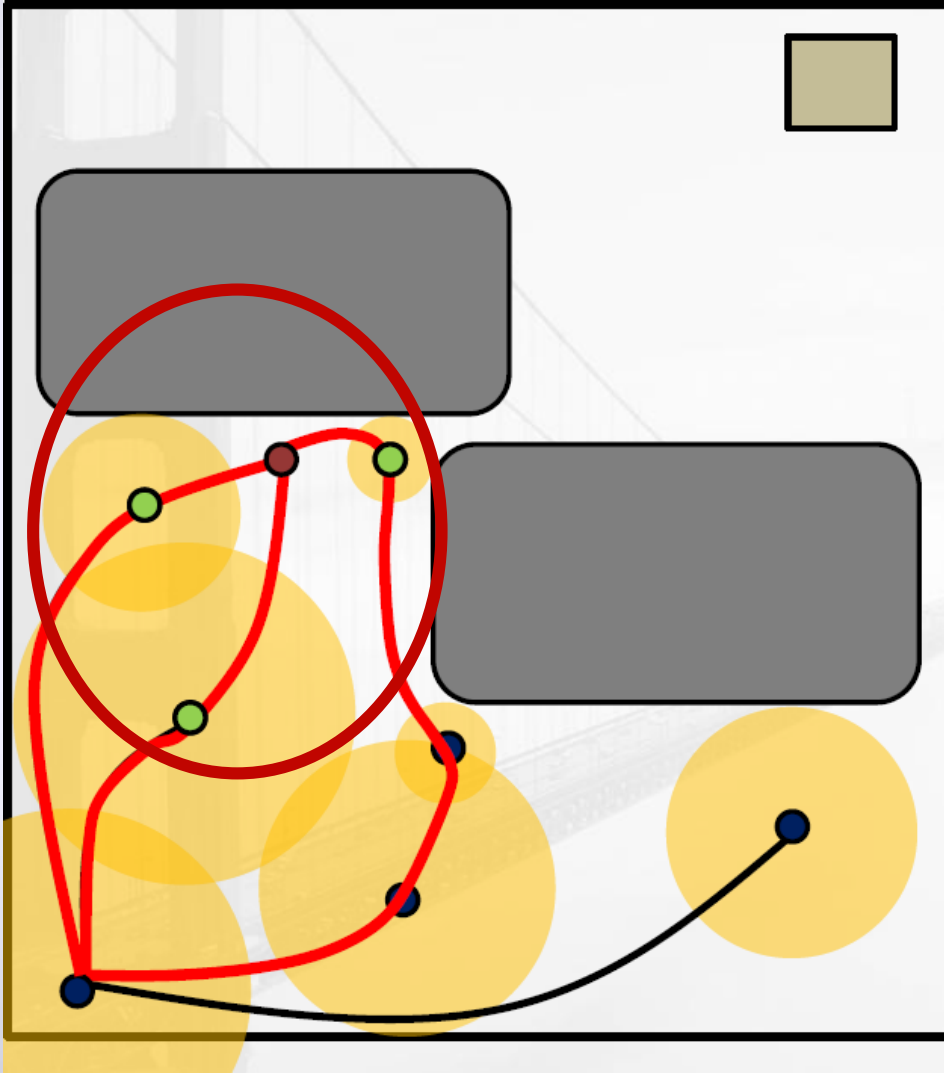
Generate a new sample

# Example: Re-Wiring Operation



Identify nodes in a ball

