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# 실사 렌더링 Physically based Rendering

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**Sung-Eui Yoon**  
(윤성익)

<http://sglab.kaist.ac.kr/~sungeui>

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## About the Instructor

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- Joined KAIST at 2007
- Main research focus
  - Handling of massive geometric data for various computer graphics and geometric problems
- Research history for rendering
  - Did volume rendering at M.S.
  - Did large-scale, real-time, rasterization based rendering at Ph.D.
  - Have been doing high-quality rendering at KAIST

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## Overview

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- We will discuss various parts of computer graphics



**Computer vision** inverts the process  
**Image processing** deals with images

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## Application of Rendering/Computer Graphics

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- Games
- Movies and film special effects
- Product design and analysis
- Medical applications
- Scientific visualization

4

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3

# Games



2D game



3D shooting game

# Movies and Film Special Effects



Avatar

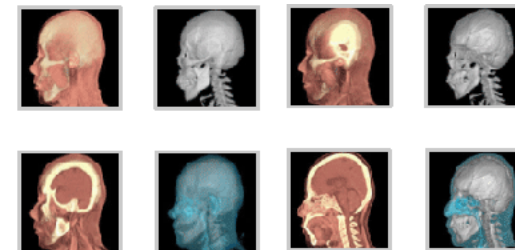
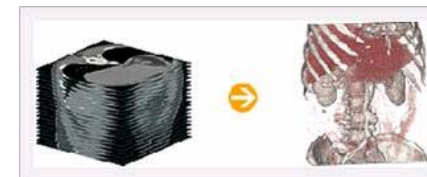
# Product Design and Analysis

- Computer-aided design (CAD)



# Medical Applications

- Visualizing data of CT, MRI, etc



Rapida homepage

## About the Course

- We will focus on:
  - Photo-Realistic Rendering
  - Study basic concepts of physically-based rendering



## Photo-Realistic Rendering

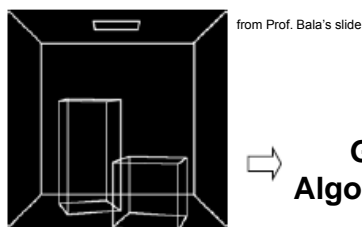
- Achieved by simulating light and material interactions



- Rendering equation
  - Mathematical formulation of light and material interactions

## Global Illumination (GI)

- GI algorithms solve the rendering equation
  - Generate 2D image from 3D scene



GI  
Algorithm



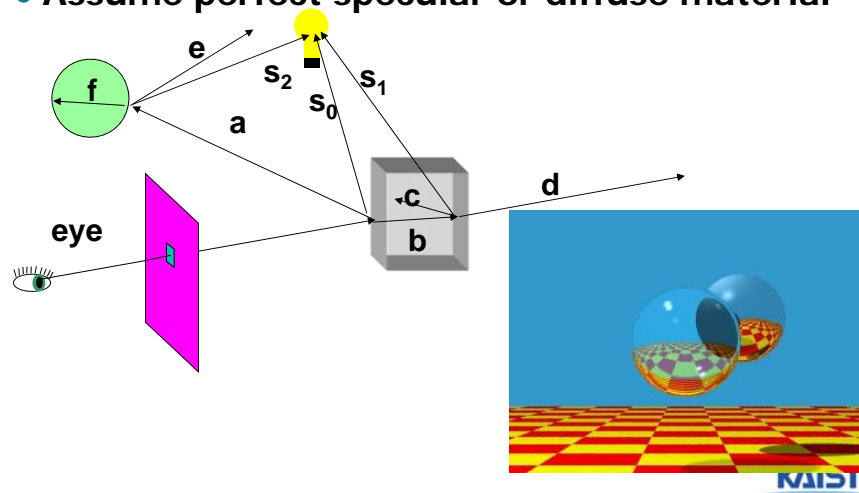
+  
Emission (light sources)  
Geometry (objects)  
BRDF (materials)

## Classic Methods of GI

- Ray tracing
  - Introduced by Whitted in 1980
- Radiosity
  - Introduced in 1984
- Monte Carlo rendering

## Ray Tracing

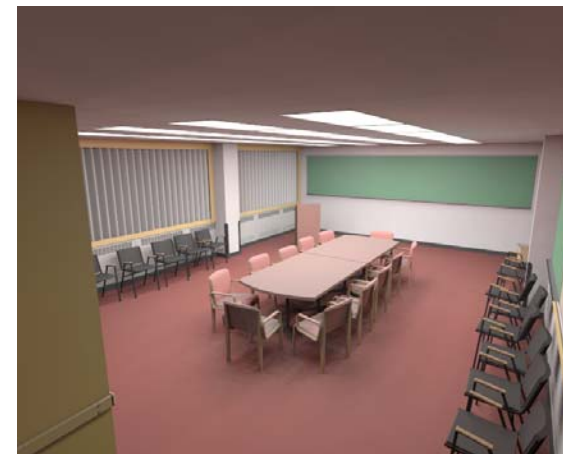
- Assume perfect specular or diffuse material



13

## Radiosity

- Assume diffuse inter-reflections



14

## Advanced Global Illumination

- Extend to handle more realistic materials than just perfect specular/diffuse
  - Classic ray tracing and classic radiosity are basic building blocks



from photon map paper



from Pixar movie

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15

## Scalable GI

- How can we handle complexity?
  - Many objects
  - Many triangles
  - Many lights
  - Complex BRDFs
  - Dynamic scenes, etc.
- Can we achieve interactive GI on commodity hardware?

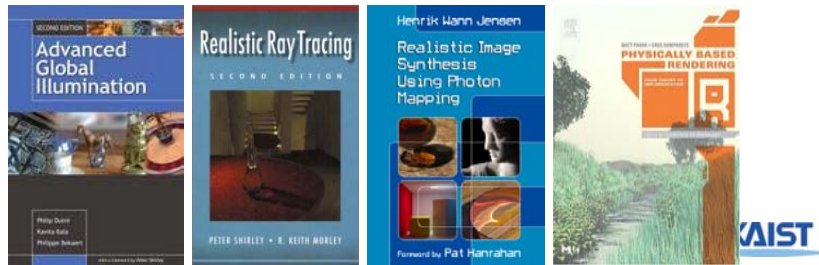
16

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## Resource

### • Reference

- Physically based rendering, Matt Pharr et al.
- Advanced Global Illumination, Philip Dutre et al. 2nd edition
- Realistic Image Synthesis Using Photon Mapping, Henrik Jensen
- Realistic Ray Tracing, 2nd edition, Peter Shirley et al.



## Other Reference

### • Technical papers

- Graphics-related conference (SIGGRAPH, etc)
- <http://kesen.huang.googlepages.com/>

### • SIGGRAPH course notes and video encore

### • Course homepages

### • Google or Google scholar



18

## Classic Rendering Pipeline

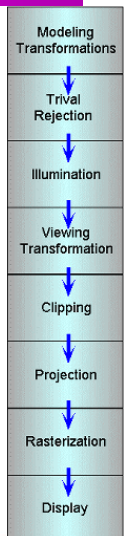
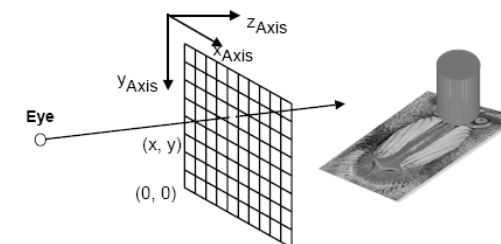
Sung-Eui Yoon  
(윤성익)

Course URL:  
<http://sglab.kaist.ac.kr/~sungeui/GCG/>



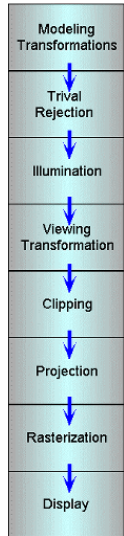
## Course Objectives

- Understand classic rendering pipeline
  - Just high-level concepts, not all the details
  - Brief introduction of common under. CG
- Know its pros and cons



