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# CS686: RRT

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**Course URL:**  
<http://sgvr.kaist.ac.kr/~sungeui/MPA>

**KAIST**



# Class Objectives

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- **Understand the RRT technique and its recent advancements**
  - **RRT\***
  - **Kinodynamic planning**
- **Last time**
  - **Probabilistic roadmap techniques**
  - **Sampling and re-sampling techniques**

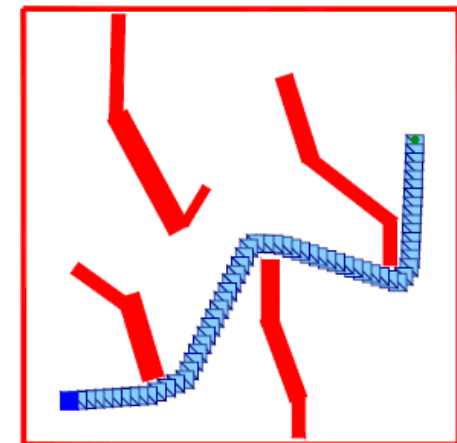
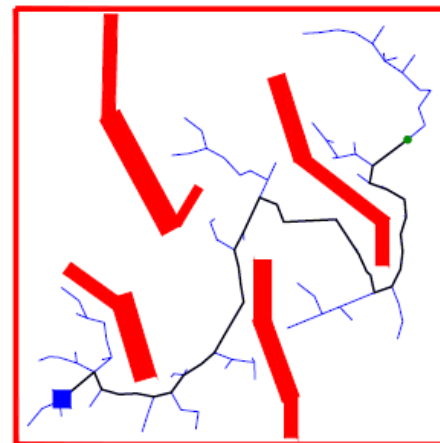
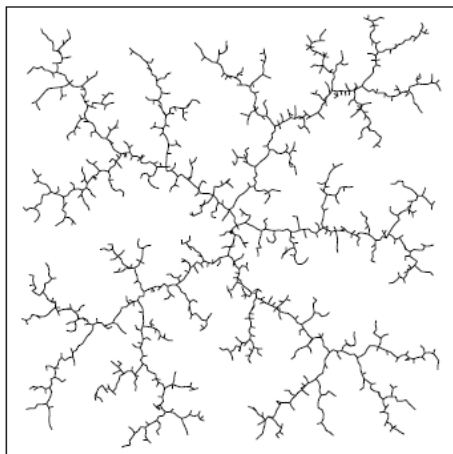
# Question

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- **PRM assumes that we know the global map, but how can we handle the case where we have only a partial map due to the limited sensor range?**
  - **지난시간에 배운 PRM 기법들은 글로벌 맵을 알고 있어야 문제 해결이 가능한데, 전체 맵의 일부분(센서 탐지거리 제약 등으로)만을 알고 있는 상황에서 PRM 알고리즘을 적용하려면 어떤 방식으로 해야 하는지요?**

# Rapidly-exploring Random Trees (RRT) [LaValle 98]

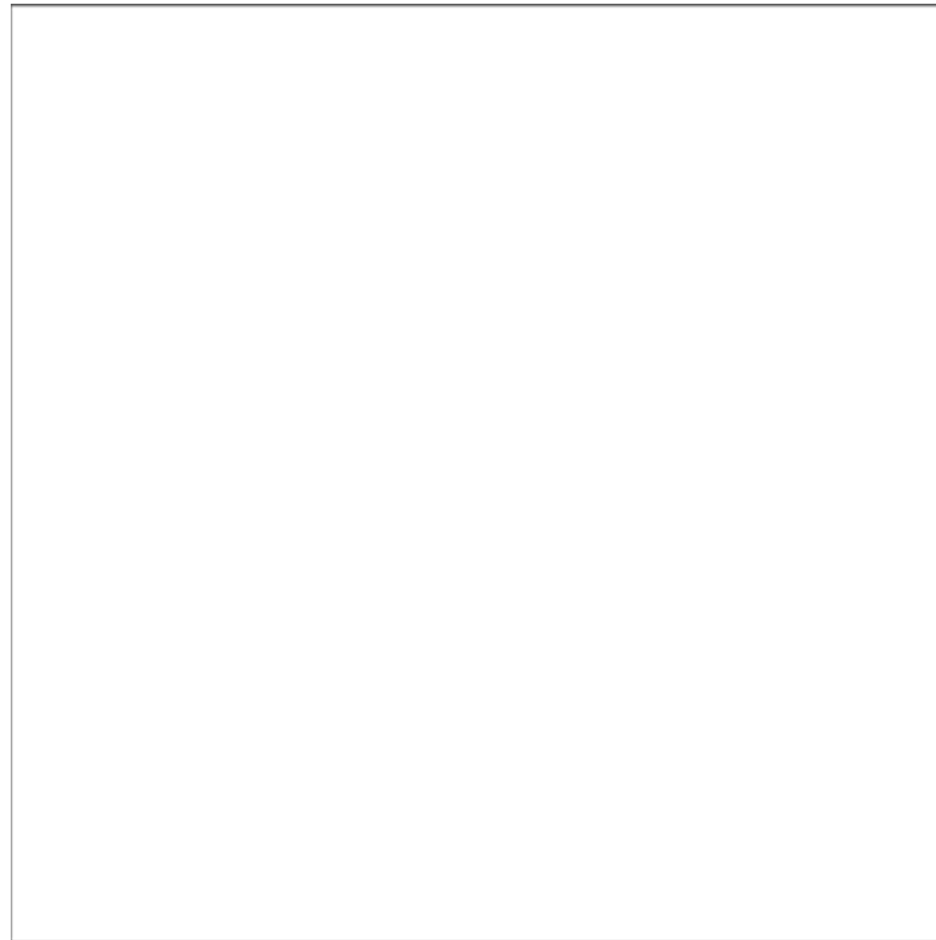
- Present an efficient randomized path planning algorithm for single-query problems
  - Converges quickly
  - Probabilistically complete
  - Works well in high-dimensional C-space



# Rapidly-Exploring Random Tree

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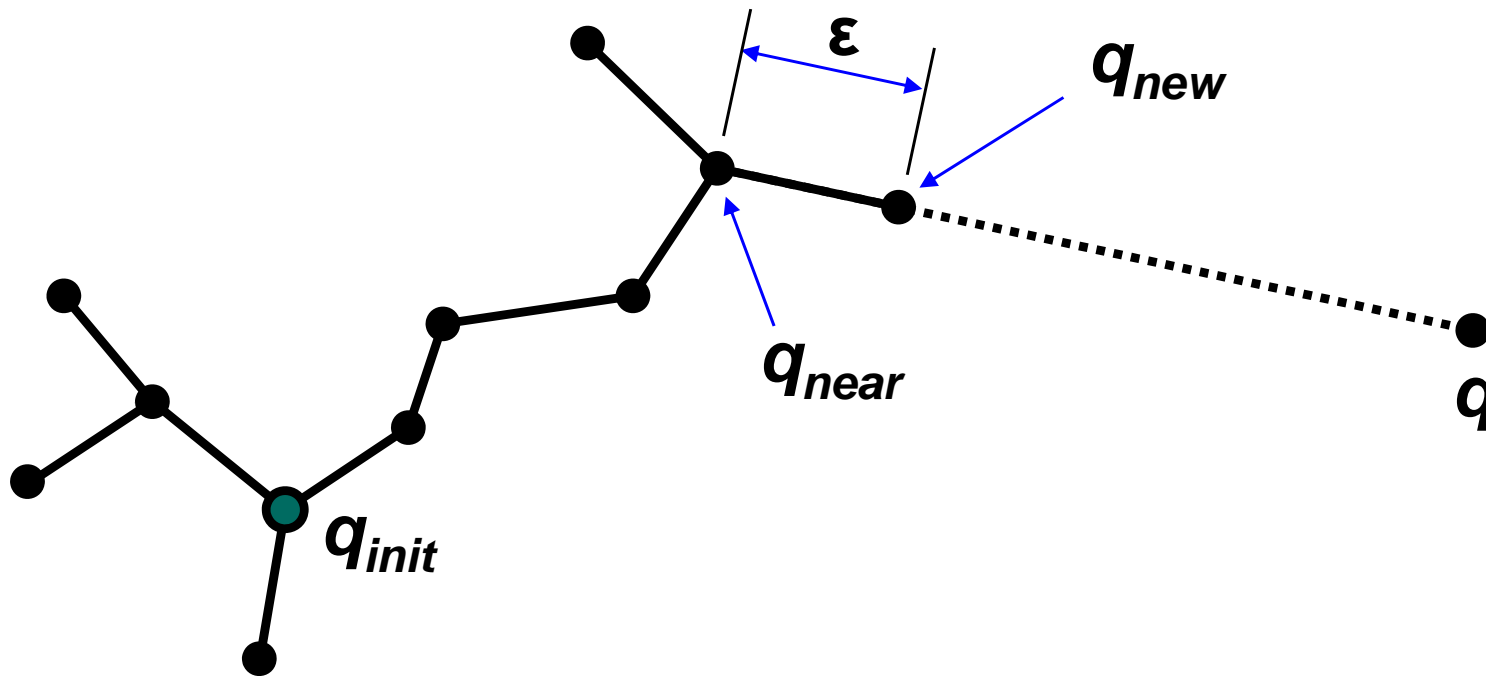
- A growing tree from an initial state



# RRT Construction Algorithm

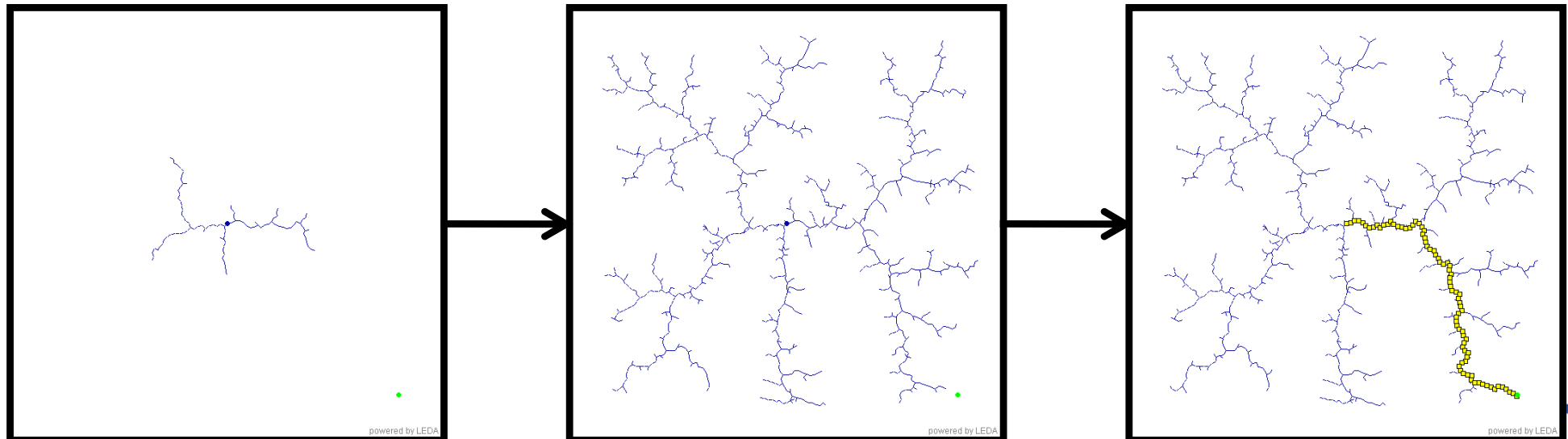
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- **Extend a new vertex in each iteration**
  - **Alternatively, one can simply connect**



# Overview – Planning with RRT

- **Extend RRT until a nearest vertex is close enough to the goal state**
  - **Can handle nonholonomic constraints and high degrees of freedom**
- **Probabilistically complete, but does not converge to the optimal one**



# RRT Construction Algorithm

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BUILD\_RRT( $q_{init}$ )

```
1   $\mathcal{T}$ .init( $q_{init}$ );
2  for  $k = 1$  to  $K$  do
3       $q_{rand} \leftarrow$  RANDOM_CONFIG();
4      EXTEND( $\mathcal{T}$ ,  $q_{rand}$ );
5  Return  $\mathcal{T}$ 
```

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EXTEND( $\mathcal{T}$ ,  $q$ )

```
1   $q_{near} \leftarrow$  NEAREST_NEIGHBOR( $q$ ,  $\mathcal{T}$ );
2  if NEW_CONFIG( $q$ ,  $q_{near}$ ,  $q_{new}$ ) then
3       $\mathcal{T}$ .add_vertex( $q_{new}$ );
4       $\mathcal{T}$ .add_edge( $q_{near}$ ,  $q_{new}$ );
5      if  $q_{new} = q$  then
6          Return Reached;
7      else
8          Return Advanced;
9  Return Trapped;
```