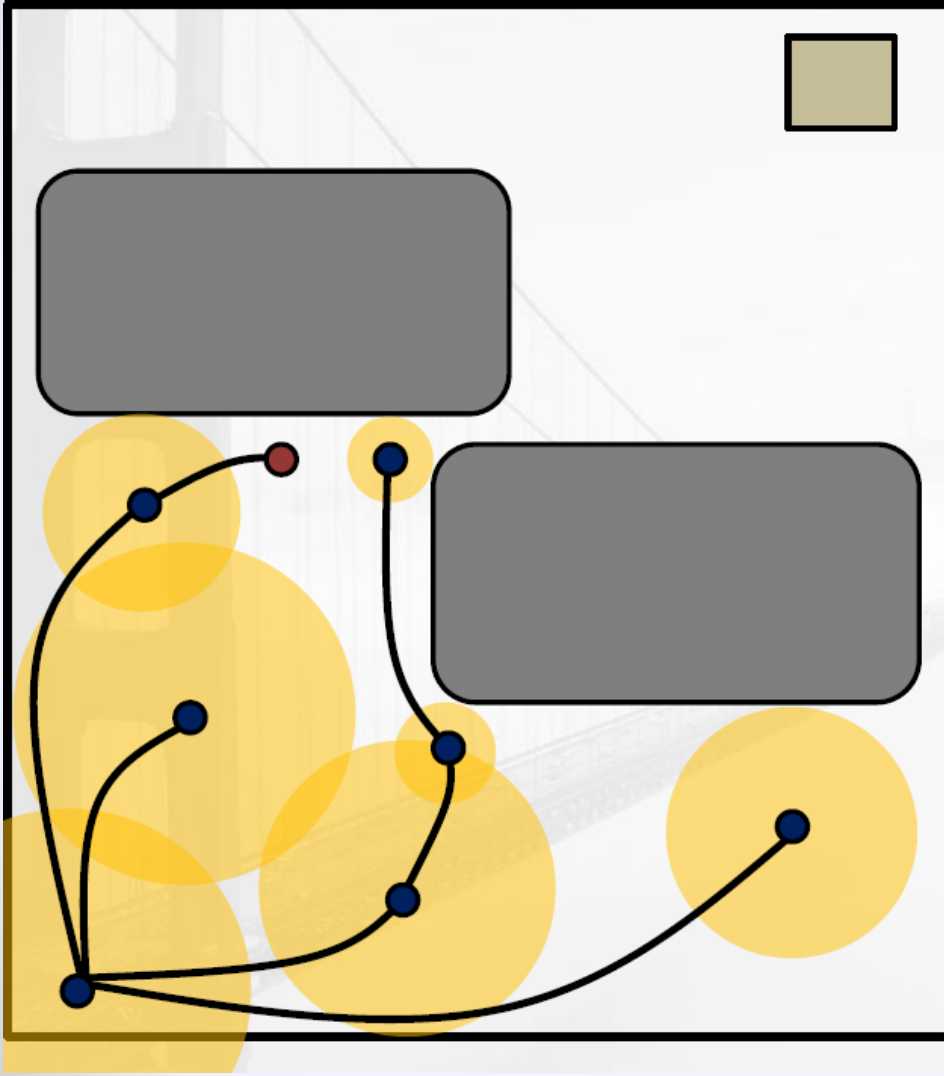
# Example: Re-Wiring Operation



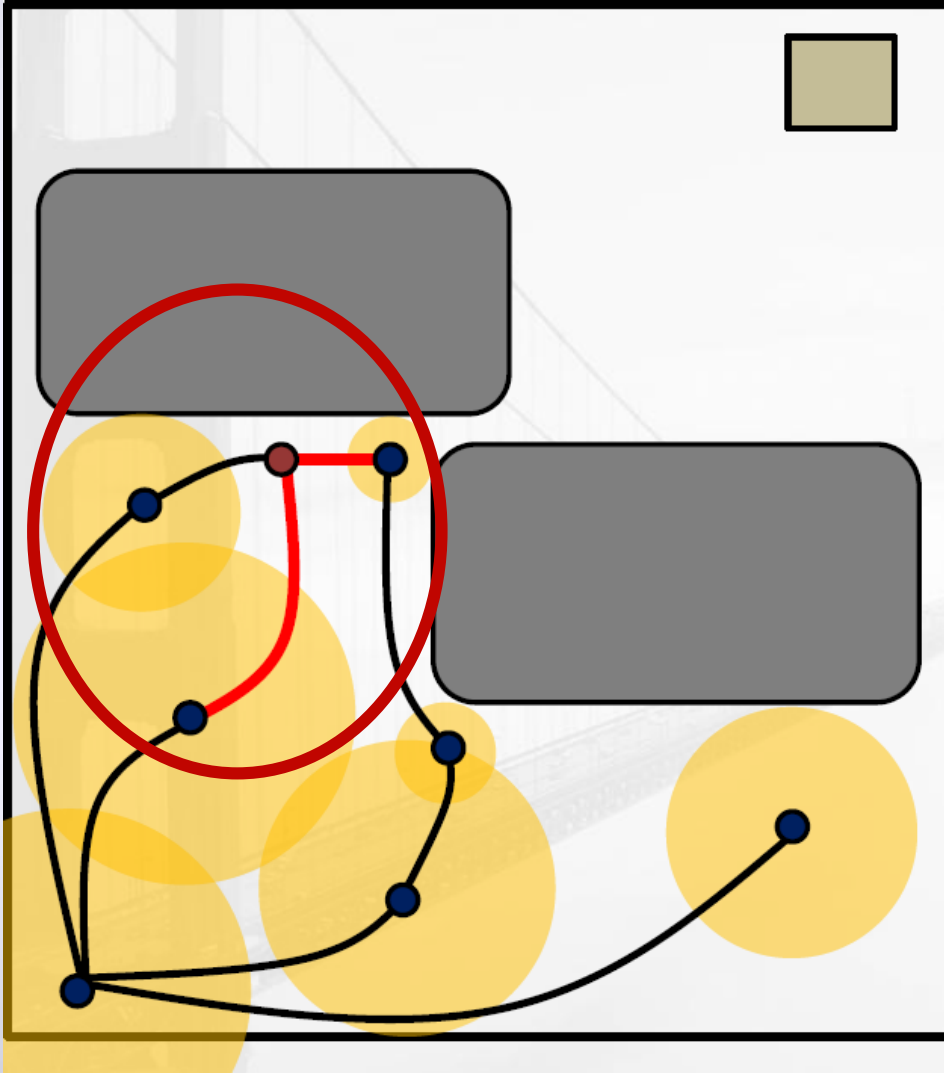
Identify which parent gives the lowest cost