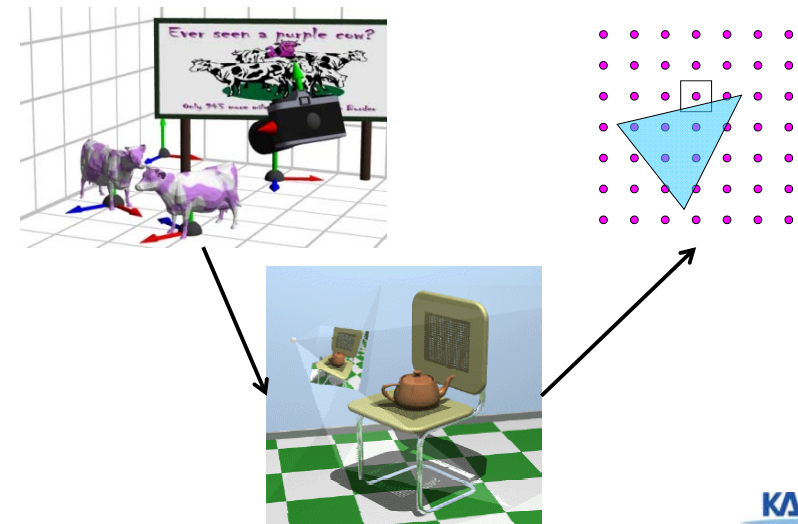
20

# The Classic Rendering Pipeline

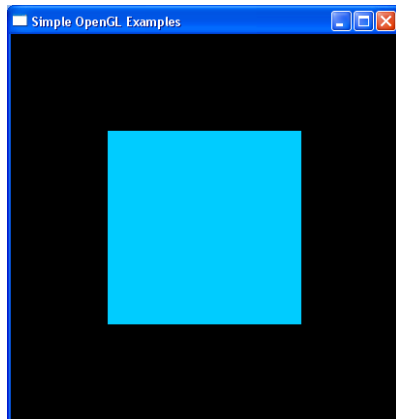


- Adopted in OpenGL and DirectX
  - Most of games are based on this pipeline
- Object **primitives** defined by vertices fed in at the top
- Pixels come out in the display at the bottom

# The Classic Rendering Pipeline



# Code Example



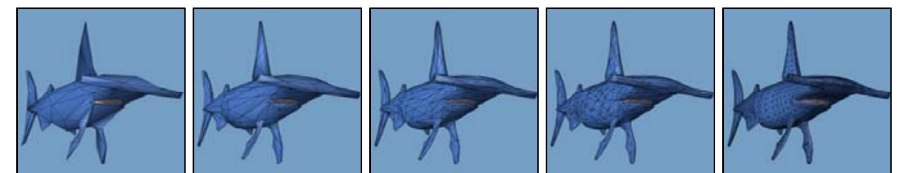
OpenGL Code:

```
glColor3d(0.0, 0.8, 1.0);

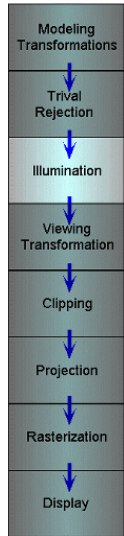
glBegin(GL_POLYGON);
    glVertex2d(-0.5, -0.5);
    glVertex2d( 0.5, -0.5);
    glVertex2d( 0.5,  0.5);
    glVertex2d(-0.5,  0.5);
glEnd();
```

# Triangle Representation, Mesh

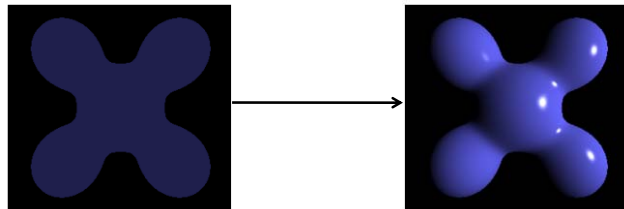
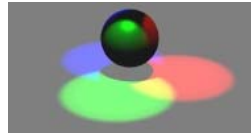
- Triangles can approximate any 2-dimensional shape (or 3D surface)
  - Polygons are a locally linear (planar) approximation
- Improve the quality of fit by increasing the number edges or faces



# Illumination

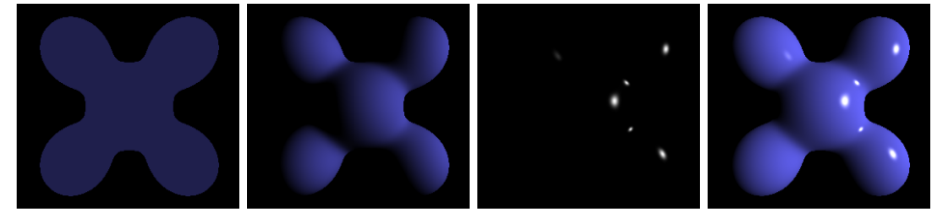


- Illuminate potentially visible objects
- Final rendered color is determined by object's orientation, its material properties, and the light sources in the scene



# OpenGL's Illumination Model

$$I_r = \sum_{j=1}^{\text{numLights}} (k_a^j I_a^j + k_d^j I_d^j \max((\hat{N} \cdot \hat{L}_j), 0) + k_s^j I_s^j \max((\hat{V} \cdot \hat{R})^{n_s}, 0))$$



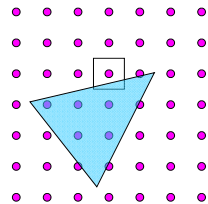
Ambient + Diffuse + Specular = Phong Reflection

From Wikipedia

# Rasterization and Display



- Transform to screen space
- Rasterization converts objects pixels

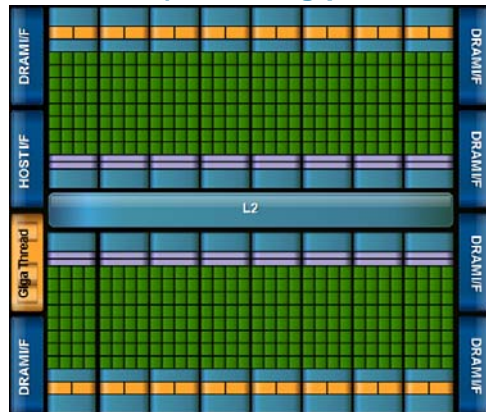


# Why we are using rasterization?

- Efficiency
- Reasonably quality

# Fermi GPU Architecture

16 SM (streaming processors)



512 CUDA cores

Memory interfaces

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29

# Where Rasterization Is

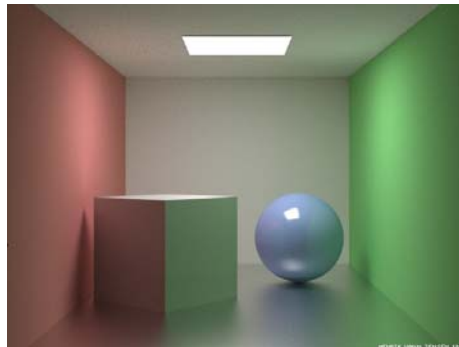


From Battlefield: Bad Company, EA Digital Illusions  
CE AB

From Eric Haines KAIST

# But what about other visual cues?

- Lighting
  - Shadows
  - Shading: glossy, transparency
- Color bleeding, etc.
- Generality



from Henrik's homepage KAIST

31



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## Ray Tracing

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Sung-Eui Yoon  
(윤성의)

<http://sglab.kaist.ac.kr/~sungeui>

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## Class Objectives

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- Understand a basic ray tracing
- Implement its acceleration data structure and know how to use it

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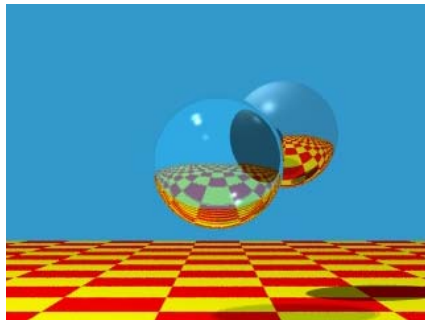
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## Recursive Ray Casting

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- Gained popularity in when Turner Whitted (1980) recognized that *recursive* ray casting could be used for global illumination effects



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## Ray Casting and Ray Tracing

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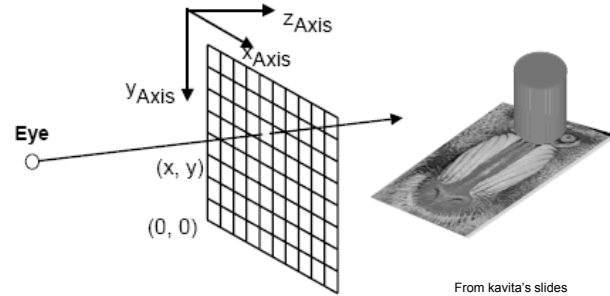
- Trace rays from eye into scene
  - Backward ray tracing
- Ray casting used to compute visibility at the eye
- Perform ray tracing for arbitrary rays needed for shading
  - Reflections
  - Refraction and transparency
  - Shadows

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## Basic Algorithms

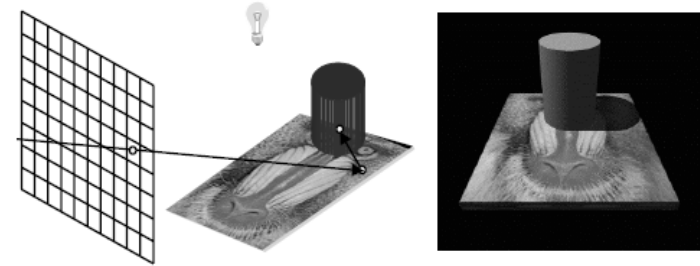
- Rays are cast from the eye point through each pixel in the image