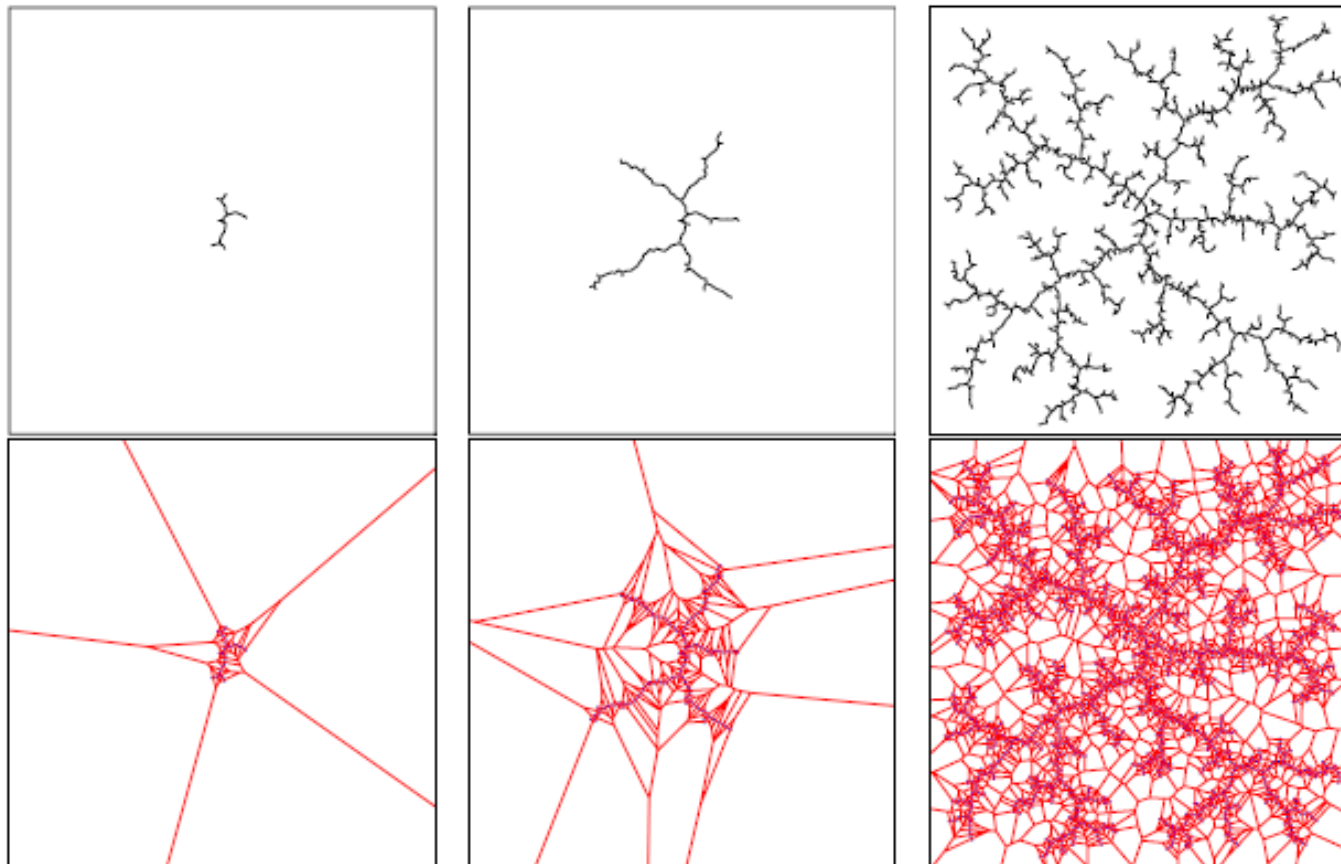
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# Voronoi Region

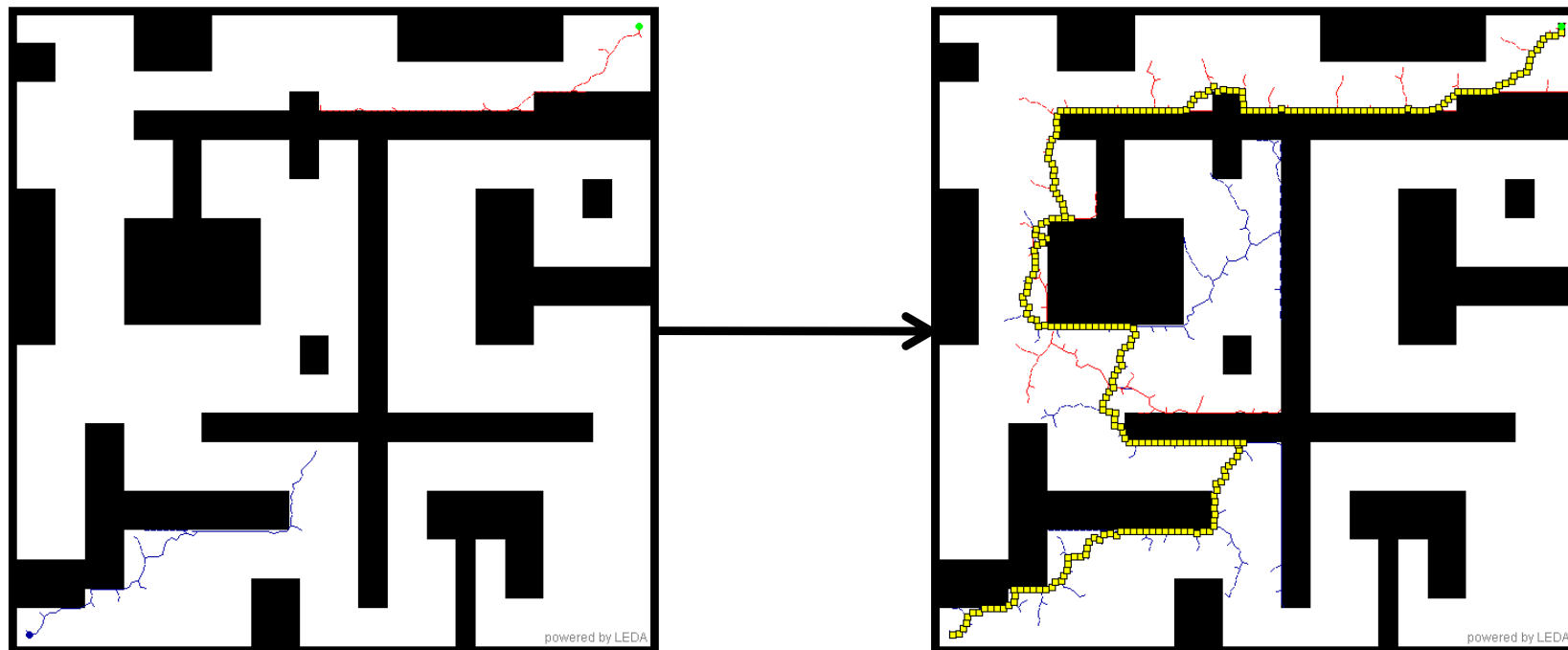
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- **An RRT is biased by large Voronoi regions to rapidly explore, before uniformly covering the space**



# Overview – With Dual RRT

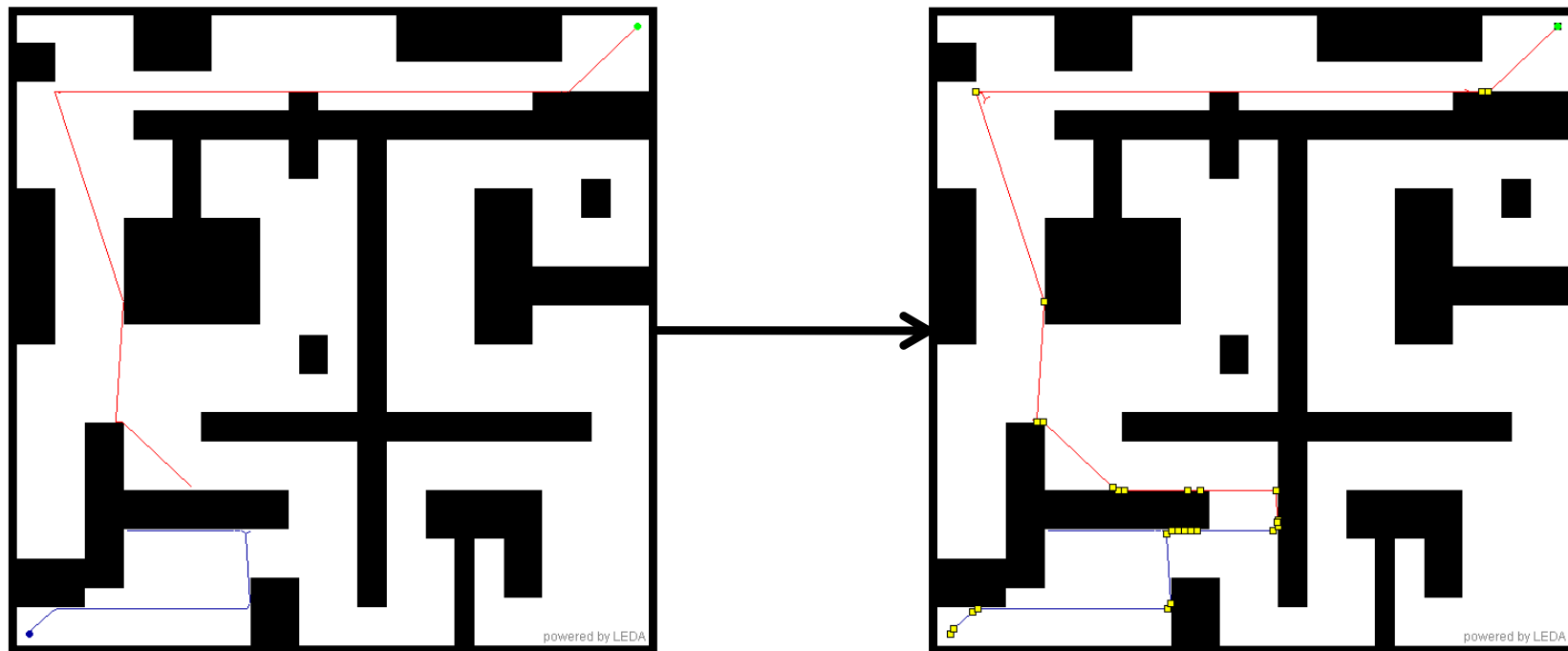
- **Extend RRTs from both initial and goal states**
- **Find path much more quickly**



737 nodes are used

# Overview – With RRT-Connect

- **Aggressively connect the dual trees using a greedy heuristic**
- **Extend & connect trees alternatively**

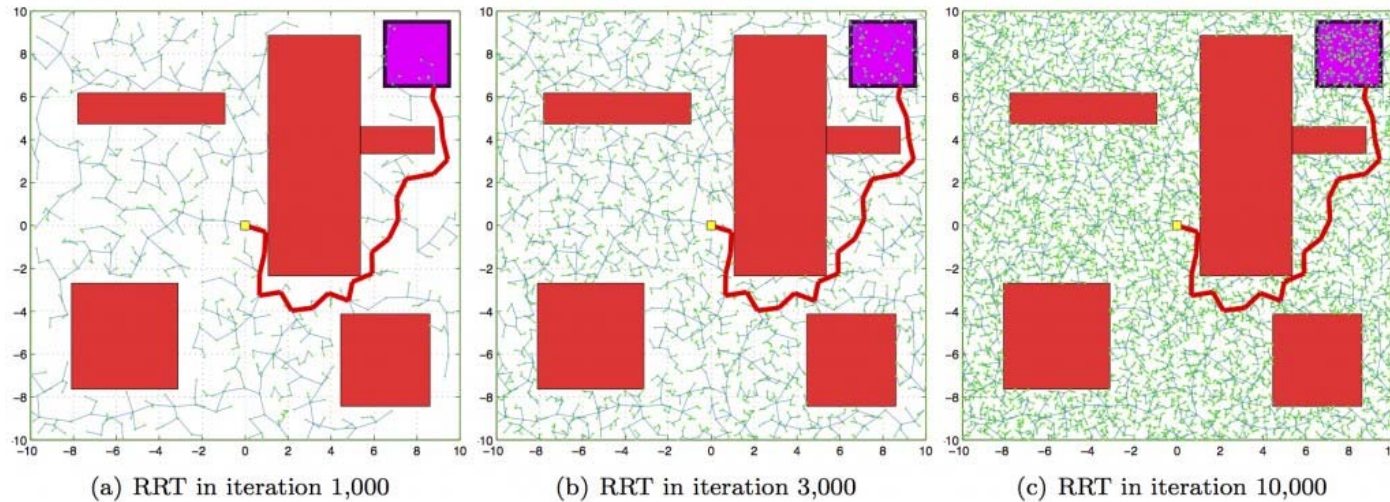


42 nodes are used

# RRT\*

- RRT does not converge to the optimal solution

RRT



RRT\*

# RRT\*

- **Asymptotically optimal without a substantial computational overhead**

**Theorem [Karaman & Frazzoli, IJRR 2011]**

(i) The RRT\* algorithm is asymptotically optimal

$$\mathbb{P}\left(\left\{\lim_{n \rightarrow \infty} Y_n^{\text{RRT}^*} = c^*\right\}\right) = 1$$

(ii) RRT\* algorithm has no substantial computational overhead when compared to the RRT:

$$\lim_{n \rightarrow \infty} \mathbb{E} \left[ \frac{M_n^{\text{RRT}^*}}{M_n^{\text{RRT}}} \right] = \text{constant}$$

- $Y_n^{\text{RRT}^*}$  : cost of the best path in the RRT\*
- $c^*$  : cost of an optimal solution
- $M_n^{\text{RRT}}$  : # of steps executed by RRT at iteration n
- $M_n^{\text{RRT}^*}$  : # of steps executed by RRT\* at iteration n