# Example: Re-Wiring Operation

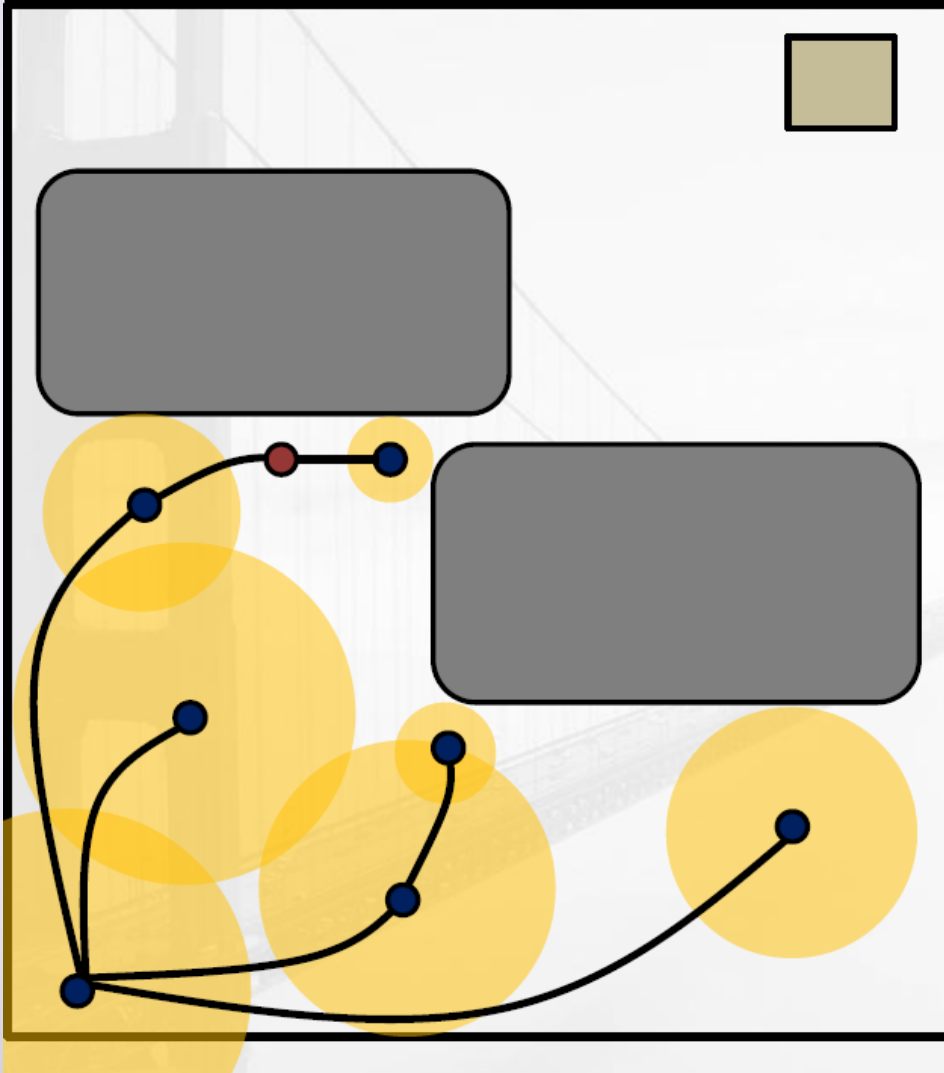


# Example: Re-Wiring Operation



Identify which child gives the lowest cost

# Example: Re-Wiring Operation



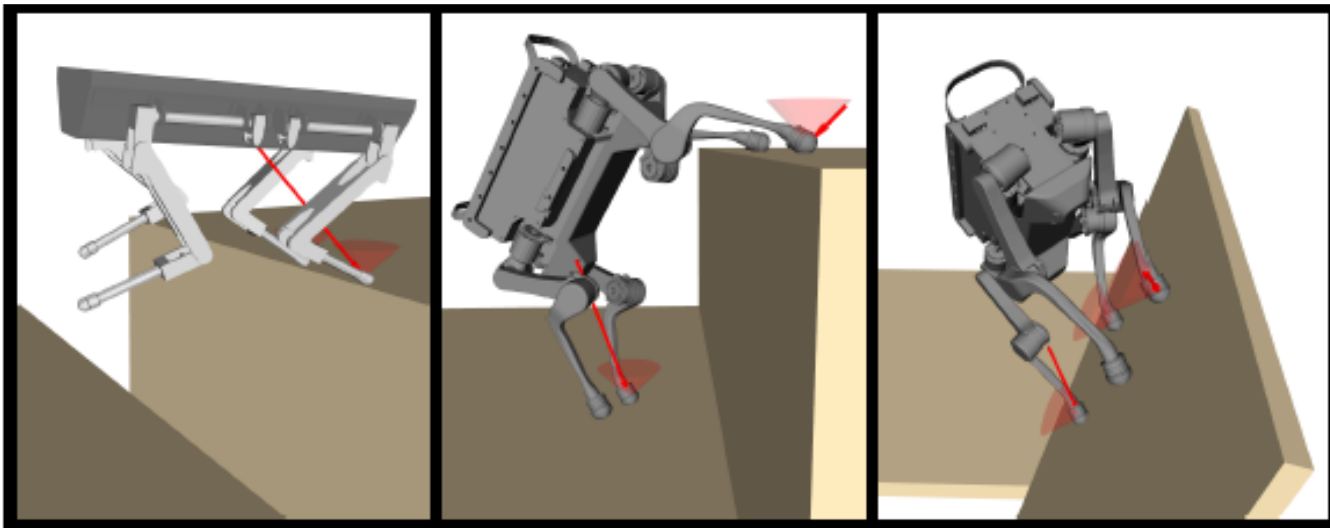
Video showing benefits  
with real robot

# Kinodynamic Path Planning

ALSO GIVEN:  $h_i(q, \dot{q}, \ddot{q}) \leq 0, h_i(q, \dot{q}, \ddot{q}) = 0, \dots$

FIND:  $\tau$  that satisfies  $f_i(q), g_i(q, \dot{q}), h_i(q, \dot{q}, \ddot{q})$

- Consider kinematic + dynamic constraints



Gait and Trajectory Optimization for Legged Systems through  
Phase-based End-Effector Parameterization

# State Space Formulation

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- **Kinodynamic planning**  $\rightarrow$  **2n-dimensional state space**

$C$  denote the  $C$ -space

$X$  denote the state space

$$x = (q, \dot{q}), \text{ for } q \in C, x \in X$$

$$x = \left[ q_1 \quad q_2 \quad \dots \quad q_n \quad \frac{dq_1}{dt} \quad \frac{dq_2}{dt} \quad \dots \quad \frac{dq_n}{dt} \right]$$