From kavita's slides

## Shadows

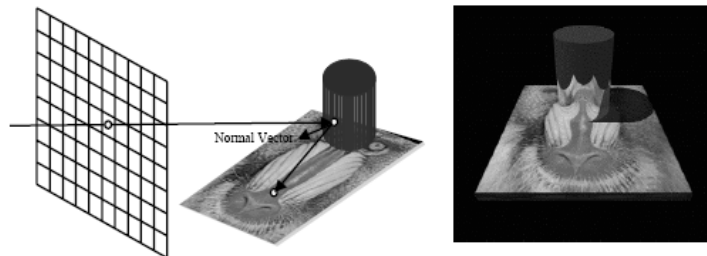
- Cast ray from the intersection point to each light source
  - Shadow rays



From kavita's slides

## Reflections

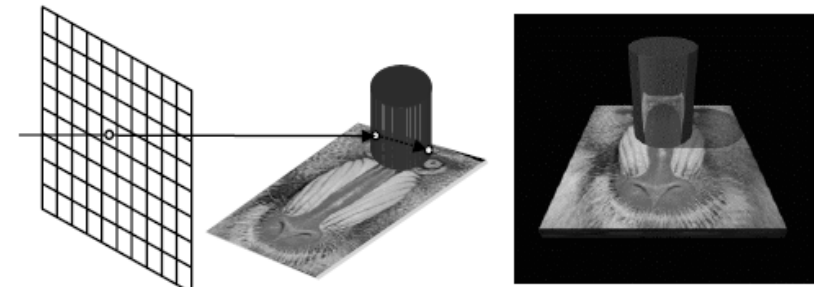
- If object specular, cast secondary reflected rays



From kavita's slides

## Refractions

- If object transparent, cast secondary refracted rays



From kavita's slides

# An Improved Illumination Model [Whitted 80]

- Phong illumination model

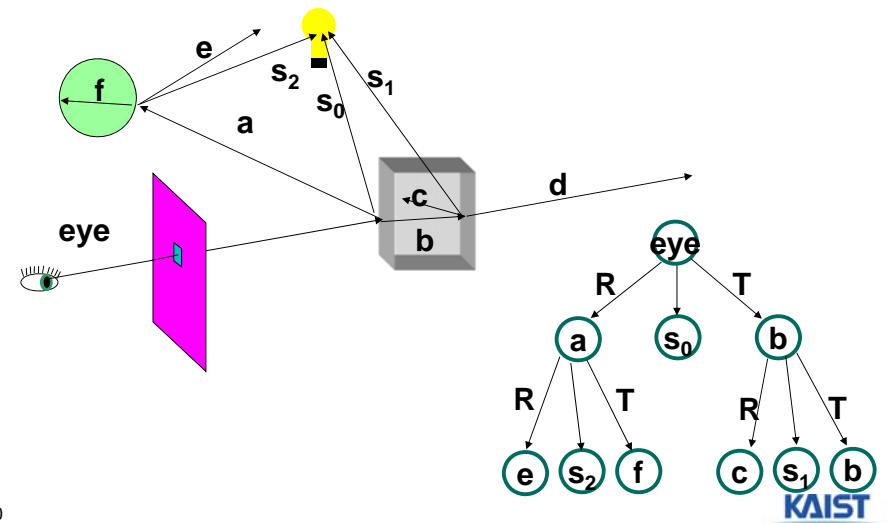
$$I_r = \sum_{j=1}^{\text{numLights}} (k_a^j I_a^j + k_d^j I_d^j (\hat{N} \cdot \hat{L}_j) + k_s^j I_s^j (\hat{V} \cdot \hat{R})^{n_s})$$

- Whitted model

$$I_r = \sum_{j=1}^{\text{numLights}} (k_a^j I_a^j + k_d^j I_d^j (\hat{N} \cdot \hat{L}_j)) + k_s S + k_t T$$

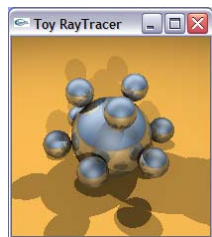
- S and T are intensity of light from reflection and transmission rays
- Ks and Kt are specular and transmission coefficient

# Ray Tree



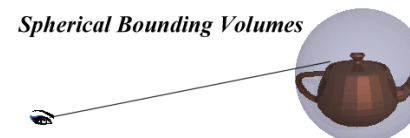
# Acceleration Methods for Ray Tracing

- Rendering time for a ray tracer depends on the number of ray intersection tests per pixel
  - The number of pixels X the number of primitives in the scene
- Early efforts focused on accelerating the ray-object intersection tests
  - Ray-triangle intersection tests
- More advanced methods required to make ray tracing practical
  - Bounding volume hierarchies
  - Spatial subdivision (e.g., kd-trees)



# Bounding Volumes

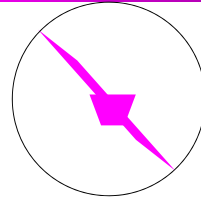
- Enclose complex objects within a simple-to-intersect objects
  - If the ray does not intersect the simple object then its contents can be ignored
  - The likelihood that it will strike the object depends on how tightly the volume surrounds the object.
- Spheres are simple, but not tight
- Axis-aligned bounding boxes often better
  - Can use nested or hierarchical bounding volumes



## Bounding Volumes

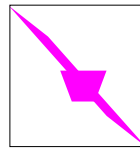
- **Sphere [Whitted80]**

- Cheap to compute
- Cheap test
- Potentially very bad fit



- **Axis-Aligned Bounding Box**

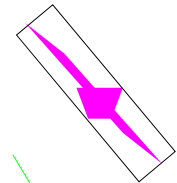
- Very cheap to compute
- Cheap test
- Tighter than sphere



## Bounding Volumes

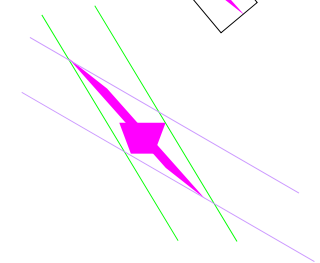
- **Oriented Bounding Box**

- Fairly cheap to compute
- Fairly Cheap test
- Generally fairly tight



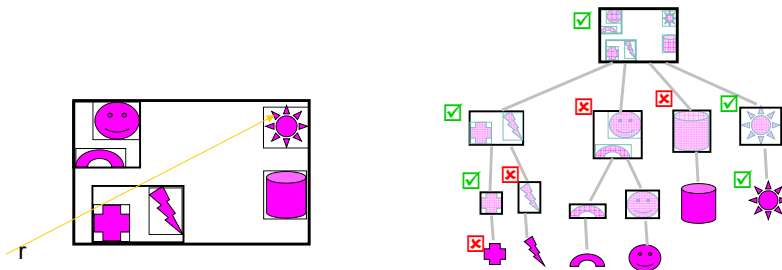
- **Slabs / K-dops**

- More expensive to compute
- Fairly cheap test
- Can be tighter than OBB



## Hierarchical Bounding Volumes

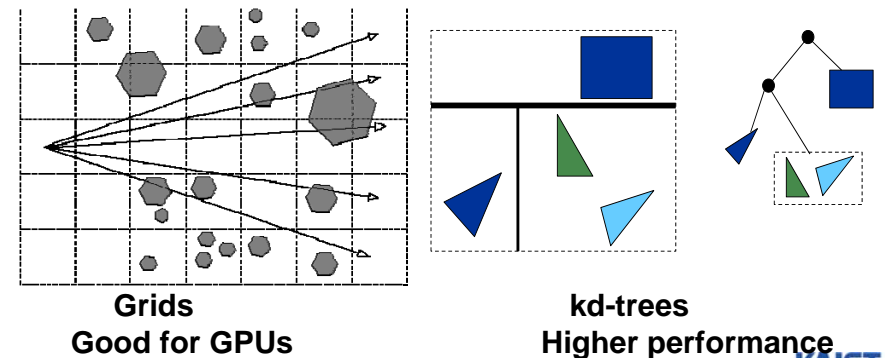
- **Organize bounding volumes as a tree**
  - Choose a partitioning plane and distribute triangles into left and right nodes
- Each ray starts with the scene BV and traverses down through the hierarchy



## Spatial Subdivision

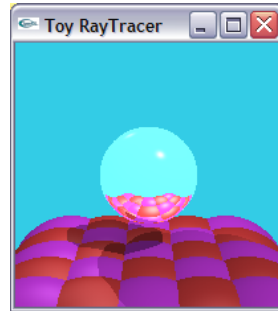
Idea: Divide space in to subregions

- Grids and kd-trees are also commonly used



# Classic Ray Tracing

- **Gathering approach**
  - From lights, reflected, and refracted directions
- **Pros of ray tracing**
  - Simple and improved realism over the rendering pipeline
- **Cons:**
  - Simple light model, material, and light propagation
  - Not a complete solution
  - Hard to accelerate with special-purpose H/W



# History

- **Problems with classic ray tracing**
  - Not realistic
  - View-dependent
- **Radiosity (1984)**
  - Global illumination in diffuse scenes
- **Monte Carlo ray tracing (1986)**
  - Global illumination for any environment

# Class Objectives were:

- Understand a basic ray tracing
- Implement its acceleration data structure and know how to use it

# PA1

- Get to know pbrt



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## Radiosity

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(윤성의)