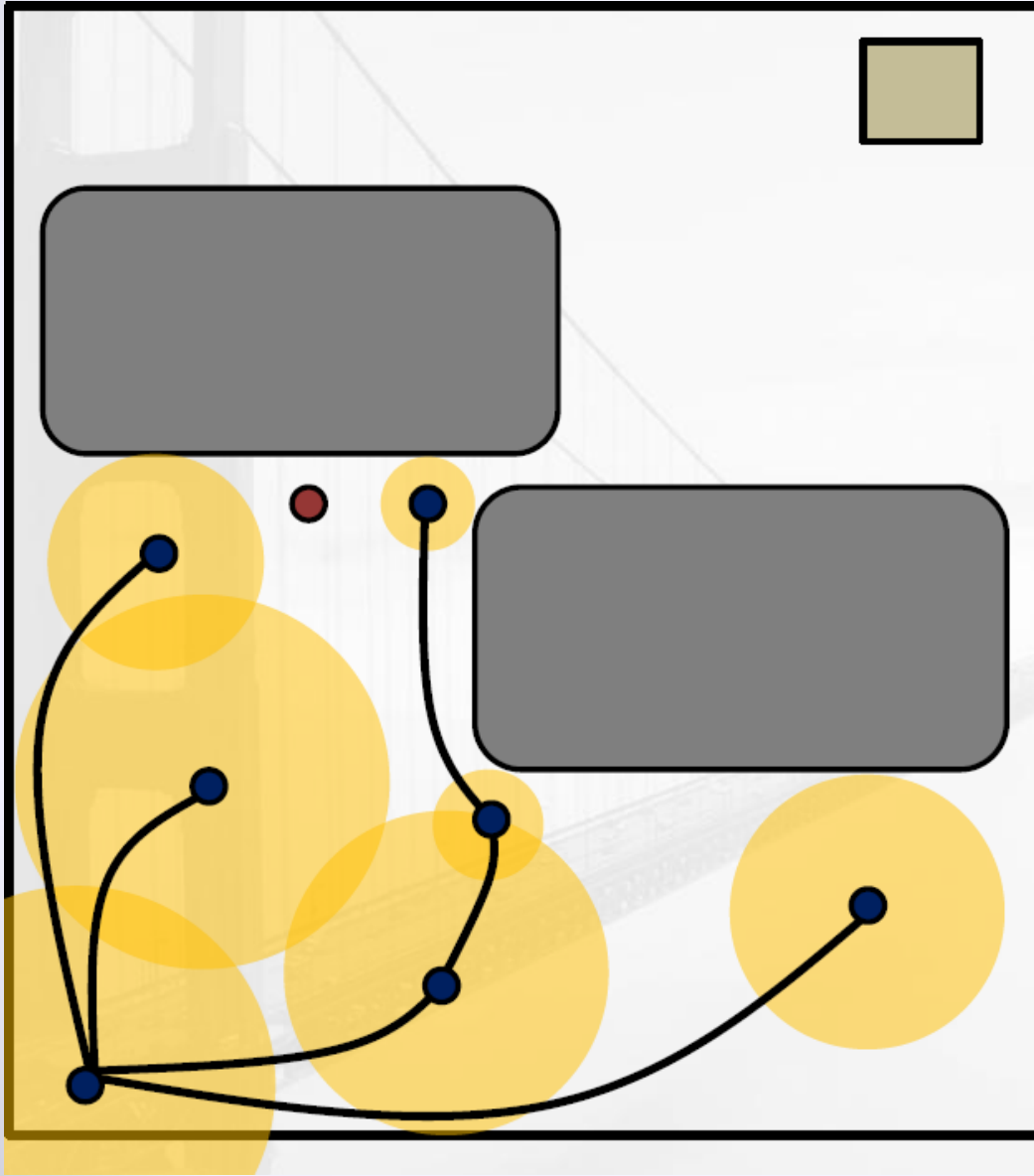
# Key Operation of RRT\*

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- **RRT**
  - **Just connect a new node to its nearest neighbor node**
- **RRT\*: refine the connection with re-wiring operation**
  - **Given a ball, identify neighbor nodes to the new node**
  - **Refine the connection to have a lower cost**