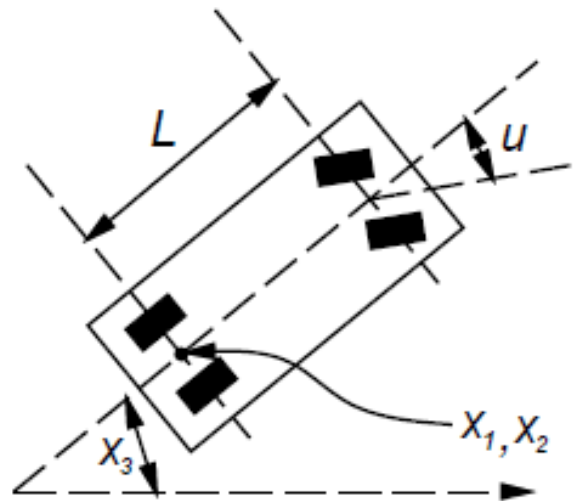
# Constraints in State Space

$h_i(q, \dot{q}, \ddot{q}) = 0$  becomes  $G_i(x, \dot{x}) = 0$ ,  
for  $i = 1, \dots, m$  and  $m < 2n$

- **Constraints can be written in:**

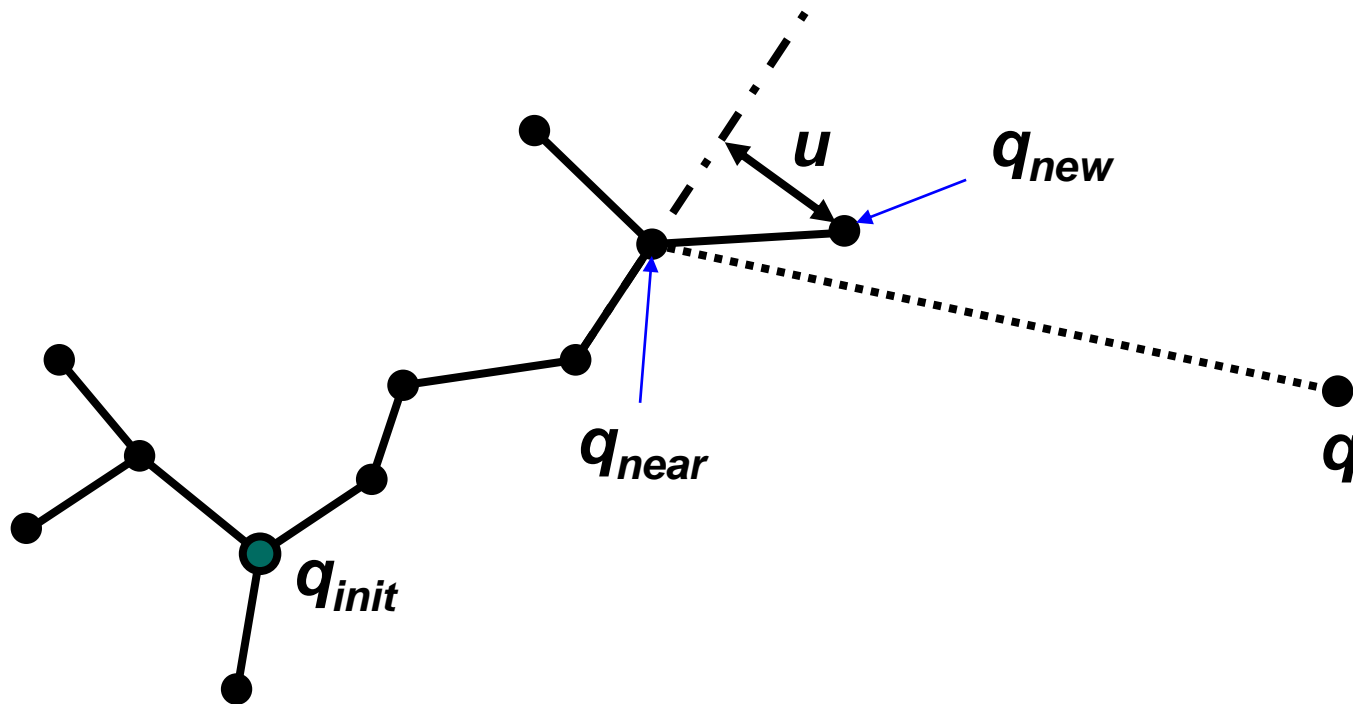
$$\dot{x} = f(x, u)$$

$u \in U$ ,  $U$  : Set of allowable controls or inputs

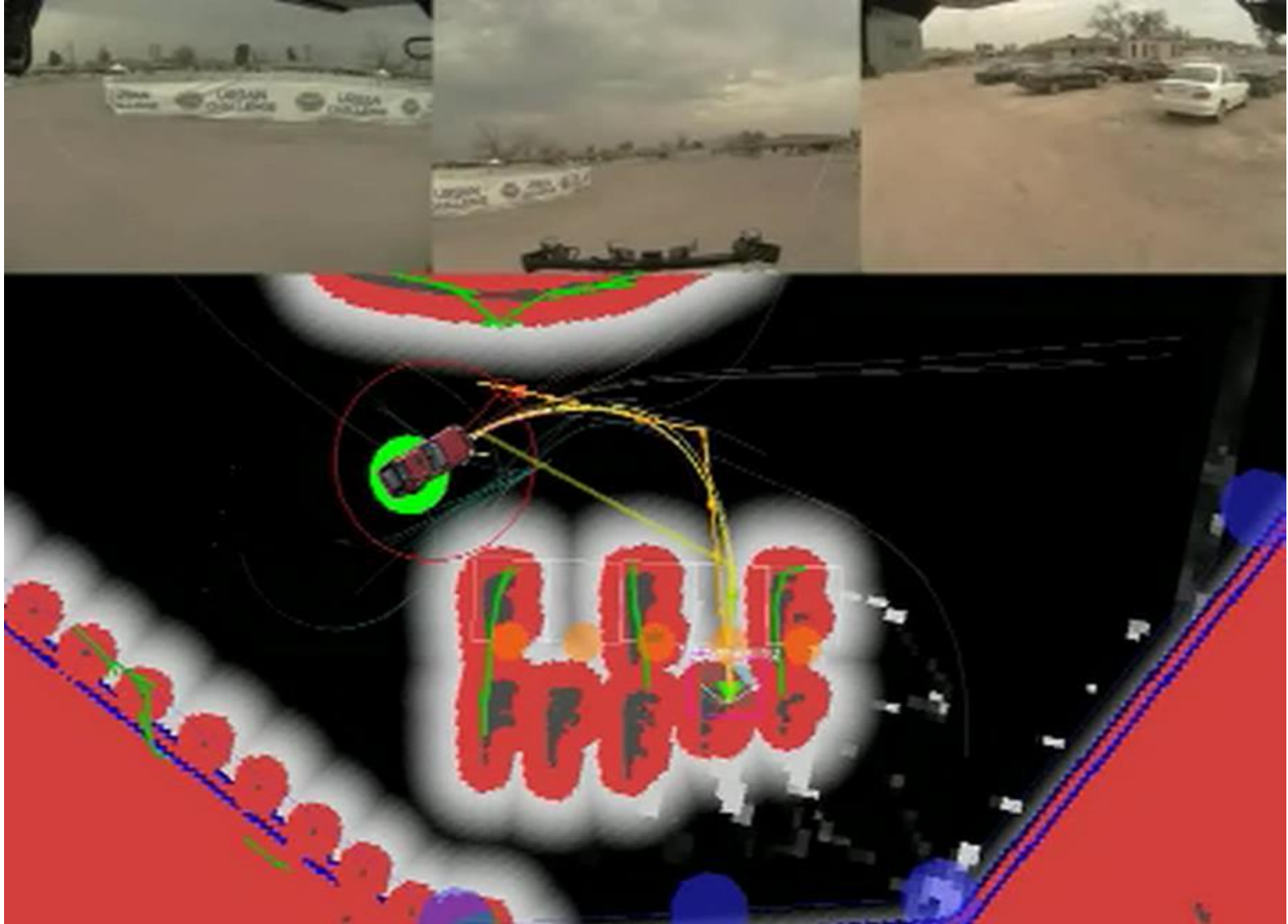