<http://sglab.kaist.ac.kr/~sungeui>

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## Class Objective

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- Understand radiosity
  - Radiosity equation
  - Solving the equation

22

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## Radiosity

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- Physically based method for diffuse environments
  - Support diffuse interactions, color bleeding, indirect lighting and penumbra
  - Account for very high percentage of total energy transfer
  - Finite element method

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23

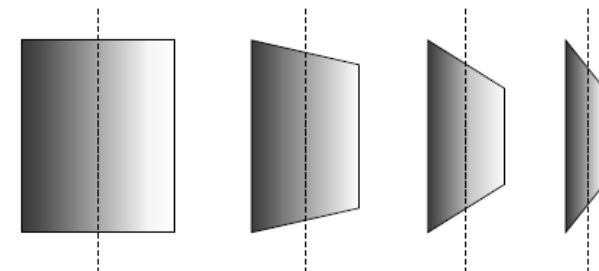
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## Key Idea #1: Diffuse Only

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From kavita's slides

- Radiance independent of direction
  - Surface looks the same from any viewpoint
  - No specular reflection

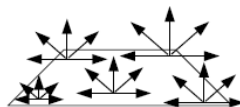
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24

## Diffuse Surfaces

- Diffuse emitter

- $L(x \rightarrow \Theta) = \text{constant over } \Theta$



- Diffuse reflector

- Constant reflectivity

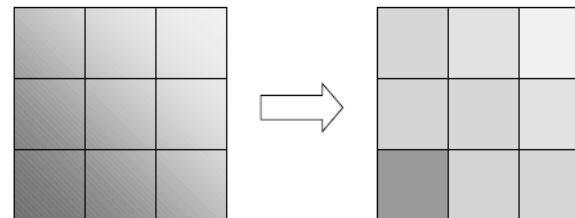


From kavita's slides

## Key Idea #2: Constant Polygons

- Radiosity is an approximation

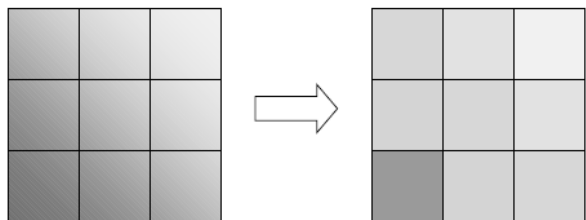
- Due to discretization of scene into patches



From kavita's slides

- Subdivide scene into small polygons

## Constant Radiance Approximation

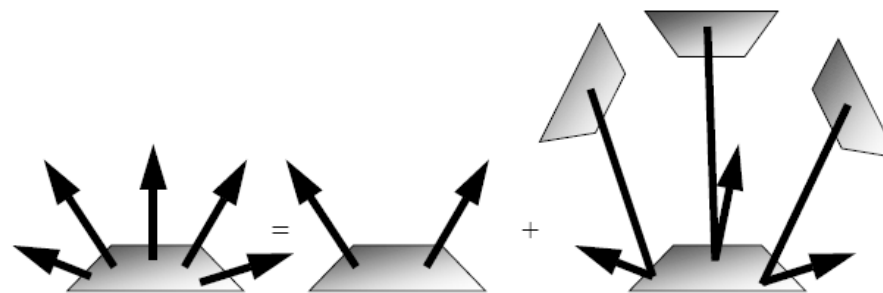


From kavita's slides

- Radiance is constant over a surface element

- $L(x) = \text{constant over } x$

## Radiosity Equation



Emitted radiosity = self-emitted radiosity + received & reflected radiosity

$$Radiosity_i = Radiosity_{self,i} + \sum_{j=1}^N a_{j \rightarrow i} Radiosity_j$$

# Radiosity Equation

- Radiosity equation for each polygon  $i$

$$Radiosity_1 = Radiosity_{self,1} + \sum_{j=1}^N a_{j \rightarrow 1} Radiosity_j$$

$$Radiosity_2 = Radiosity_{self,2} + \sum_{j=1}^N a_{j \rightarrow 2} Radiosity_j$$

...

$$Radiosity_N = Radiosity_{self,N} + \sum_{j=1}^N a_{j \rightarrow N} Radiosity_j$$

- $N$  equations;  $N$  unknown variables

© Kavita Bala, Computer Science, Cornell University

# Radiosity Algorithm

- Maps well to rasterization pipeline
  - Subdivide the scene in small polygons
  - Compute a constant illumination value for each polygon
  - Choose a viewpoint and display the visible polygon

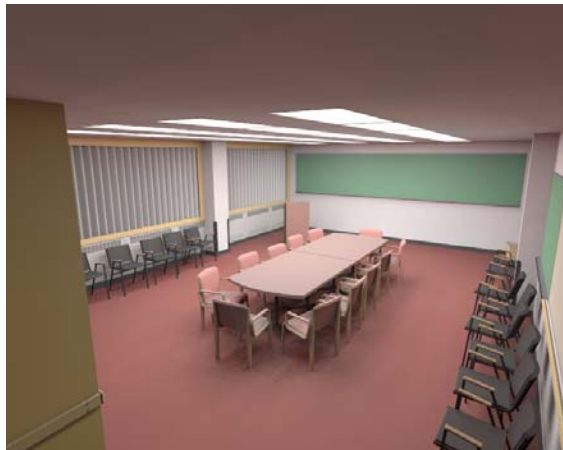


From Donald Fong's slides

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30

# Radiosity Result

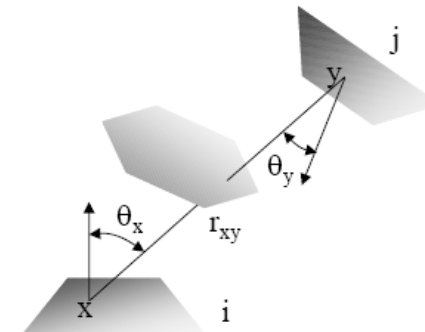


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# Compute Form Factors

$$F(j \rightarrow i) = \frac{1}{A_j} \iint_{A_j} \frac{\cos \theta_x \cdot \cos \theta_y}{\pi \cdot r_{xy}^2} \cdot V(x, y) \cdot dA_y \cdot dA_x$$



© Kavita Bala, Computer Science, Cornell University



## Radiosity Equation

- Radiosity for each polygon  $i$

$$\forall i: B_i = B_{e,i} + \rho_i \sum_{j=1}^N B_j F(i \rightarrow j)$$

- Linear system

- $B_i$  : radiosity of patch  $i$  (unknown)
- $B_{e,i}$  : emission of patch  $i$  (known)
- $\rho_i$  : reflectivity of patch  $i$  (known)
- $F(i \rightarrow j)$ : form-factor (coefficients of matrix)

## Linear System of Radiosity Equations

$$\begin{bmatrix} 1 - \rho_1 F_{1 \rightarrow 1} & -\rho_1 F_{1 \rightarrow 2} & \dots & -\rho_1 F_{1 \rightarrow n} \\ -\rho_2 F_{2 \rightarrow 1} & 1 - \rho_2 F_{2 \rightarrow 2} & \dots & -\rho_2 F_{2 \rightarrow n} \\ \dots & \dots & \dots & \dots \\ -\rho_n F_{n \rightarrow 1} & -\rho_n F_{n \rightarrow 2} & \dots & 1 - \rho_n F_{n \rightarrow n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \dots \\ B_n \end{bmatrix} = \begin{bmatrix} B_{e,1} \\ B_{e,2} \\ \dots \\ B_{e,n} \end{bmatrix}$$

known
↓
known  
 unknown

## How to Solve Linear System

- Matrix inversion

- Takes  $O(n^3)$

- Gather methods

- Jacobi iteration
- Gauss-Seidel

- Shooting

- Southwell iteration

## Iterative Approaches

- Jacobi iteration

- Start with initial guess for energy distribution (light sources)
- Update radiosity of all patches based on the previous guess

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^N B_j F(i \rightarrow j)$$

new value
old values

- Repeat until converged
- Gauss-Seidel iteration
  - New values used immediately

## Hybrid and Multipass Methods

- Ray tracing
  - Good for specular and refractive indirect illumination
  - View-dependent
- Radiosity
  - Good for diffuse
  - Allows interactive rendering
  - Does not scale well for massive models
- Hybrid methods
  - Combine both of them in a way

37

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## Instant Radiosity

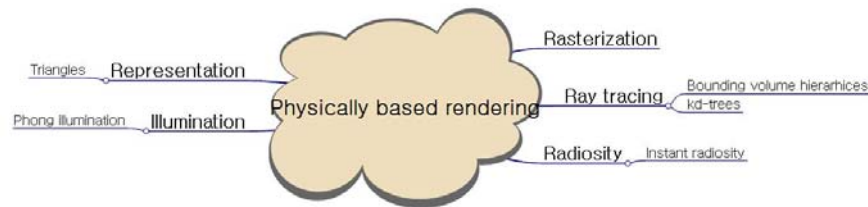
- Use the concept of Radiosity
- Map its functions to those of classic rendering pipeline
  - Utilize fast GPU
- Additional concepts
  - Virtual point lights
  - Shadow maps
- Micro-Rendering for Scalable, Parallel Final Gathering (Video)
  - Tobias Ritschel, Thomas Engelhardt, Thorsten Grosch, Hans-Peter Seidel, Jan Kautz, Carsten Dachsbacher
  - ACM Trans. Graph. 28(5) (Proc. SIGGRAPH Asia 2009), 2009.