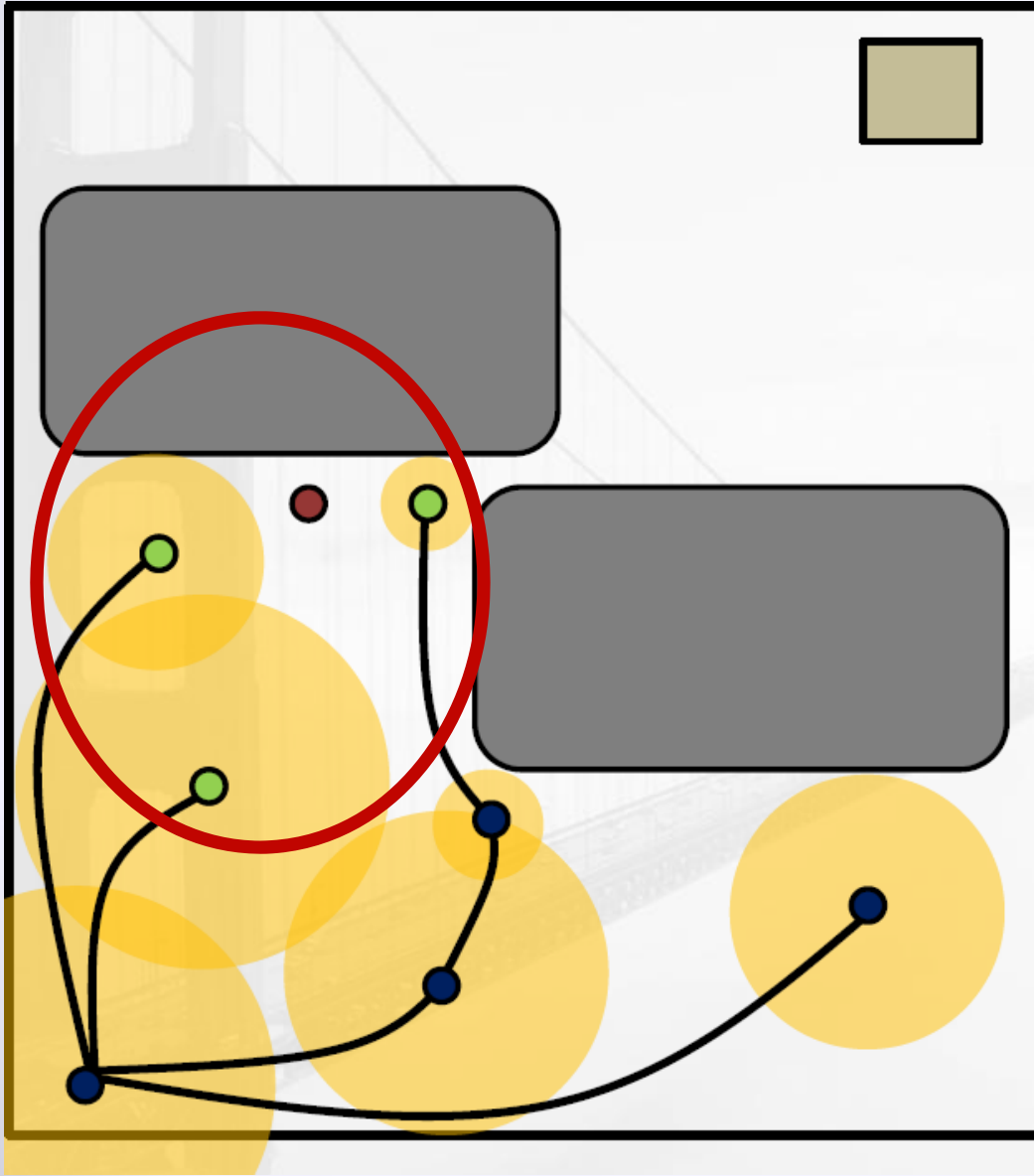
# Example: Re-Wiring Operation



Generate a new sample

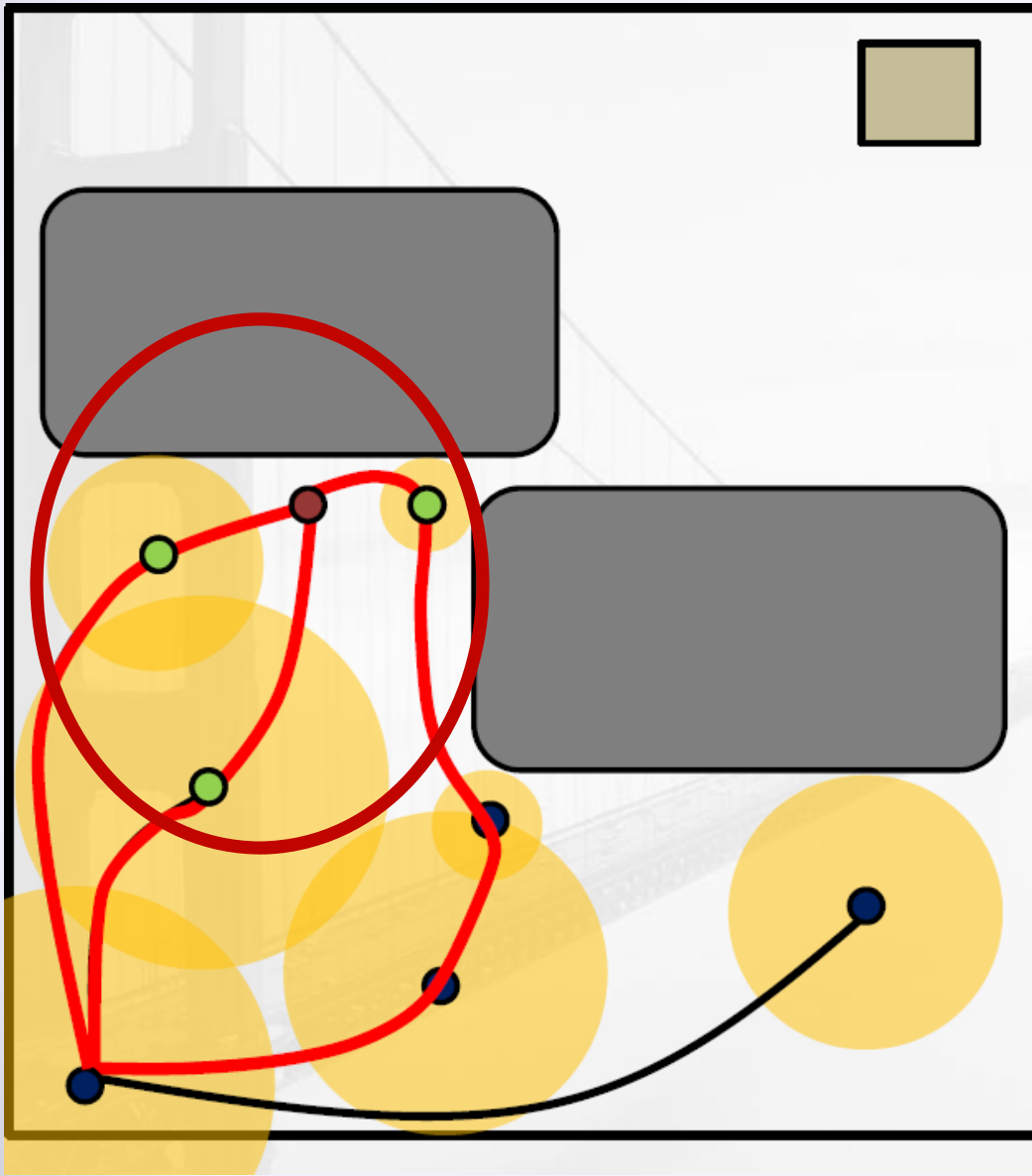


# Example: Re-Wiring Operation



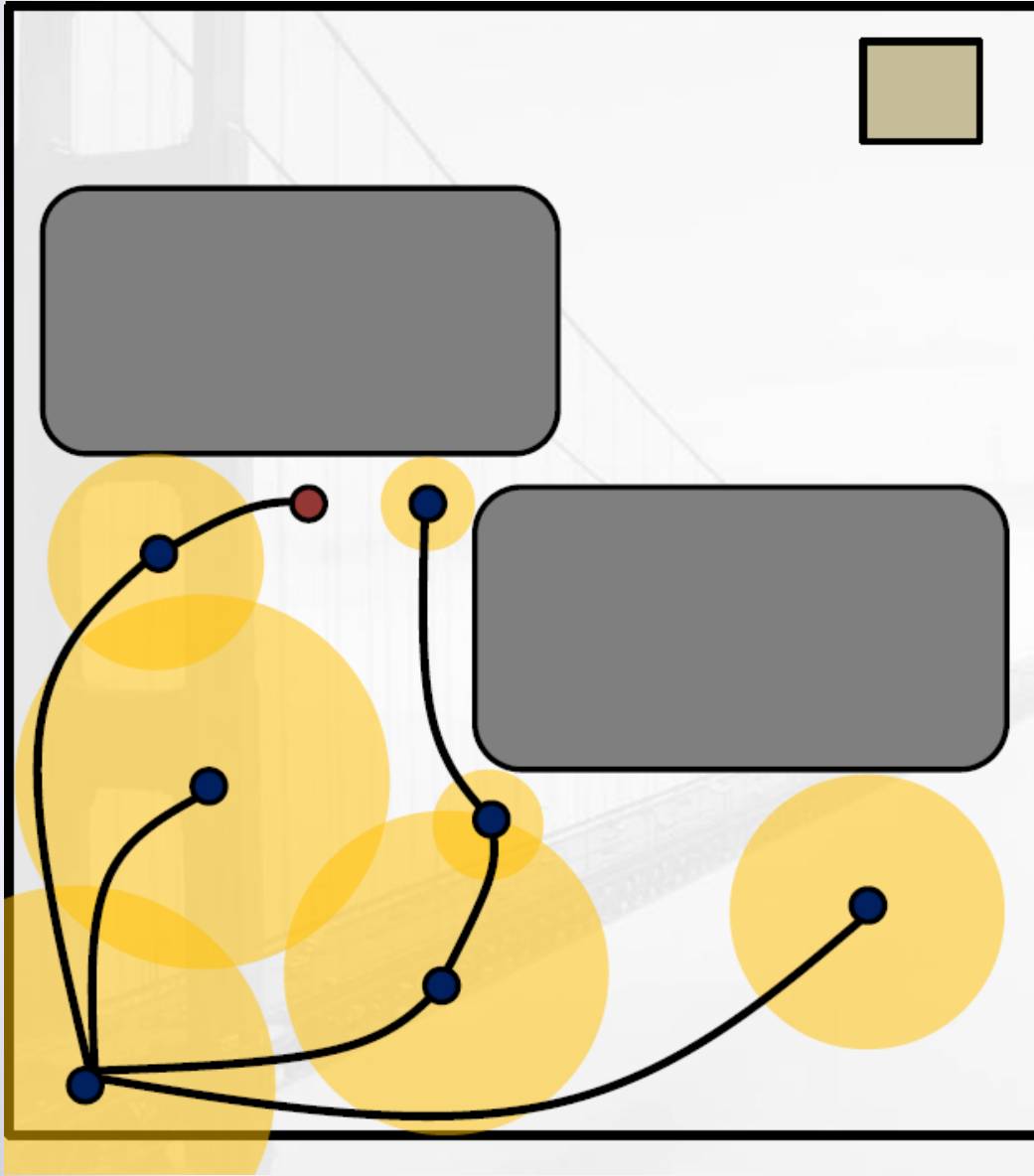
Identify nodes in a ball

# Example: Re-Wiring Operation

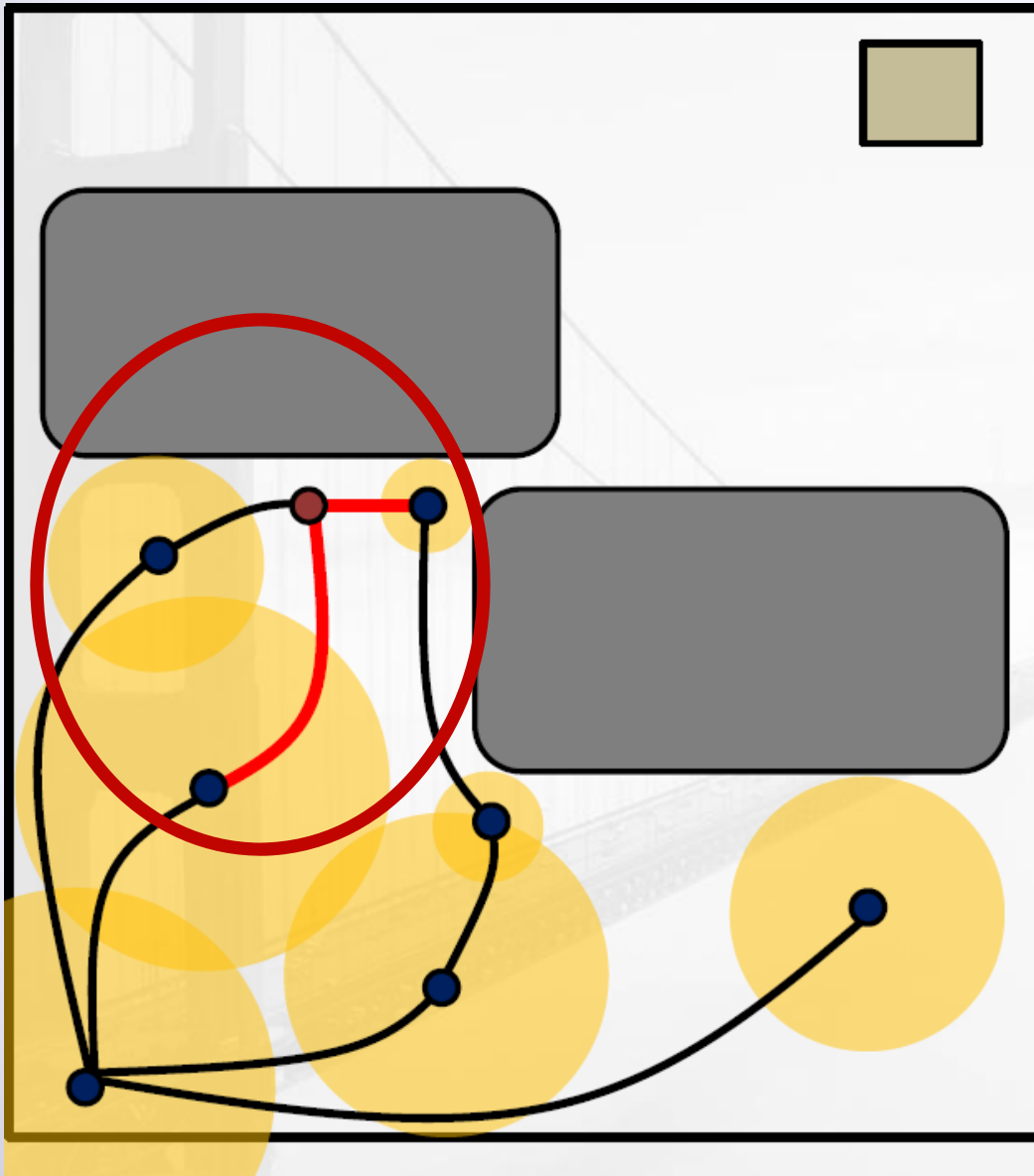


Identify which parent gives the lowest cost

# Example: Re-Wiring Operation

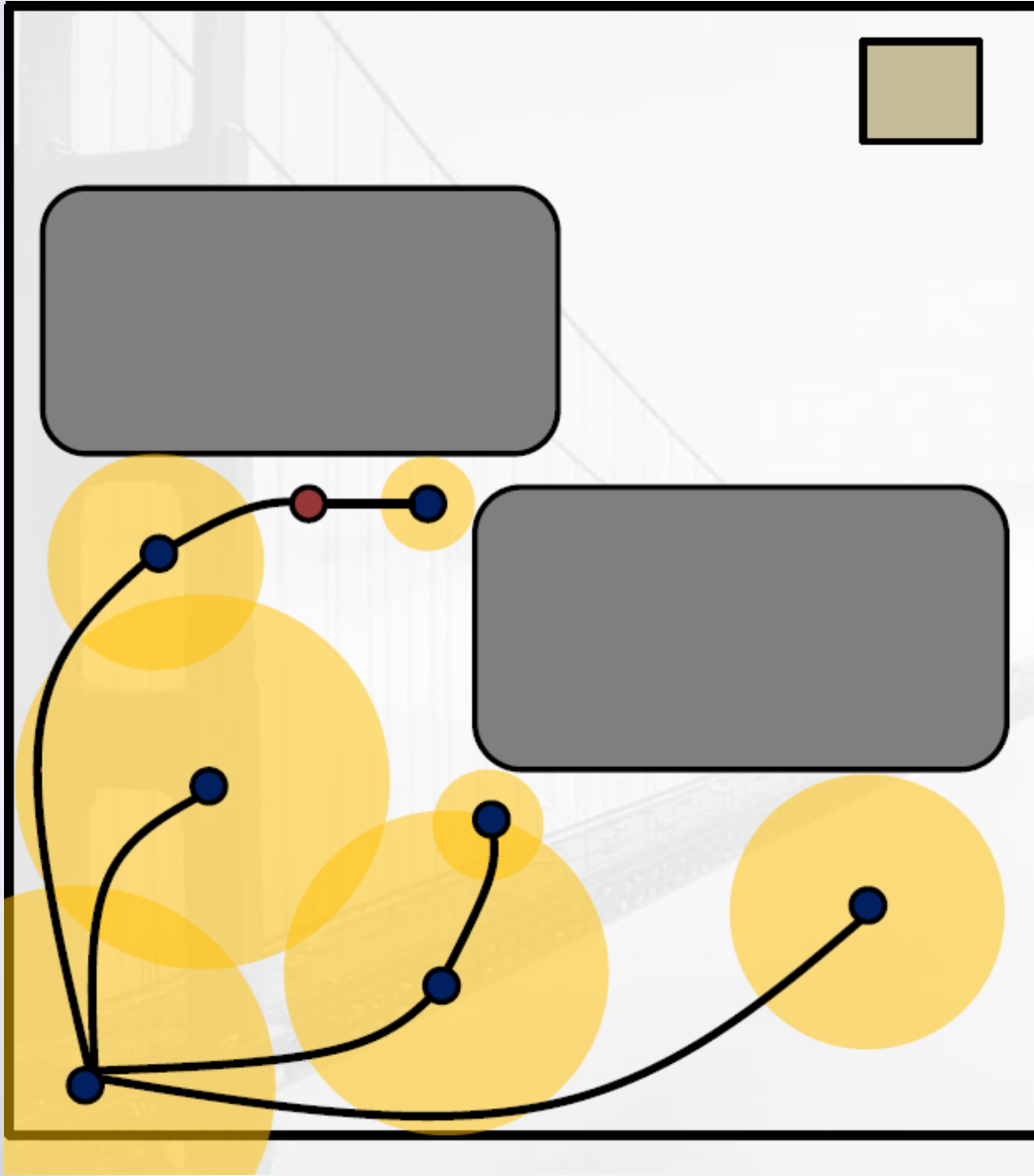


# Example: Re-Wiring Operation



Identify which child  
gives the lowest cost

# Example: Re-Wiring Operation



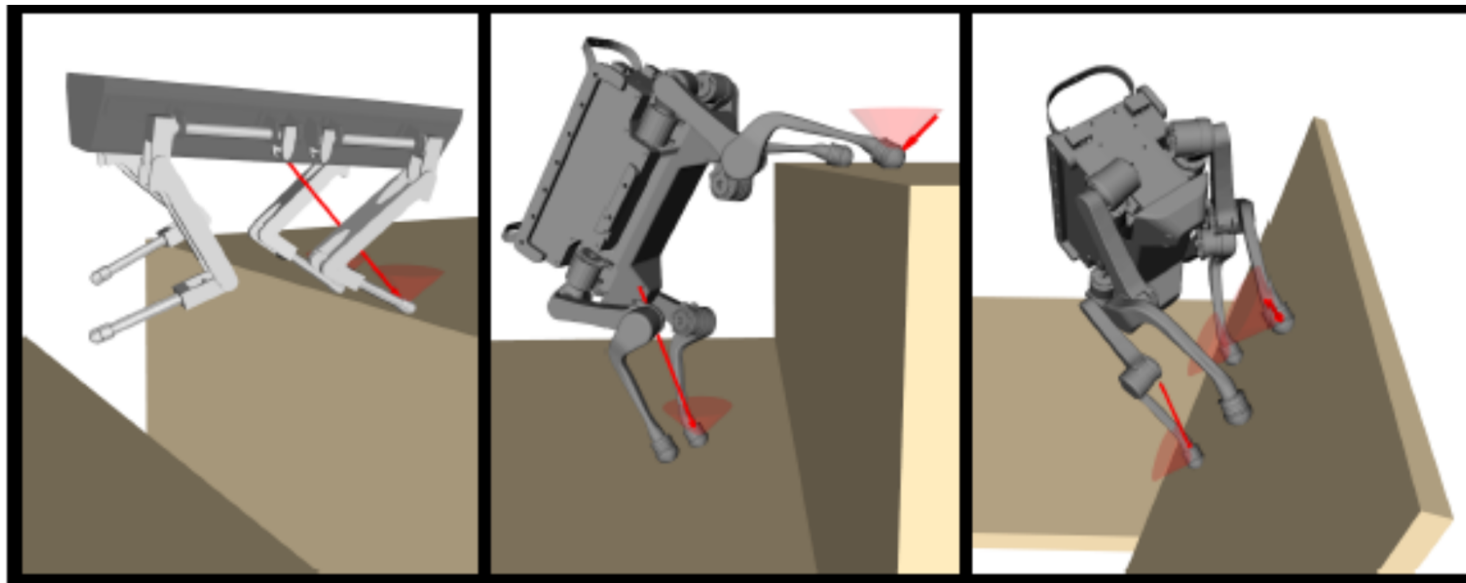
Video showing benefits  
with real robot

# Kinodynamic Path Planning

ALSO GIVEN:  $h_i(q, \dot{q}, \ddot{q}) \leq 0, h_i(q, \dot{q}, \ddot{q}) = 0, \dots$

FIND:  $\tau$  that satisfies  $f_i(q), g_i(q, \dot{q}), h_i(q, \dot{q}, \ddot{q})$

- Consider kinematic + dynamic constraints



Gait and Trajectory Optimization for Legged Systems through  
Phase-based End-Effector Parameterization

# State Space Formulation

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- **Kinodynamic planning**  $\rightarrow$  **2n-dimensional state space**

$C$  denote the  $C$ -space

$X$  denote the state space

$$x = (q, \dot{q}), \text{ for } q \in C, x \in X$$

$$x = \left[ q_1 \quad q_2 \quad \dots \quad q_n \quad \frac{dq_1}{dt} \quad \frac{dq_2}{dt} \quad \dots \quad \frac{dq_n}{dt} \right]$$

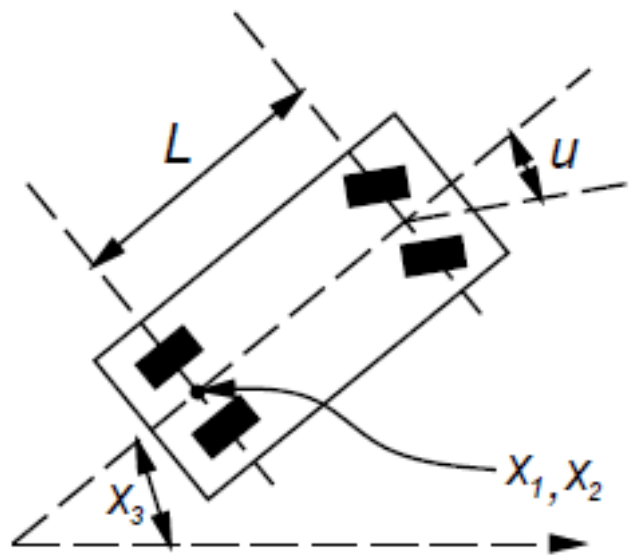
# Constraints in State Space

$h_i(q, \dot{q}, \ddot{q}) = 0$  becomes  $G_i(x, \dot{x}) = 0$ ,  
for  $i = 1, \dots, m$  and  $m < 2n$