# Rapidly-Exploring Random Tree

- Extend a new vertex in each iteration



# RRT at work: Successful Parking Maneuver



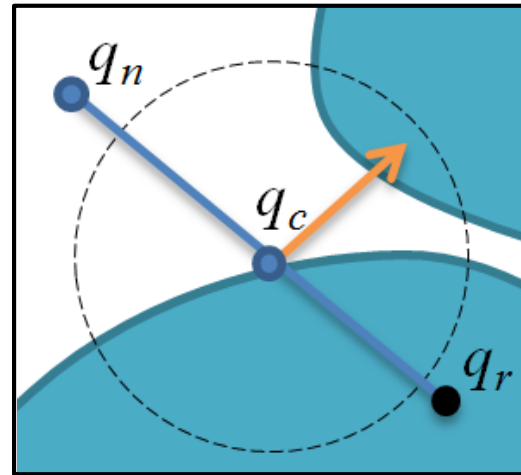
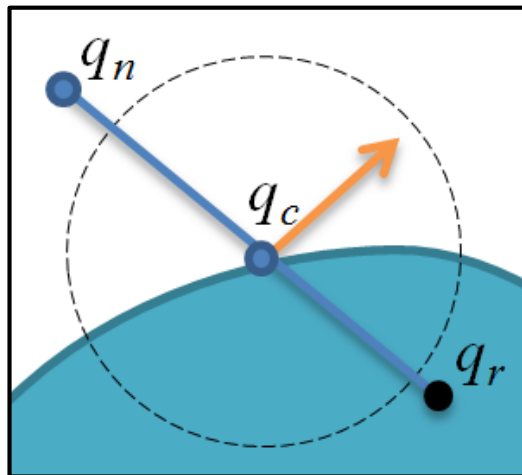


# Some Works of Our Group

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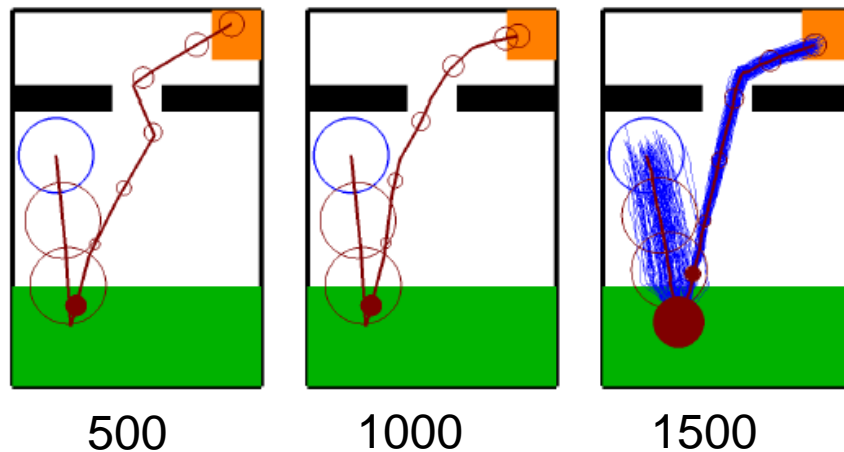
- **Narrow passages**