38

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## Class Objectives were:

- Understand radiosity
  - Radiosity equation
  - Solving the equation



39

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# Radiometry and Rendering Equation

Sung-Eui Yoon  
(윤성익)

<http://sglab.kaist.ac.kr/~sungeui>



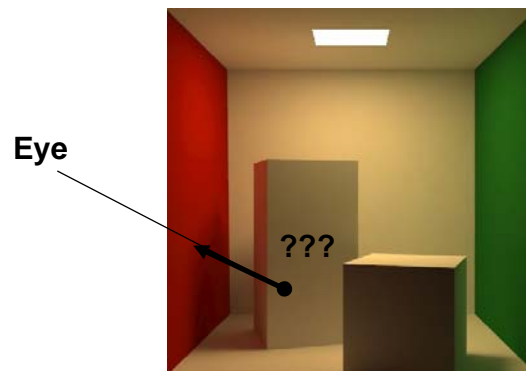
# Class Objectives

- Know terms of:
  - Hemispherical coordinates and integration
  - Various radiometric quantities (e.g., radiance)
  - Basic material function, BRDF
  - Understand the rendering equation

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# Motivation

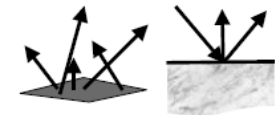


3



# Light and Material Interactions

- Physics of light
- Radiometry
- Material properties
- Rendering equation



From kavita's slides

4



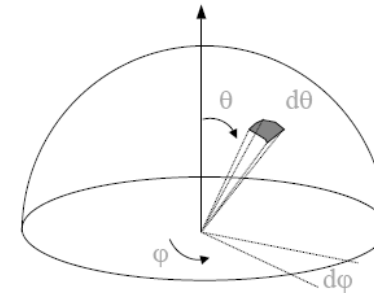
# Models of Light

- **Quantum optics**
  - Fundamental model of the light
  - Explain the dual wave-particle nature of light
- **Wave model**
  - Simplified quantum optics
  - Explains diffraction, interference, and polarization
- **Geometric optics**
  - Most commonly used model in CG
  - Size of objects  $\gg$  wavelength of light
  - Light is emitted, reflected, and transmitted



# Hemispheres

- **Hemisphere**
  - Two-dimensional surfaces
- **Direction**
  - Point on (unit) sphere



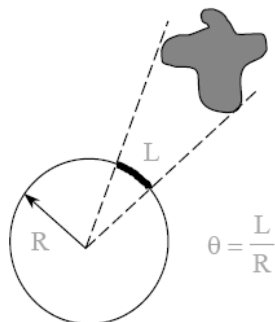
$$\theta \in [0, \frac{\pi}{2}]$$

$$\phi \in [0, 2\pi]$$

From kavita's slides

# Solid Angles

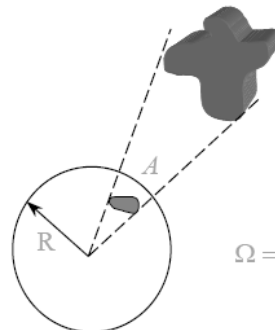
2D



$$\theta = \frac{L}{R}$$

Full circle  
=  $2\pi$  radians

3D

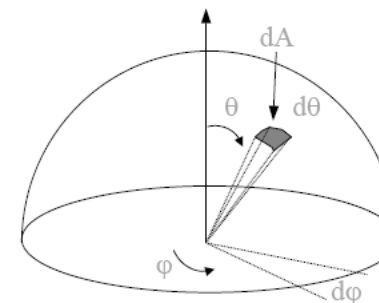


$$\Omega = \frac{A}{R^2}$$

Full sphere  
=  $4\pi$  steradians

# Hemispherical Coordinates

- **Direction,  $\Theta$** 
  - Point on (unit) sphere



$$dA = (r \sin \theta d\phi)(r d\theta)$$

From kavita's slides

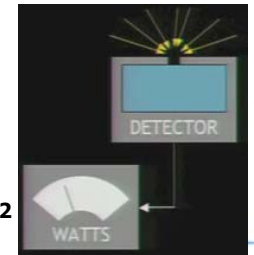
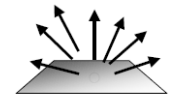
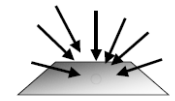
# Hemispherical Coordinates

- Differential solid angle

$$d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi$$

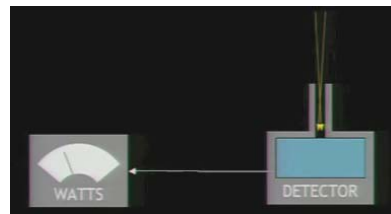
# Irradiance

- Incident radiant power per unit area ( $dP/dA$ )
  - Area density of power
- Symbol:  $E$ , unit:  $W/m^2$ 
  - Area power density existing a surface is called radiance exitance ( $M$ ) or radiosity ( $B$ )
- For example
  - A light source emitting 100 W of area 0.1  $m^2$
  - Its radiant exitance is 1000  $W/m^2$



# Radiance

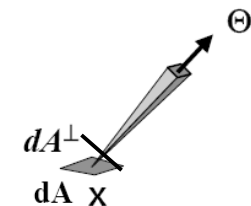
- Radiant power at  $x$  in direction  $\theta$ 
  - $L(x \rightarrow \Theta)$  : 5D function
    - Per unit area
    - Per unit solid angle



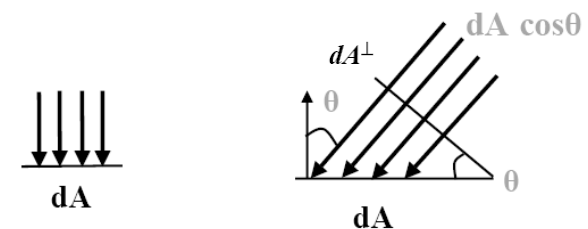
- Important quantity for rendering

# Radiance: Projected Area

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^\perp d\omega_\Theta} = \frac{d^2 P}{d\omega_\Theta dA \cos \theta}$$

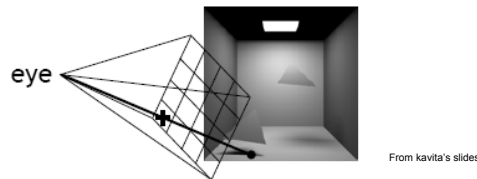


- Why per unit projected surface area



## Sensitivity to Radiance

- Responses of sensors (camera, human eye) is proportional to radiance



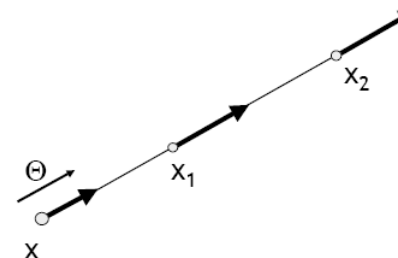
- Pixel values in image proportional to radiance received from that direction

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## Properties of Radiance

- Invariant along a straight line (in vacuum)



14

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## Invariance of Radiance

We can prove it based on the assumption the conservation of energy.

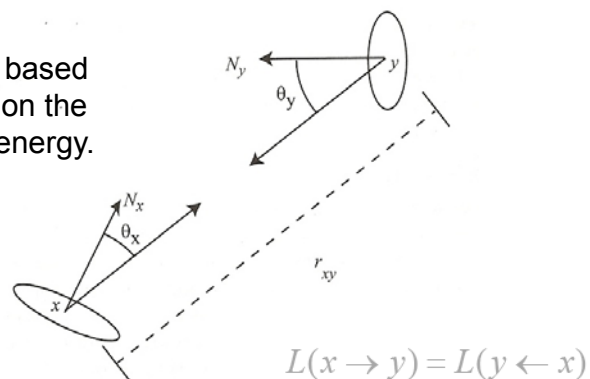


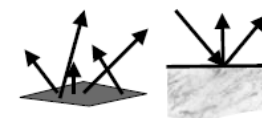
Figure 2.3. Invariance of radiance.