- **Constraints can be written in:**

$$\dot{x} = f(x, u)$$

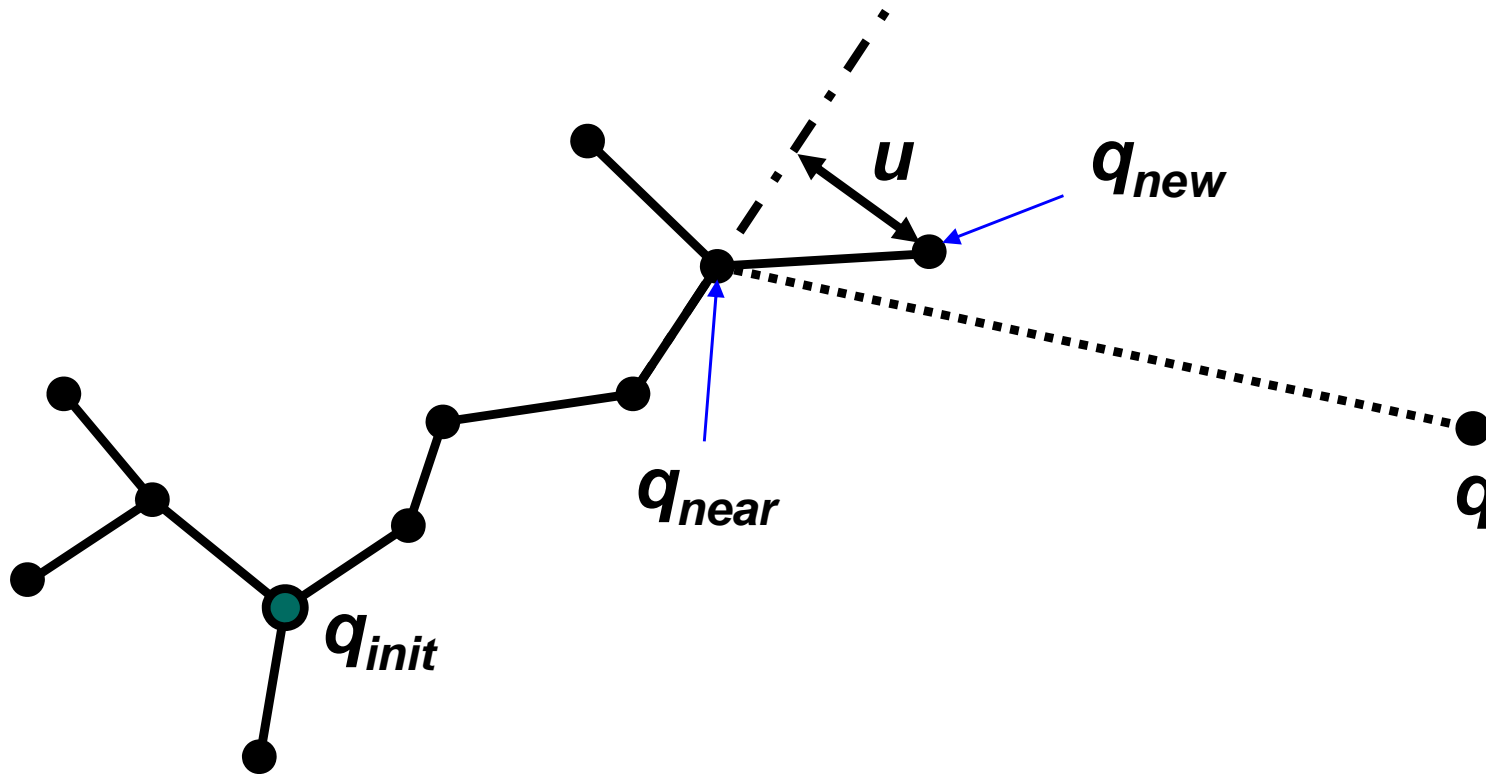
$u \in U$ ,  $U$  : Set of allowable controls or inputs



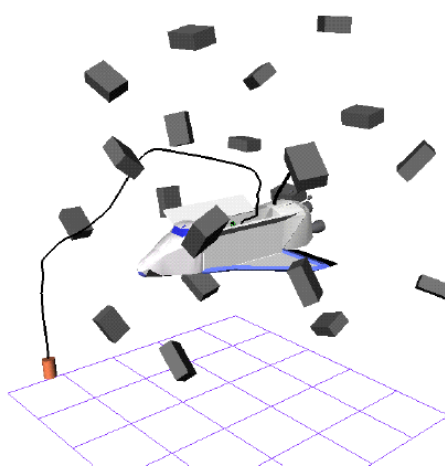
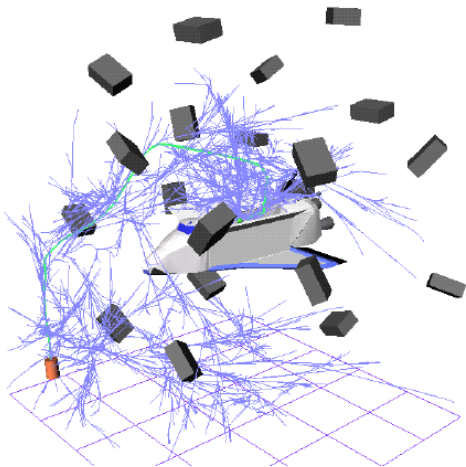
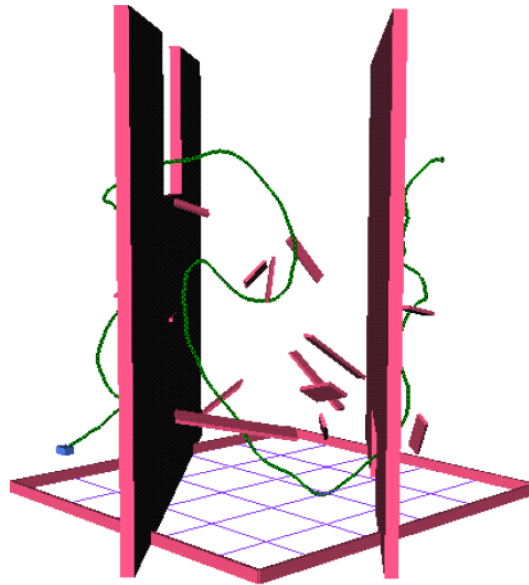
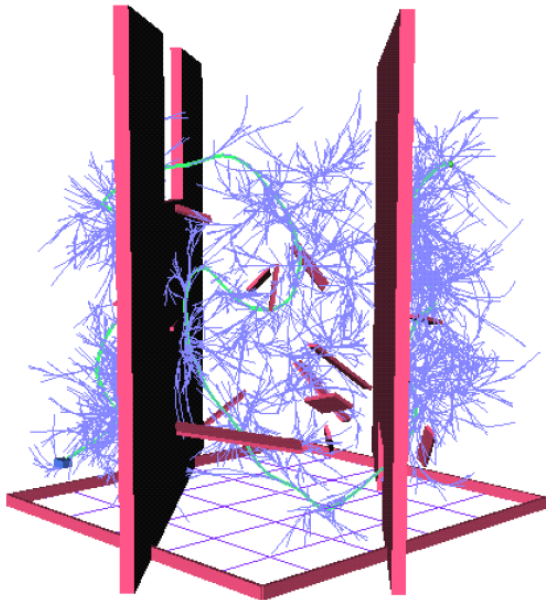


# Rapidly-Exploring Random Tree

- Extend a new vertex in each iteration

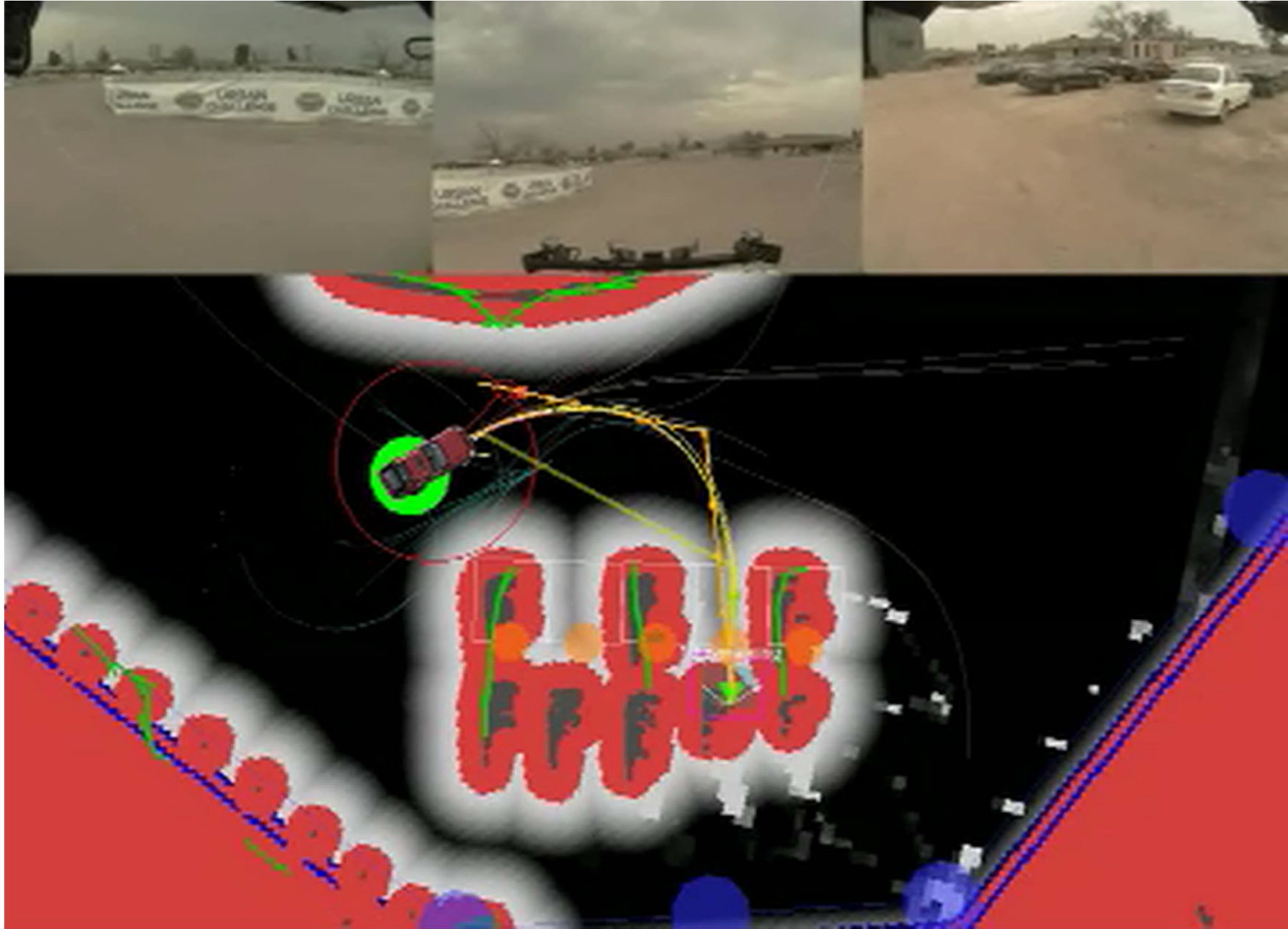


# Results – 200MHz, 128MB



- **3D translating**
- **X=6 DOF**
- **16,300 nodes**
- **4.1min**
  
- **3D TR+RO**
- **X=12 DOF**
- **23,800 nodes**
- **8.4min**

# RRT at work: Successful Parking Maneuver



# Some Works of Our Group

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- **Narrow passages**
  - **Identify narrow passage with a simple one-dimensional line test, and selectively explore such regions**
  - **Selective retraction-based RRT planner for various environments, Lee et al., T-RO 14**
  - **<http://sglab.kaist.ac.kr/SRRRT/T-RO.html>**

# Retraction-based RRT

## [Zhang & Manocha 08]

- Retraction-based RRT technique **handling narrow passages**

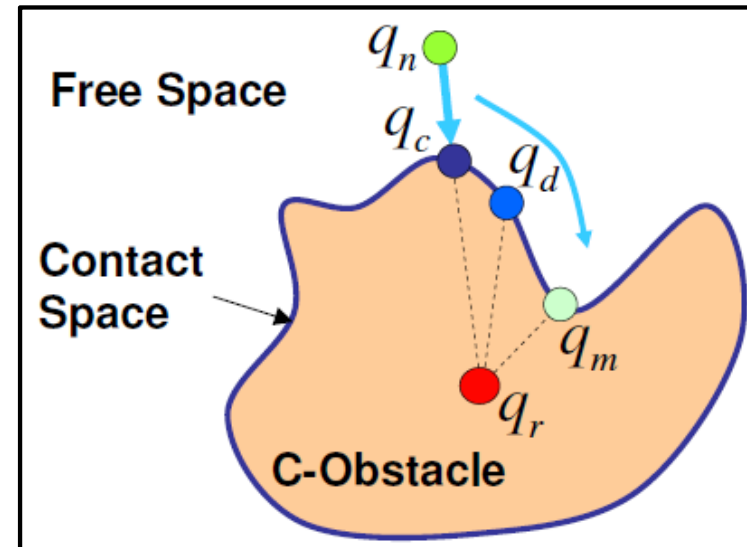
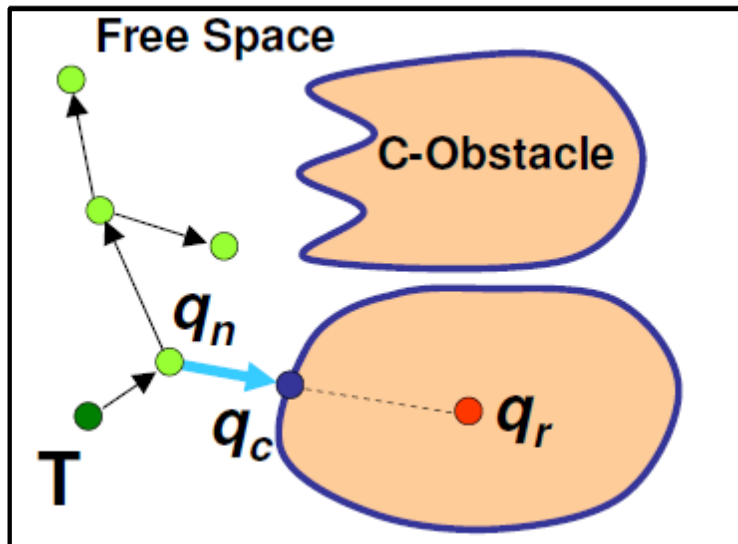
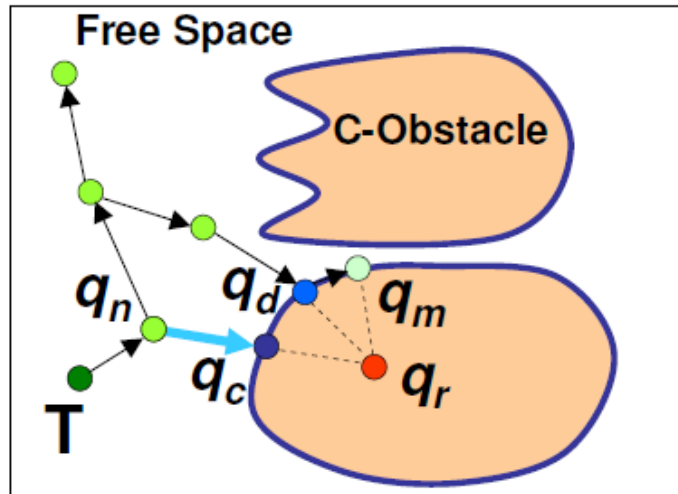