- **Identify narrow passage with a simple one-dimensional line test, and selectively explore such regions**
- **Selective retraction-based RRT planner for various environments, Lee et al., T-RO 14**
- **<http://sglab.kaist.ac.kr/SRRRT/T-RO.html>**



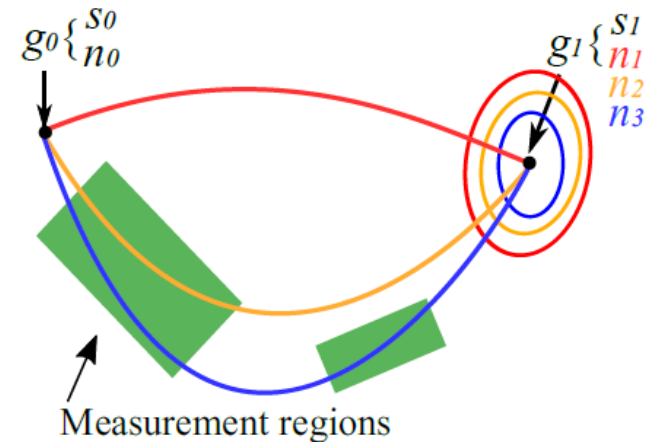
# Handling uncertainty and dynamic objects

- Anytime RRBT for handling uncertainty and dynamic objects, IROS 16



Number of iteration

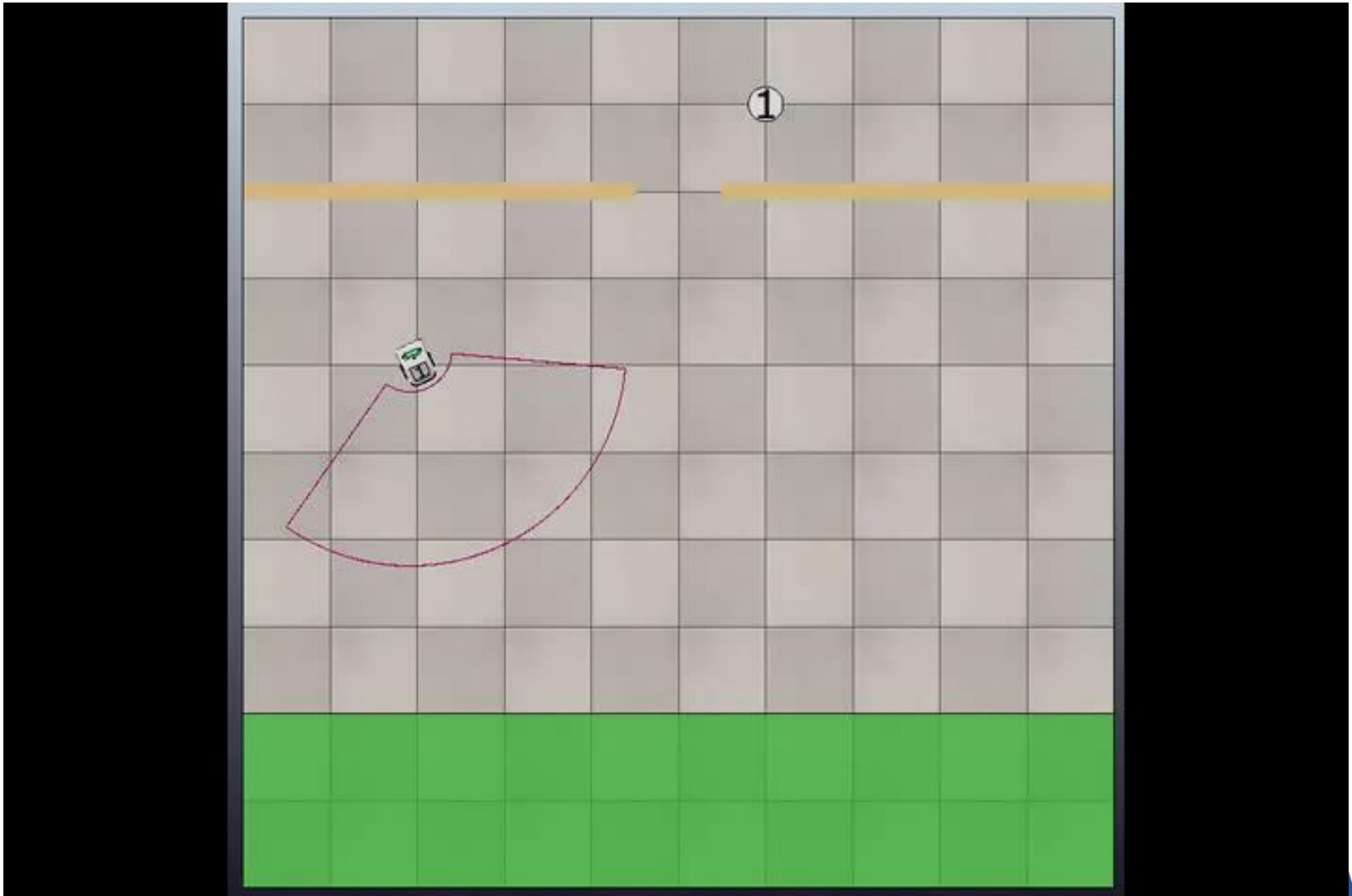
Multiple belief nodes in the same vertex