15

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## Light and Material Interactions

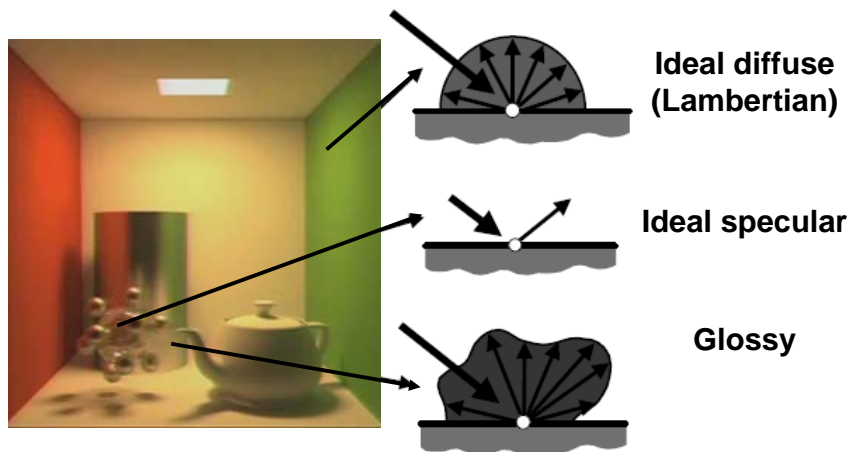
- Physics of light
- Radiometry
- **Material properties**
- Rendering equation



16

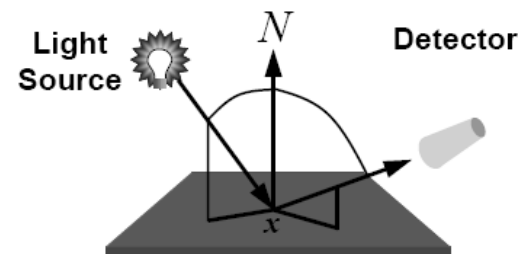
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# Materials



From kavita's slides

# Bidirectional Reflectance Distribution Function (BRDF)

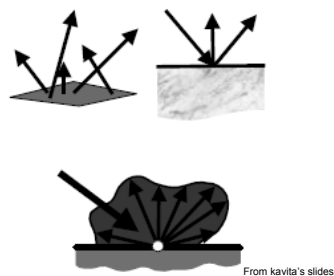


$$f_r(x, \Psi \rightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{dE(x \leftarrow \Psi)} = \frac{dL(x \rightarrow \Theta)}{L(x \leftarrow \Psi) \cos(N_x, \Psi) d\omega_\Psi}$$

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# Light and Material Interactions

- Physics of light
- Radiometry
- Material properties
- **Rendering equation**



From kavita's slides

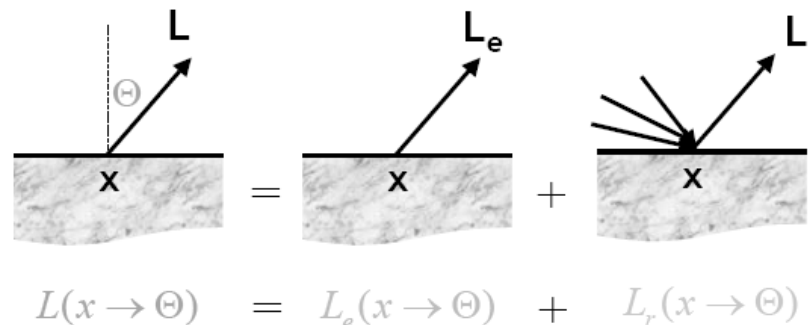
# Light Transport

- **Goal**
  - Describe steady-state radiance distribution in the scene
- **Assumptions**
  - Geometric optics
  - Achieves steady state instantaneously

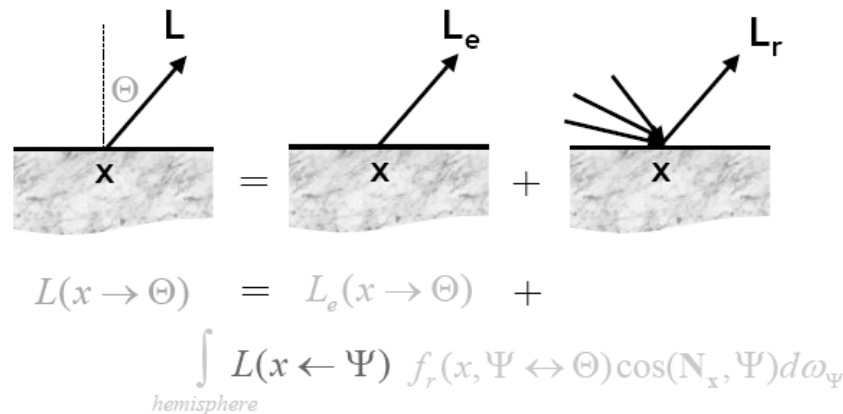
# Rendering Equation

- Describes energy transport in the scene
- Input
  - Light sources
  - Surface geometry
  - Reflectance characteristics of surfaces
- Output
  - Value of radiances at all surface points in all directions

# Rendering Equation

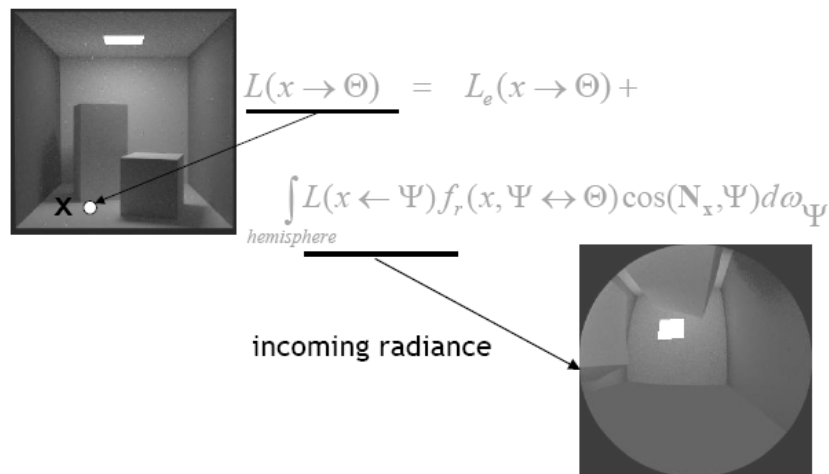


# Rendering Equation



- Applicable for each wavelength

# Rendering Equation





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# Monte Carol Integration

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Sung-Eui Yoon  
(윤성의)

<http://sglab.kaist.ac.kr/~sungeui>



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# Class Objectives

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- Sampling approach for solving the rendering equation
  - Monte Carlo integration
  - Estimator and its variance

2



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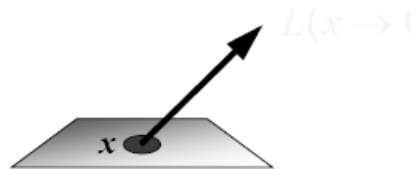
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# Radiance Evaluation

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- Fundamental problem in GI algorithm
  - Evaluate radiance at a given surface point in a given direction
  - Invariance defines radiance everywhere else



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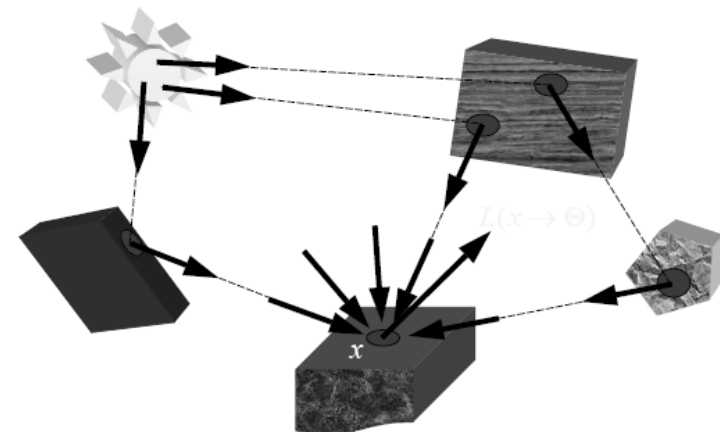
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# Radiance Evaluation

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... find paths between sources and surfaces to be shaded

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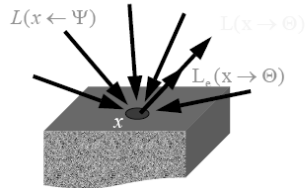
3

4

# Why Monte Carlo?

- Radiance is hard to evaluate

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos(\Psi, n_x) \cdot d\omega_\Psi$$



From kavita's slides

- Sample many paths
  - Integrate over all incoming directions
- Analytical integration is difficult
  - Need numerical techniques

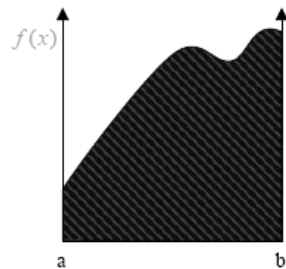
# Monte Carlo Integration

- Numerical tool to evaluate integrals
  - Use sampling
- Stochastic errors
- Unbiased
  - On average, we get the right answer

# Numerical Integration

- A one-dimensional integral:

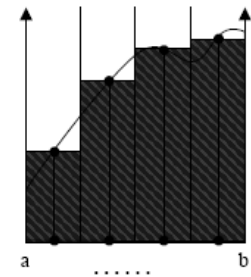
$$I = \int_a^b f(x) dx$$



# Deterministic Integration

- Quadrature rules:

$$I = \int_a^b f(x) dx \approx \sum_{i=1}^N w_i f(x_i)$$

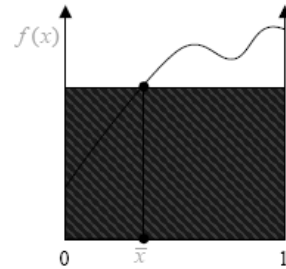


# Monte Carlo Integration

Primary estimator:

$$I = \int_a^b f(x) dx$$

$$I_{prim} = f(\bar{x})$$

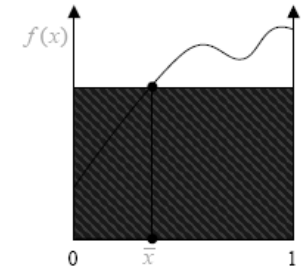


# Monte Carlo Integration

Primary estimator:

$$I = \int_a^b f(x) dx$$

$$I_{prim} = f(\bar{x})$$



$$E(I_{prim}) = \int_0^1 f(x) p(x) dx = \int_0^1 f(x) 1 dx = I$$

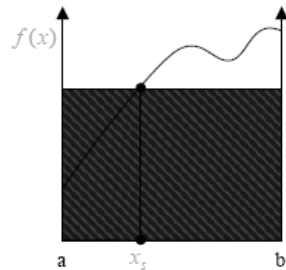
Unbiased estimator!