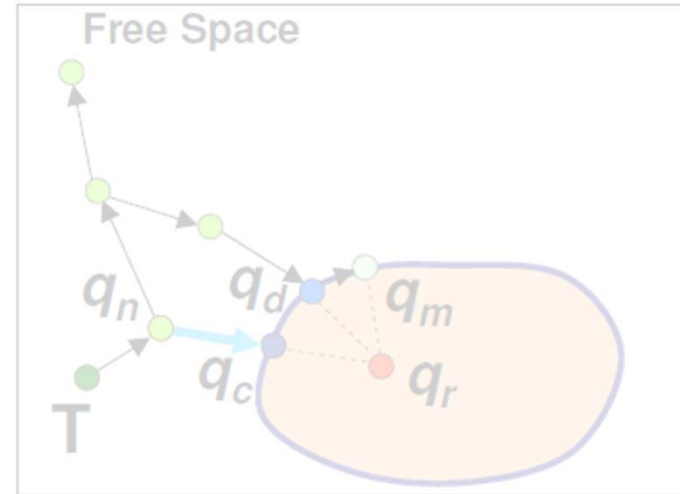
image from [Zhang & Manocha 08]

- General characteristic:**  
Generates more samples near the boundary of obstacles

# RRRT: Pros and Cons

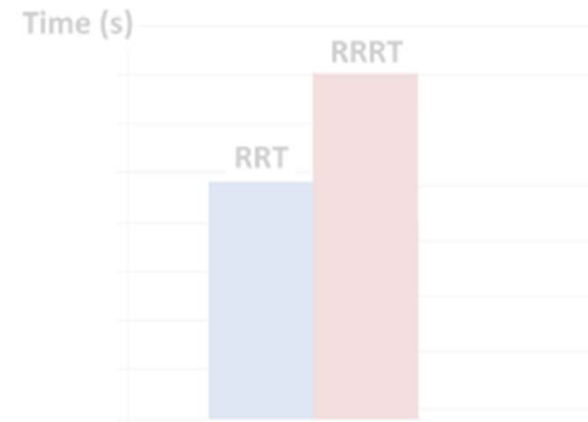
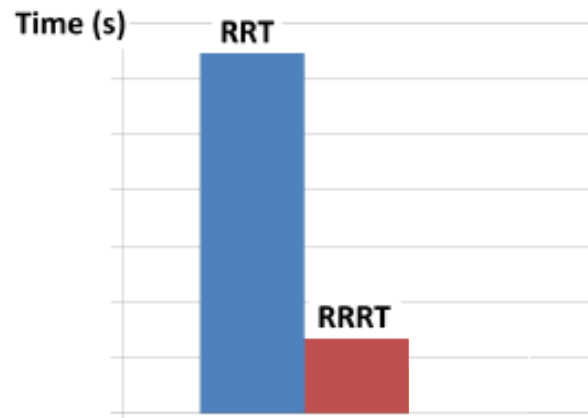


with narrow passages

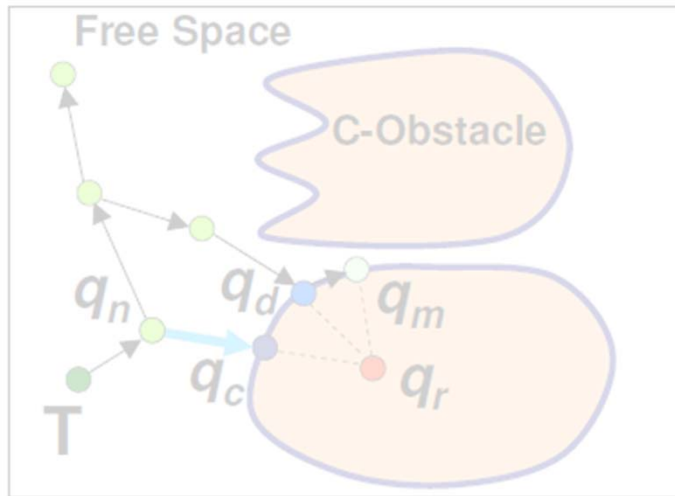


without narrow passages

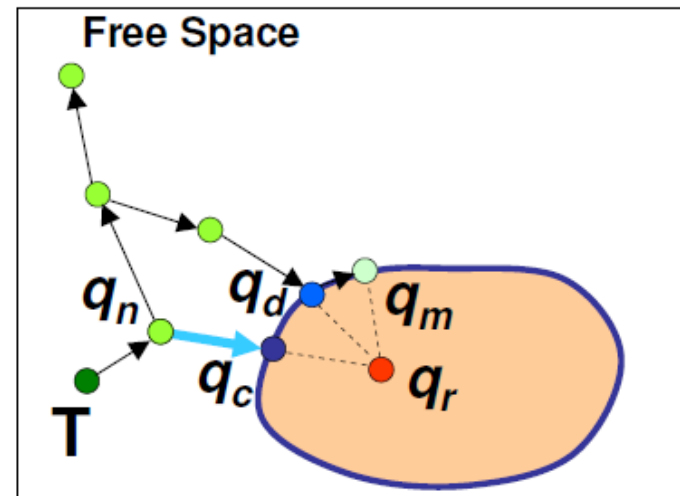
images from [Zhang & Manocha 08]



# RRRT: Pros and Cons

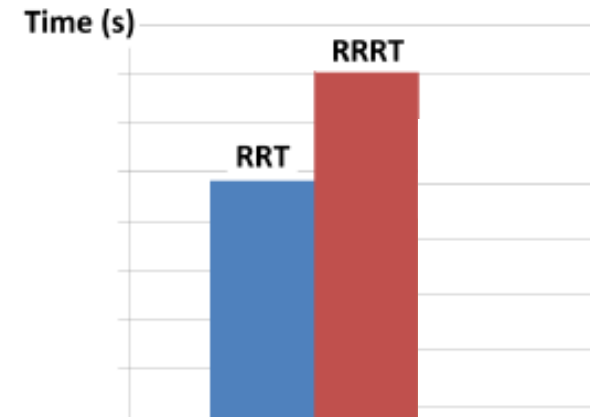
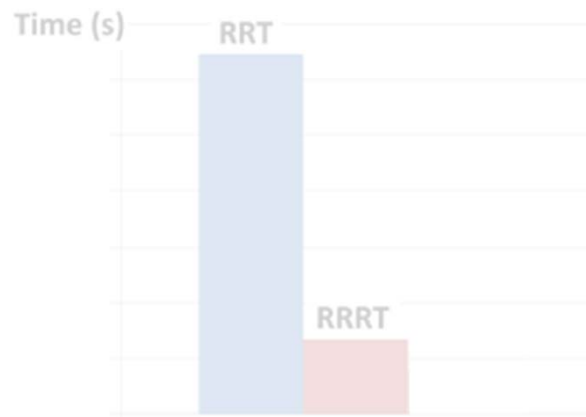


with narrow passages



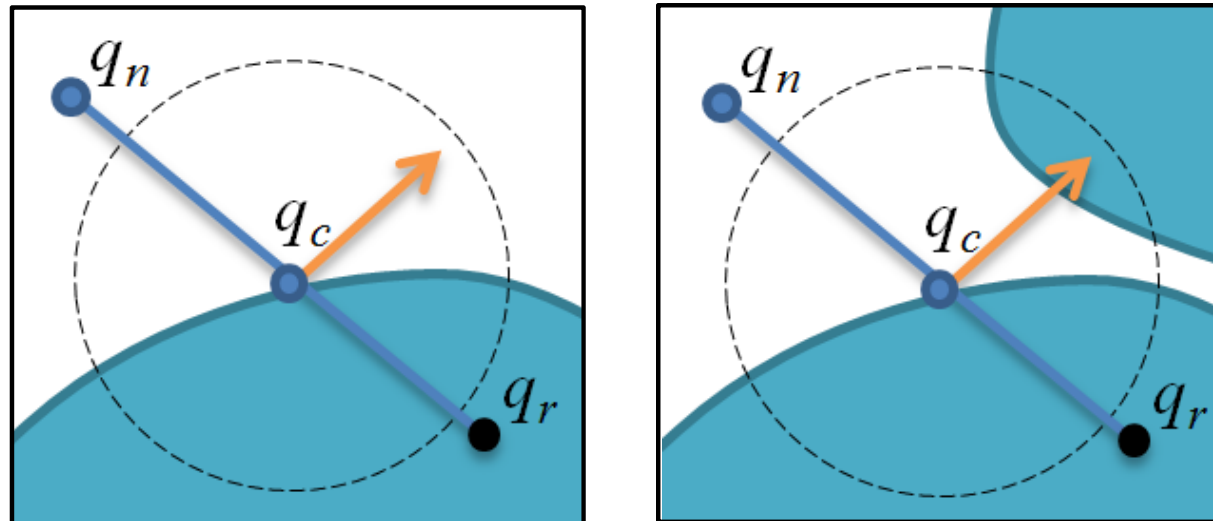
without narrow passages

images from [Zhang & Manocha 08]



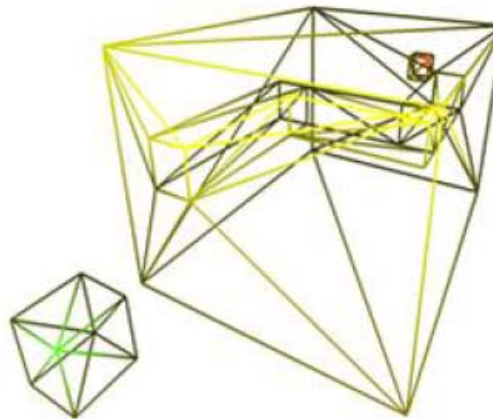
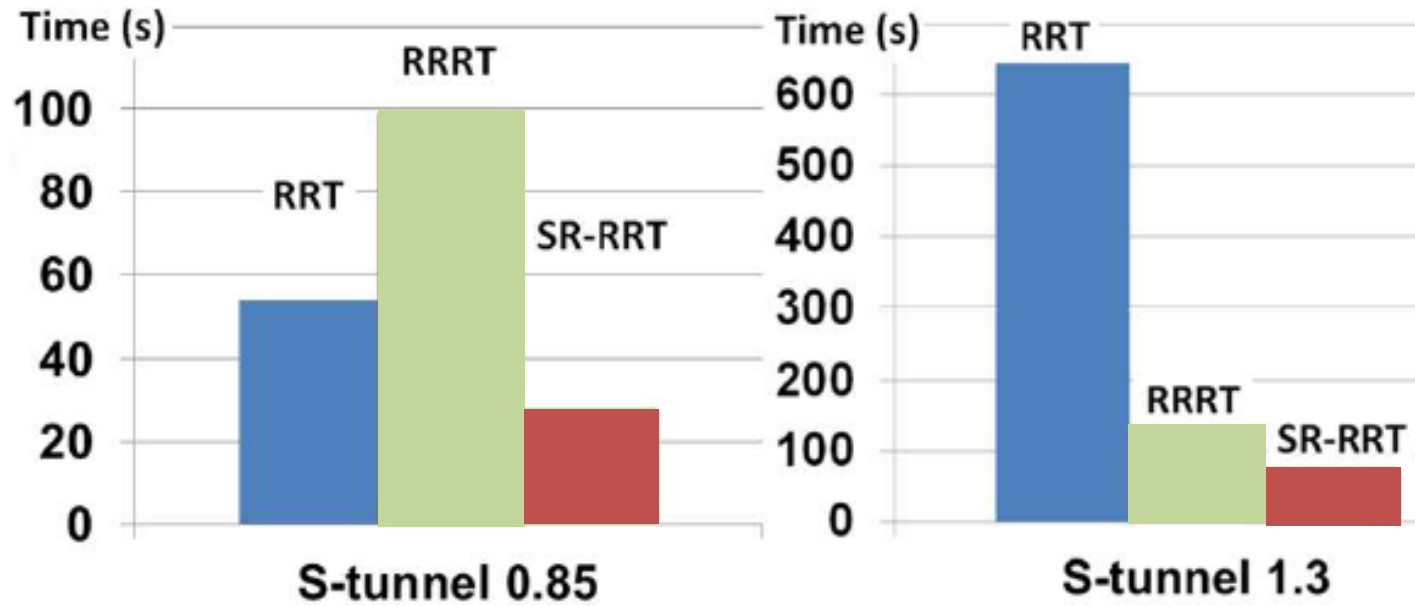
# Bridge line-test [Lee et al., T-RO 14]