Preserve optimal path

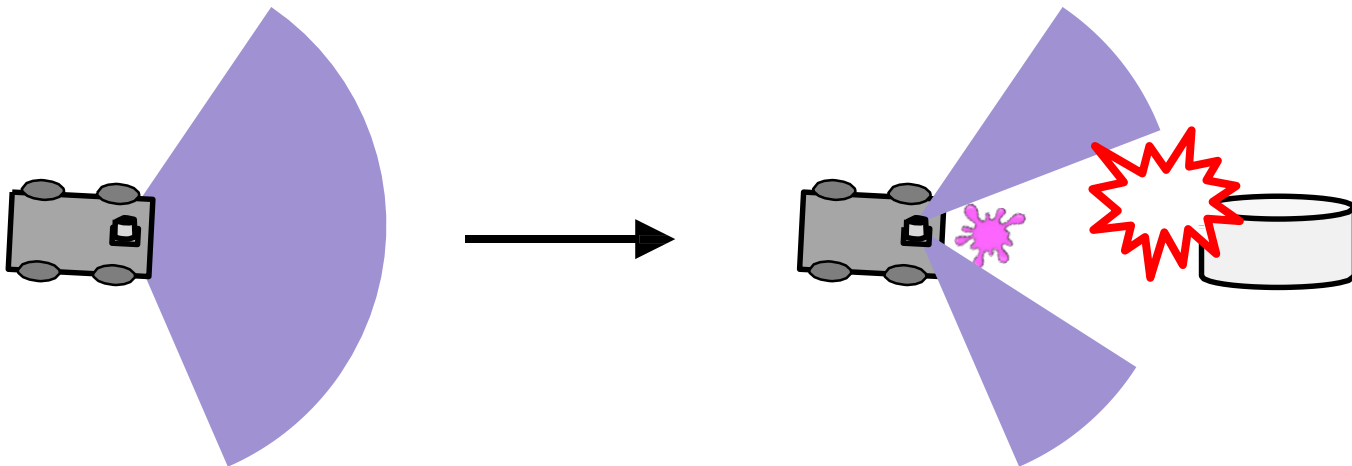
# Main Contribution: Anytime Extension

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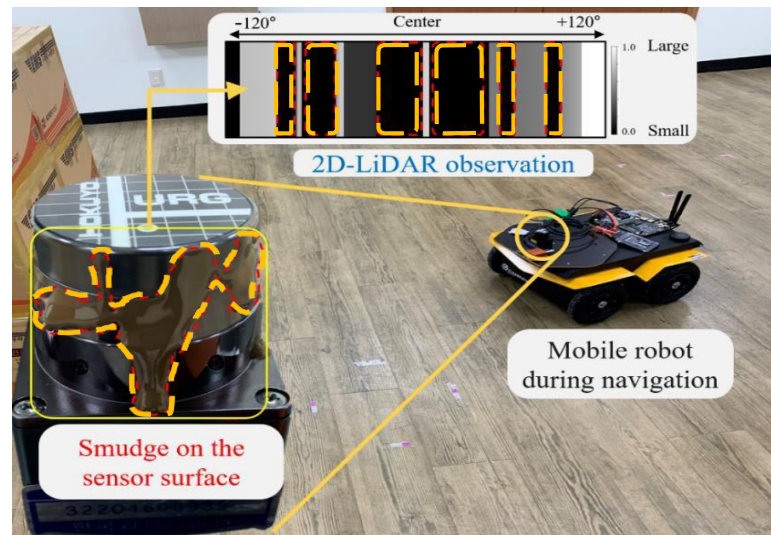
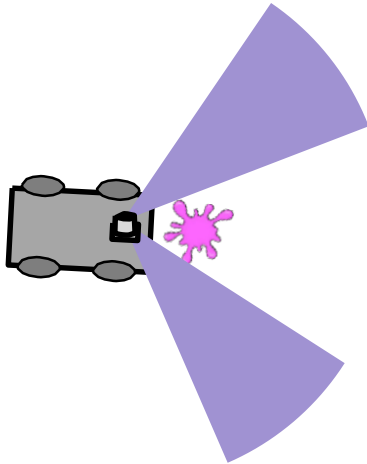
# Confidence-based Robot Navigation under Sensor Occlusion w/ Deep Reinforcement Learning, ICRA 22

- **Robot navigation under sensor occlusion**
  - **LiDAR based navigation often suffer from unexpected occlusion on (e.g., dust, water, or smudge) sensor surface**
  - **Such occlusion lowers the visibility of the sensor and might cause potential collisions.**



# Confidence-based Robot Navigation under Sensor Occlusion w/ Deep Reinforcement Learning, ICRA 22

- **Our goal**
  - **Build a robot navigation policy robust to such sensor occlusion**



Occlusions on the real sensor surface

Received Outstanding Navigation Award Finalist

# Hybrid Planning Techniques

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- **Traditional methods have been carefully designed and worked quite well in many cases**
- **Learning approaches are showing interesting success, yet have limitations such as data hungry, high computation, and handling global information**
- **Interesting to combine those two orthogonal approaches together!**

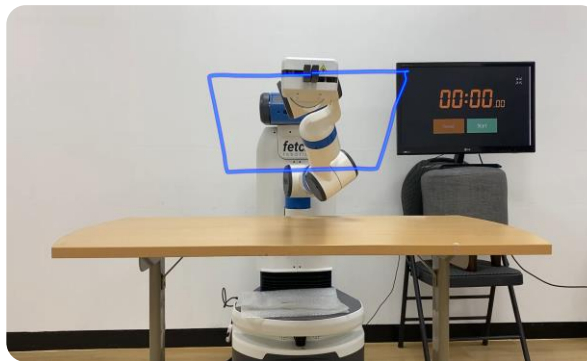
# Learning-based Initialization of Trajectory Optimization for Path-following Problems of Redundant Manipulators



- Problem Statement of Path-following Problems
- Generate a joint trajectory precisely following a given 6-dimensional Cartesian path (i.e., target path) with an end-effector.