# Monte Carlo Integration

Primary estimator:

$$I = \int_a^b f(x) dx$$

$$I_{prim} = f(x_s)(b - a)$$



$$E(I_{prim}) = \int_a^b f(x)(b - a)p(x) dx = \int_a^b f(x)(b - a) \frac{1}{(b - a)} dx = I$$

Unbiased estimator!

# Monte Carlo Integration: Error

Variance of the estimator → a measure of the stochastic error

$$\sigma_{prim}^2 = \int_a^b \left[ \frac{f(x)}{p(x)} - I \right]^2 p(x) dx$$

- Consider  $p(x)$  for estimate
- We will study it as importance sampling later

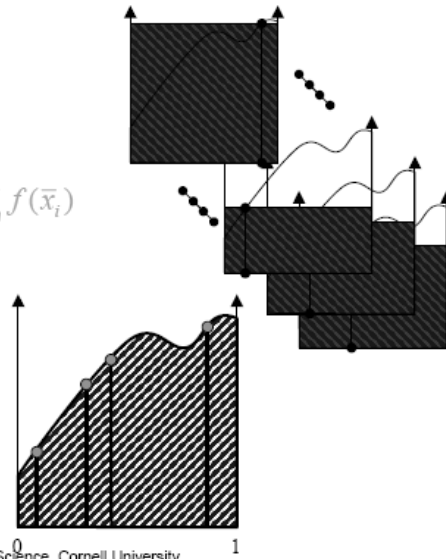
## More samples

### Secondary estimator

Generate N random samples  $x_i$

$$\text{Estimator: } \langle I \rangle = I_{\text{sec}} = \frac{1}{N} \sum_{i=1}^N f(\bar{x}_i)$$

$$\text{Variance } \sigma_{\text{sec}}^2 = \sigma_{\text{prim}}^2 / N$$



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## Monte Carlo Integration

- Expected value of estimator

$$\begin{aligned} E[\langle I \rangle] &= E\left[\frac{1}{N} \sum_i \frac{f(x_i)}{p(x_i)}\right] = \frac{1}{N} \int \left(\sum_i \frac{f(x_i)}{p(x_i)}\right) p(x) dx \\ &= \frac{1}{N} \sum_i \int \left(\frac{f(x)}{p(x)}\right) p(x) dx \\ &= \frac{N}{N} \int f(x) dx = I \end{aligned}$$

– on ‘average’ get right result: **unbiased**

- Standard deviation  $\sigma$  is a measure of the stochastic error

$$\sigma^2 = \frac{1}{N} \int_a^b \left[\frac{f(x)}{p(x)} - I\right]^2 p(x) dx$$

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## MC Integration - Example

– Integral  $I = \int_0^1 5x^4 dx = 1$

– Uniform sampling

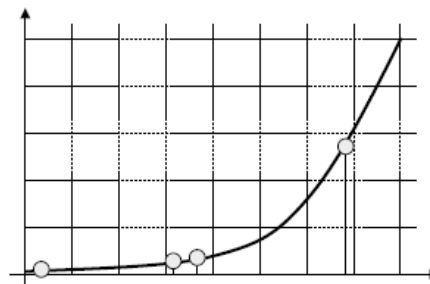
– Samples :

$$x_1 = .86 \quad \langle I \rangle = 2.74$$

$$x_2 = .41 \quad \langle I \rangle = 1.44$$

$$x_3 = .02 \quad \langle I \rangle = 0.96$$

$$x_4 = .38 \quad \langle I \rangle = 0.75$$



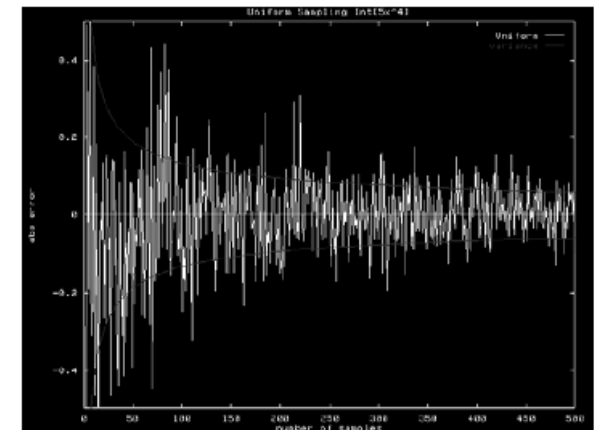
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## MC Integration - Example

- Integral

$$I = \int_0^1 5x^4 dx = 1$$

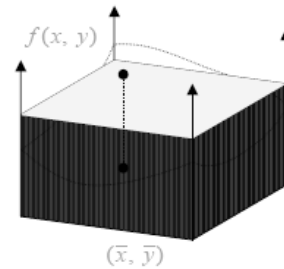
- Variance



## MC Integration: 2D

- Primary estimator:

$$\bar{I}_{prim} = \frac{f(\bar{x}, \bar{y})}{p(\bar{x}, \bar{y})}$$

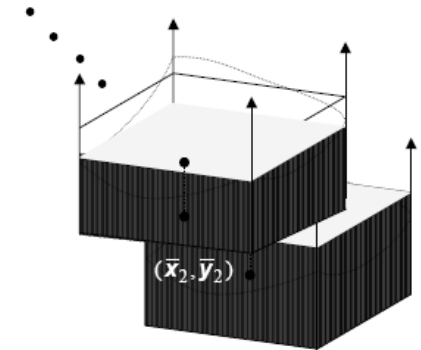


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## MC Integration: 2D

- Secondary estimator:

$$I_{sec} = \frac{1}{N} \sum_{i=1}^N \frac{f(\bar{x}_i, \bar{y}_i)}{p(\bar{x}_i, \bar{y}_i)}$$



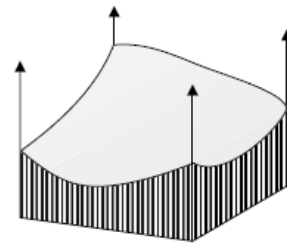
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## Monte Carlo Integration - 2D

- MC Integration works well for higher dimensions
- Unlike quadrature

$$I = \int_a^b \int_c^d f(x, y) dx dy$$

$$\langle I \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f(x_i, y_i)}{p(x_i, y_i)}$$



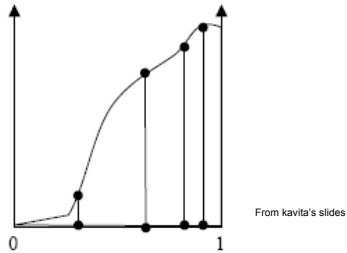
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## Advantages of MC

- Convergence rate of  $O(\frac{1}{\sqrt{N}})$
- Simple
  - Sampling
  - Point evaluation
- General
  - Works for high dimensions
  - Deals with discontinuities, crazy functions, etc.