- To identify narrow passage regions
- Bridge line-test
  1. Generate a random line
  2. Check whether the line meets any obstacle





# Results



Video

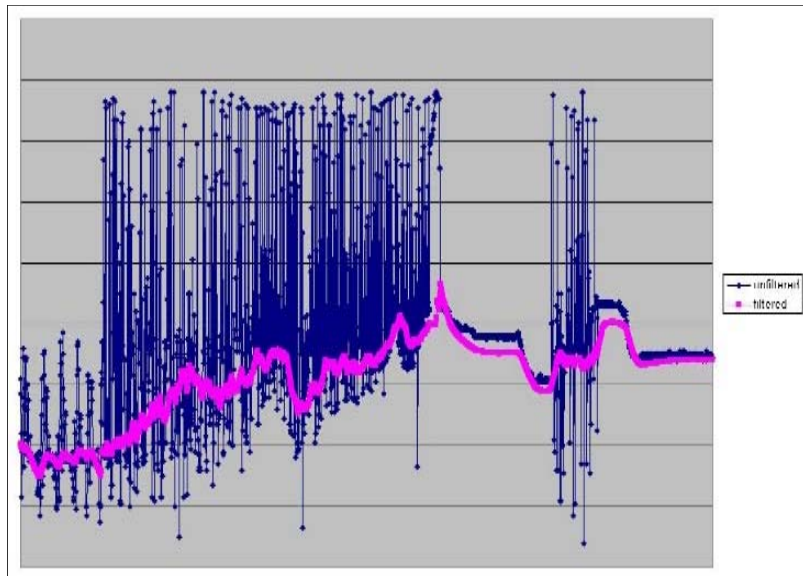
# Related Works of Our Group

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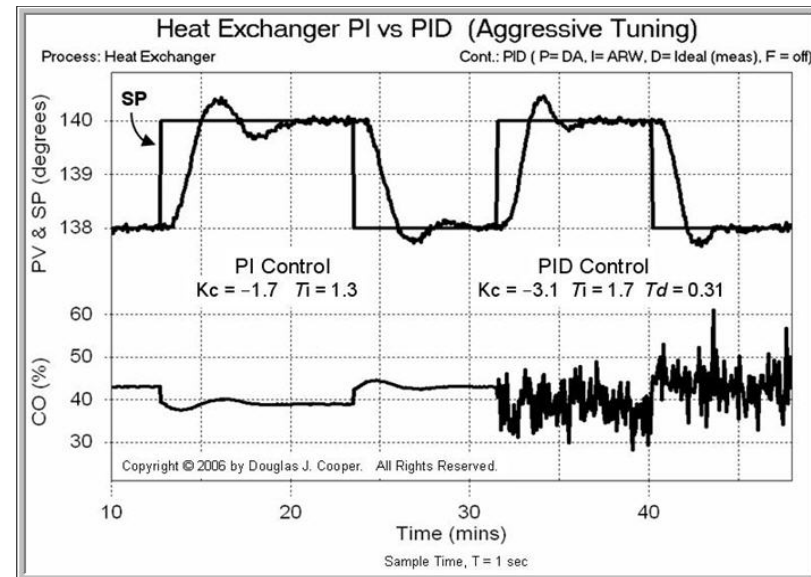
- **Handling narrow passages**
- **Handling uncertainty and dynamic objects**
  - **Anytime RRBT for handling uncertainty and dynamic objects, IROS 16**

# Handling Sensor Errors

- **Uncertainty caused by:**
  - **Various sensors**
  - **Low-level controllers**



Sensor noise

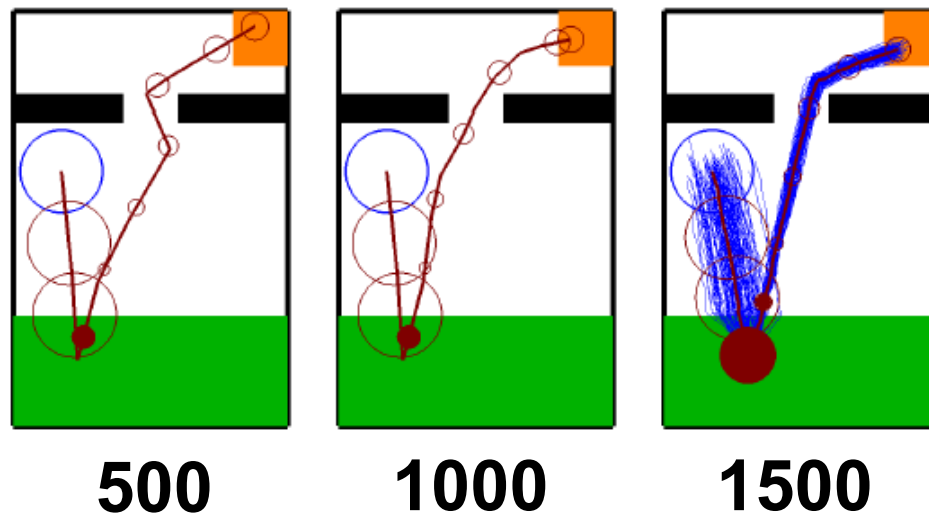


Controller noise

# Rapidly-exploring Random Belief Tree

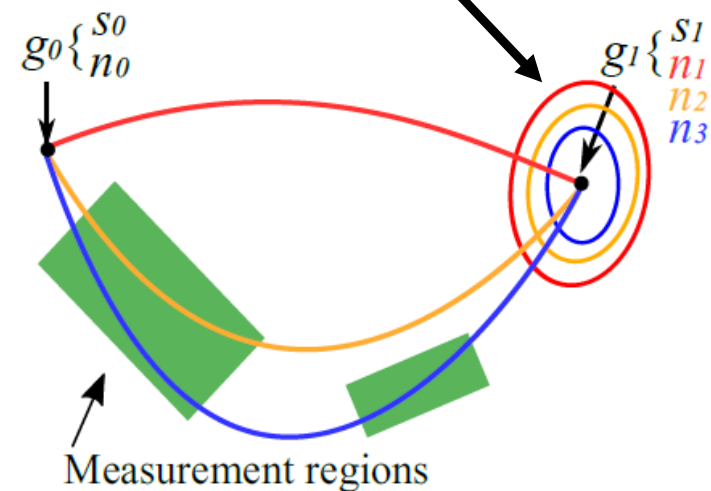
[Bry et al., ICRA 11]

- Use Kalman filter to propagate Gaussian states
- Improve solutions toward optimal



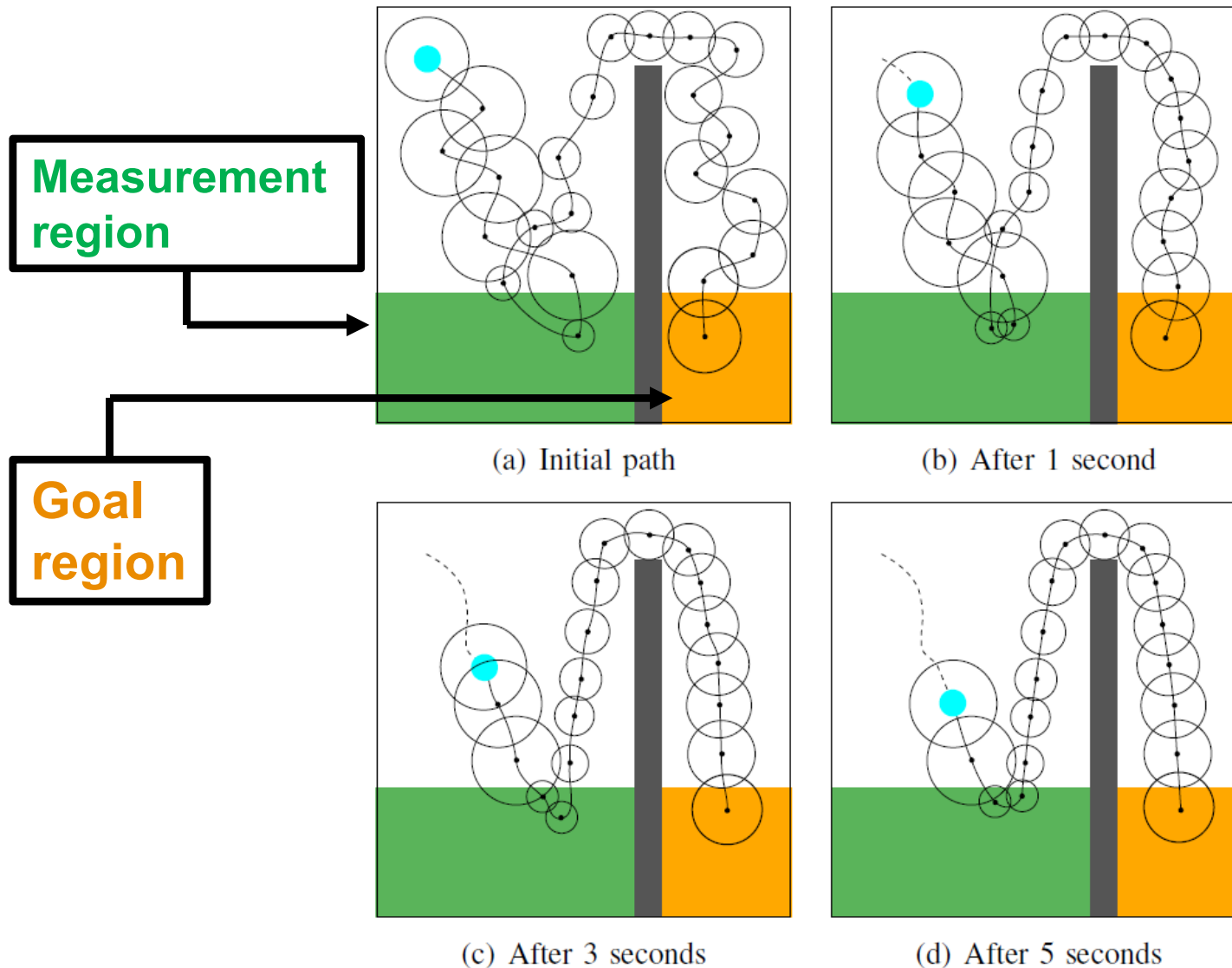
Number of iteration

Multiple belief nodes  
in the same vertex



Preserve optimal path

# Main Contribution: Anytime Extension [Yang et al., IROS 16]

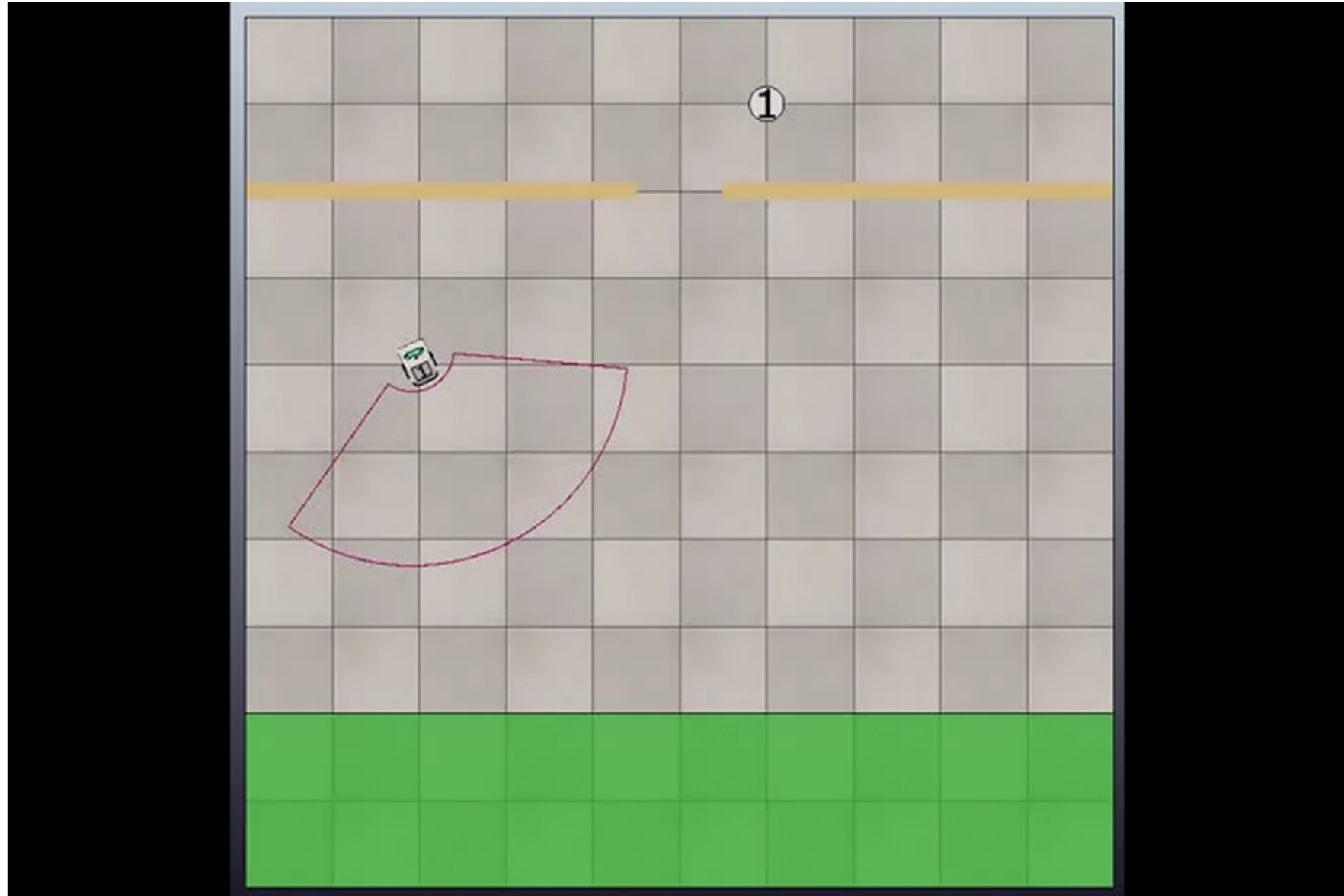


Bigger circle means higher uncertainty

The robot computes better path while executing the path

# Main Contribution: Anytime Extension

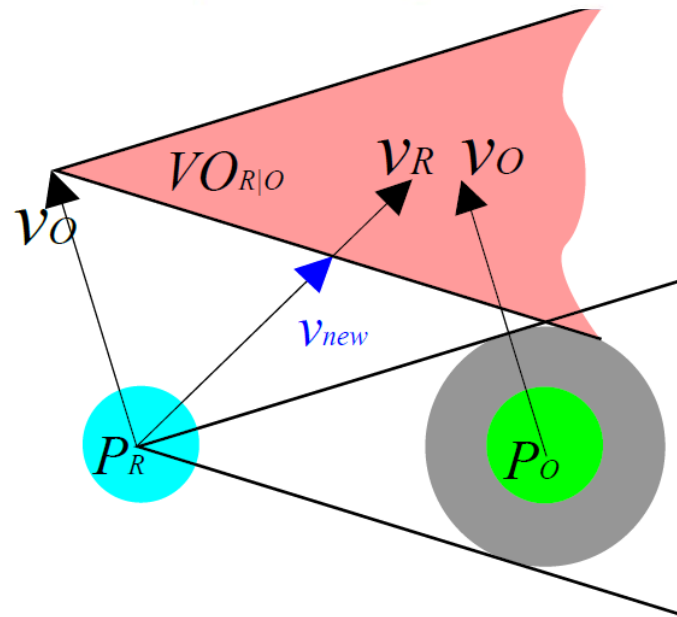
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# Velocity Obstacle: Local Geometric Analysis