Target path: 'Hello'



Target path: 'Square'



Target path: 'Zigzag'

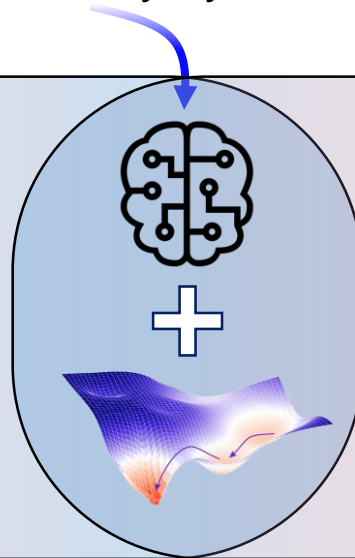
# Take-home Message



- **Integrating learning and planning** is an important strategy that works in a complementary manner.
  - Improves accuracy and efficiency by combining the two approaches.

## Learning-based methods

- may not guarantee optimality
- but offer a good starting point for optimization quickly.



## Optimization-based methods

- may struggle with high-dimensional and non-convex problems
- but find optimal solutions around starting point by iterative refinement.



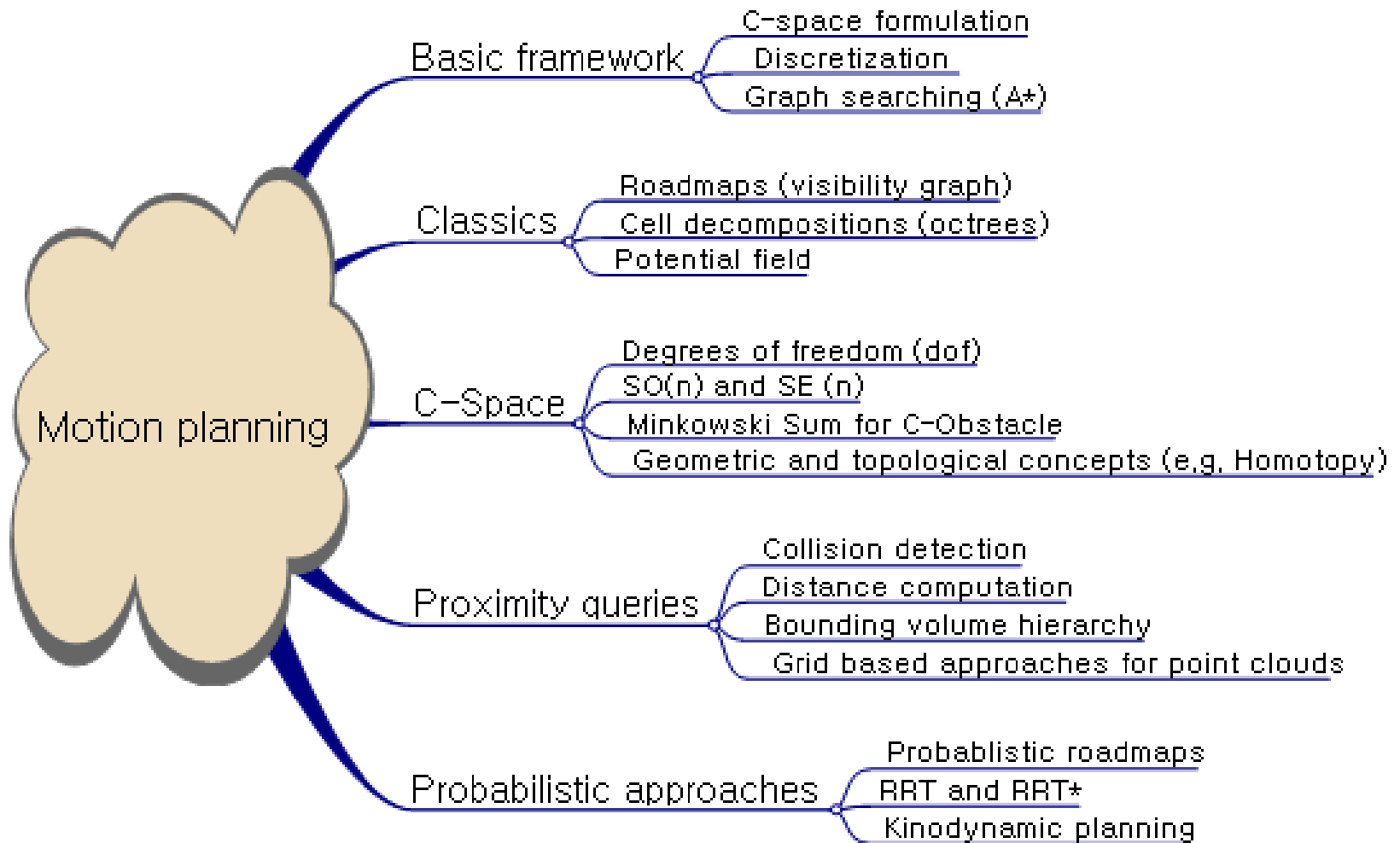


# Class Objectives were:

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- **Understand the RRT technique and its recent advancements**
  - **RRT\* for optimal path planning**
  - **Kinodynamic planning**
  - **Some related techniques to RRT**

# Summary



# Next Time..

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- **Basic concepts of reinforcement learning**

# Homework for Every Class

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- **Submit summaries of 2 ICRA/IROS/RSS/CoRL/TRO/IJRR papers**
- **Go over the next lecture slides**
- **Come up with two question submissions before the mid-term exam**