- Used for collision avoidance among multiple robots
- When  $v$  for Robot is in the VO, we will have collision

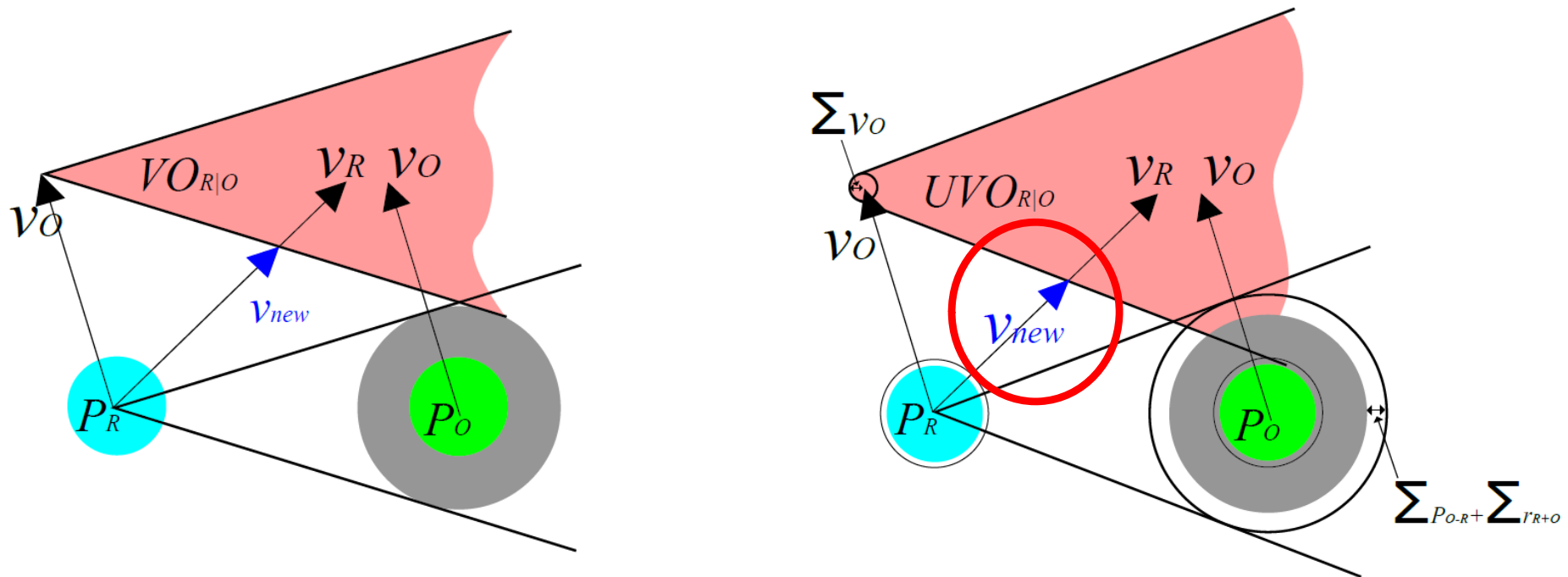
$$VO_{R|O} = \{v | \exists t > 0 : t(v - v_O) \in Disc(P_O - P_R, r_R + r_O)\}$$



“The hybrid reciprocal velocity obstacle” **TRO11** J Snape, J van den Berg, SJ Guy  
“Reciprocal velocity obstacles for real-time multi-agent navigation” J van den Berg  
“Generalized Velocity Obstacles” **IROS09**, D Wilkie, J Van den Berg

# Uncertainty-aware Velocity Obstacle as Local Geometry Analysis

## Conservative collision checking



(a) Velocity obstacle

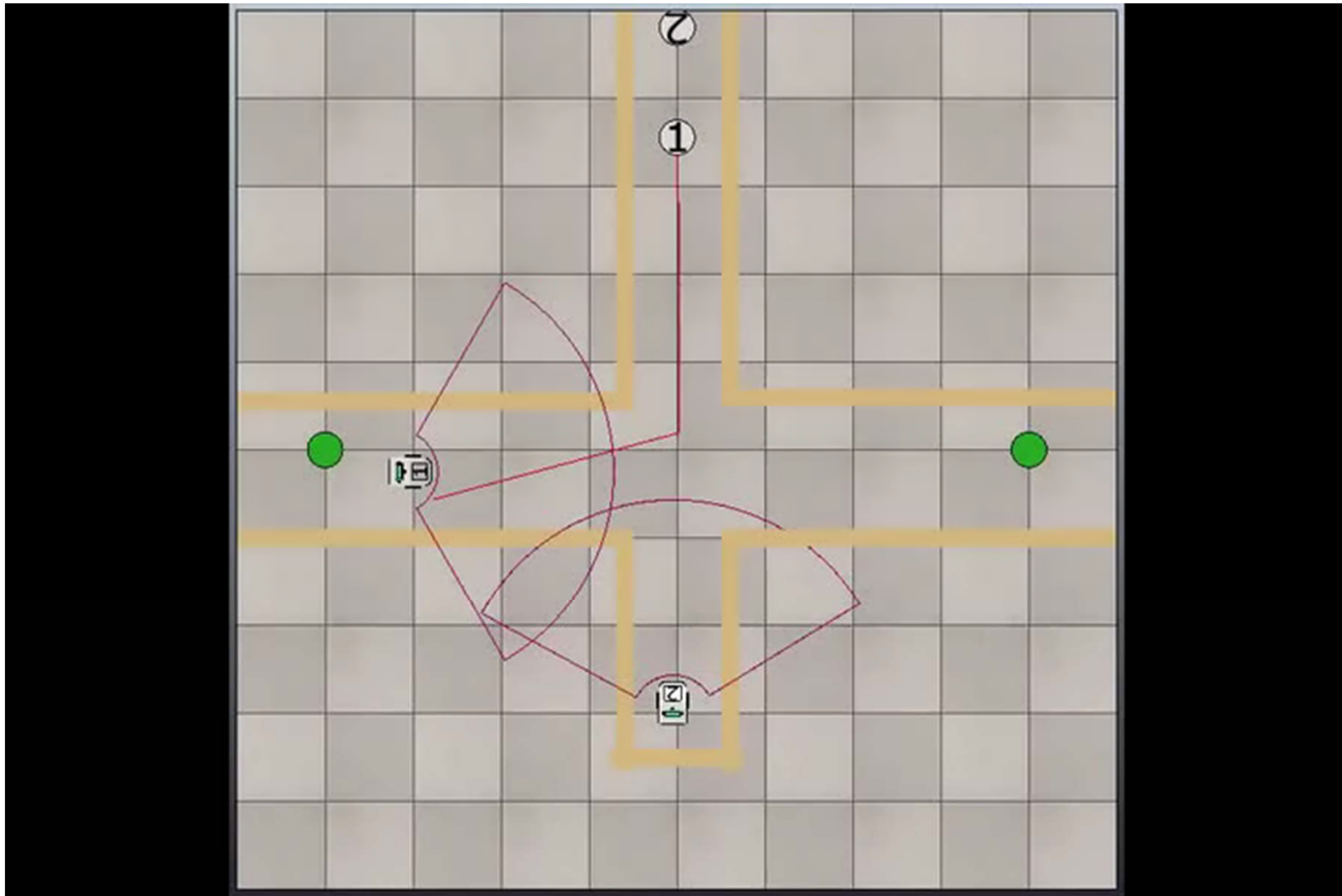
(b) Uncertainty-aware velocity obstacle

“The hybrid reciprocal velocity obstacle” **TRO11** J Snape, J van den Berg, SJ Guy  
“Reciprocal velocity obstacles for real-time multi-agent navigation” J van den Berg  
“Generalized Velocity Obstacles” **IROS09**, D Wilkie, J Van den Berg



# Intersection scene – with UVO

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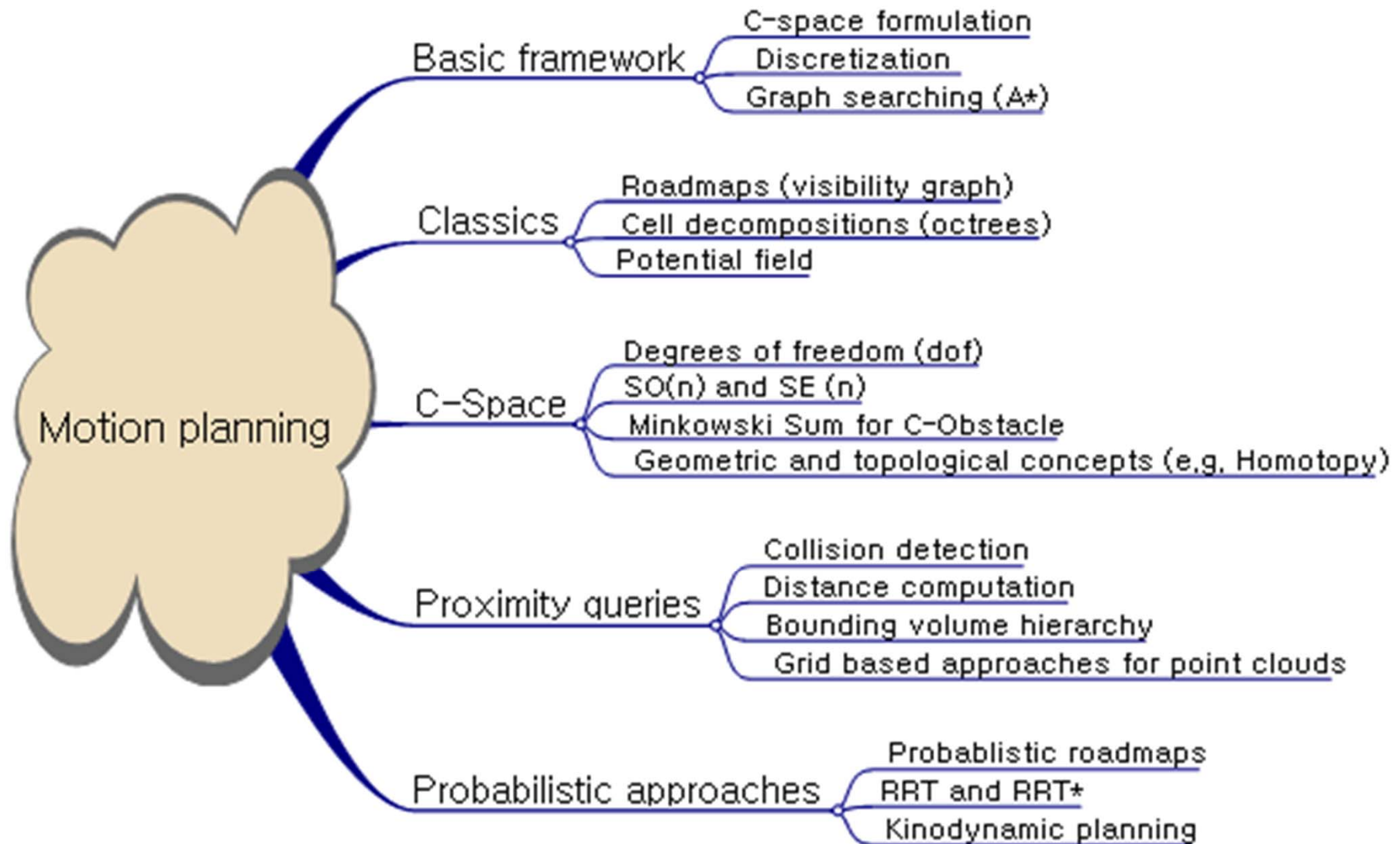


# Class Objectives were:

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- **Understand the RRT technique and its recent advancements**
  - **RRT\* for optimal path planning**
  - **Kinodynamic planning**
  - **Some related techniques to RRT**

# Summary



# Next Time..

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- **Basic concepts of reinforcement learning**

# Homework for Every Class

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- **Submit summaries of 2 ICRA/IROS/RSS/CoRL/TRO/IJRR papers**
- **Go over the next lecture slides**
- **Come up with three question before the mid-term exam**