## Importance Sampling

- Take more samples in important regions, where the function is large



## Class Objectives were:

- Sampling approach for solving the rendering equation
  - Monte Carlo integration
  - Estimator and its variance

# Monte Carlo Ray Tracing: Part I

Sung-Eui Yoon  
(윤성의)

<http://sglab.kaist.ac.kr/~sungeui>



# Class Objectives

- Understand a basic structure of Monte Carlo ray tracing
  - Russian roulette for its termination
  - Path tracing

2



# Rendering Equation

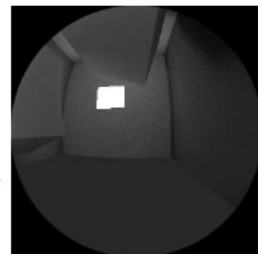
$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos(\Psi, n_x) \cdot d\omega_\Psi$$

function to integrate over all incoming directions over the hemisphere around x

Value we want



$$= L_e + \int_{\Omega_x} \text{[Hemisphere Diagram]} \cdot f_r \cdot \cos$$



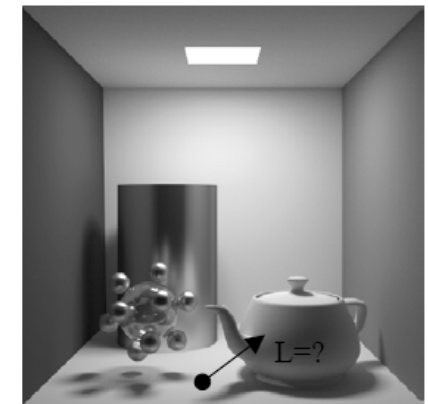
# How to compute?

$$L(x \rightarrow \Theta) = ?$$

Check for  $L_e(x \rightarrow \Theta)$

Now add  $L_r(x \rightarrow \Theta) =$

$$\int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos(\Psi, n_x) \cdot d\omega_\Psi$$



## How to compute?

- Use Monte Carlo
- Generate random directions on hemisphere  $\Omega_x$  using pdf  $p(\Psi)$

$$L(x \rightarrow \Theta) = \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos(\Psi, n_x) \cdot d\omega_\Psi$$

$$\langle L(x \rightarrow \Theta) \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f_r(\Psi_i \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi_i) \cdot \cos(\Psi_i, n_x)}{p(\Psi_i)}$$

5

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## How to compute?

Generate random directions  $\Psi_i$

$$\langle L \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f_r(\dots) \cdot L(x \leftarrow \Psi_i) \cdot \cos(\dots)}{p(\Psi_i)}$$

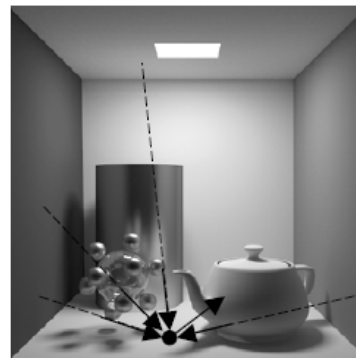
- evaluate brdf
- evaluate cosine term
- evaluate  $L(x \leftarrow \Psi_i)$



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## How to compute?

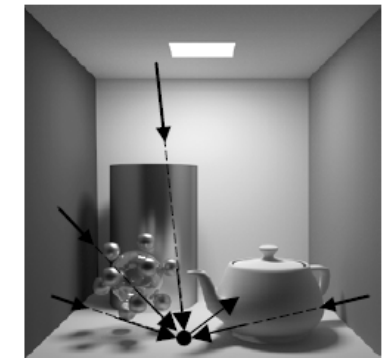
- evaluate  $L(x \leftarrow \Psi_i)$ ?
- Radiance is invariant along straight paths
- $vp(x, \Psi_i)$  = first visible point
- $L(x \leftarrow \Psi_i) = L(vp(x, \Psi_i) \rightarrow \Psi_i)$



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## How to compute? Recursion ...

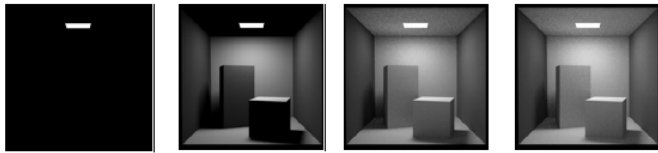
- Recursion ....
- Each additional bounce adds one more level of indirect light
- Handles ALL light transport
- “Stochastic Ray Tracing”



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## When to end recursion?



From kavita's slides

- Contributions of further light bounces become less significant
  - Max recursion
  - Some threshold for radiance value
- If we just ignore them, estimators will be biased

9

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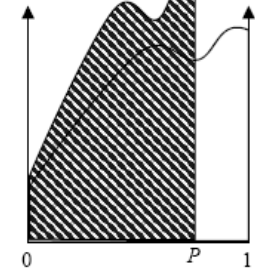
## Russian Roulette

Integral

$$I = \int_0^1 f(x) dx = \int_0^1 \frac{f(x)}{P} P dx = \int_0^P \frac{f(y/P)}{P} dy$$

Estimator

$$\langle I_{\text{roulette}} \rangle = \begin{cases} \frac{f(x_i)}{P} & \text{if } x_i \leq P, \\ 0 & \text{if } x_i > P. \end{cases}$$



Variance  $\sigma_{\text{roulette}} > \sigma$

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## Russian Roulette

- Pick absorption probability,  $\alpha = 1-P$ 
  - Recursion is terminated
- $1-\alpha$  is commonly to be equal to the reflectance of the material of the surface
  - Darker surface absorbs more paths

11

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## Algorithm so far

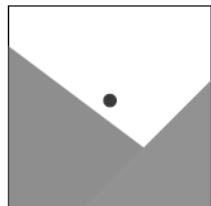
- Shoot primary rays through each pixel
- Shoot indirect rays, sampled over hemisphere
- Terminate recursion using Russian Roulette

12

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## Pixel Anti-Aliasing

- Compute radiance only at the center of pixel
  - Produce jaggies
- Simple box filter
  - The averaging method
- We want to evaluate using MC



13

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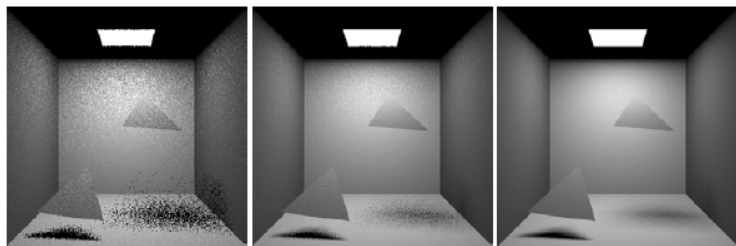
## Stochastic Ray Tracing

- Parameters
  - Num. of starting ray per pixel
  - Num. of random rays for each surface point (branching factor)
- Path tracing
  - Branching factor = 1

14

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## Path Tracing



1 ray / pixel

10 rays / pixel

100 rays / pixel

From Kavita's slides

- Pixel sampling + light source sampling folded into one method

15

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## Algorithm so far

- Shoot primary rays through each pixel
- Shoot indirect rays, sampled over hemisphere
  - Path tracing shoots only 1 indirect ray
- Terminate recursion using Russian Roulette

16

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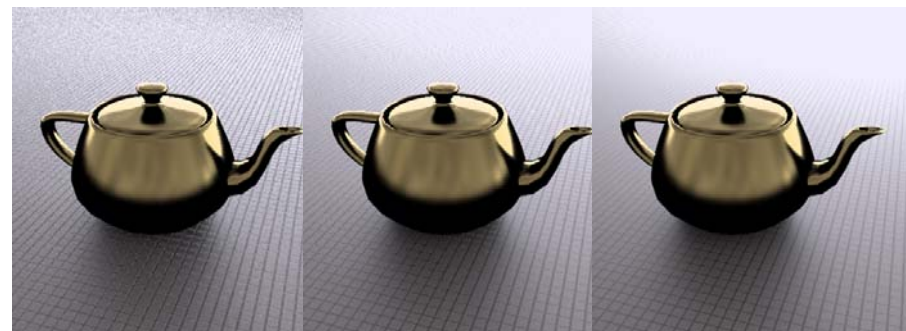
## Performance

- Want better quality with smaller # of samples
  - Fewer samples/better performance
  - Quasi Monte Carlo: well-distributed samples
  - Adaptive sampling

17

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## PA2



Uniform sampling  
(64 samples per pixel)

Adaptive sampling

Reference

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## Class Objectives were:

- Understand a basic structure of Monte Carlo ray tracing
  - Russian roulette for its termination
  - Path tracing

19

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## MC Ray Tracing: Part II, Acceleration and Biased Tech.

Sung-Eui Yoon  
(윤성익)

Course URL:  
<http://sglab.kaist.ac.kr/~sungeui/GCG>

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## Class Objectives:

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- **Extensions to the basic MC path tracer**
  - Bidirectional path tracer
  - Metropolis sampling
- **Biased techniques**
  - Irradiance caching
  - Photon mapping

21

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## General GI Algorithm

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- **Design path generators**
- **Path generators determine efficiency of GI algorithm**
- **Black boxes**
  - Evaluate BRDF, ray intersection, visibility evaluations, etc

22

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## Other Rendering Techniques

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- **Bidirectional path tracing**
- **Metropolis**
- **Biased techniques**
  - Irradiance caching
  - Photon mapping

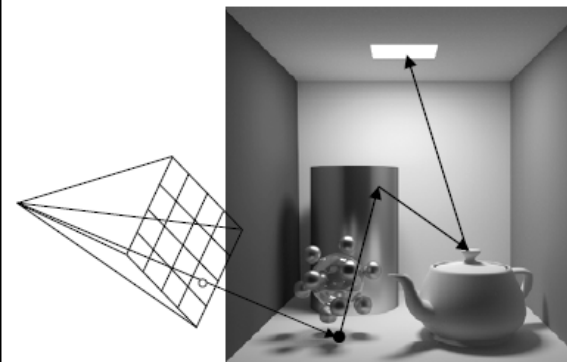
23

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## Stochastic ray tracing: limitations

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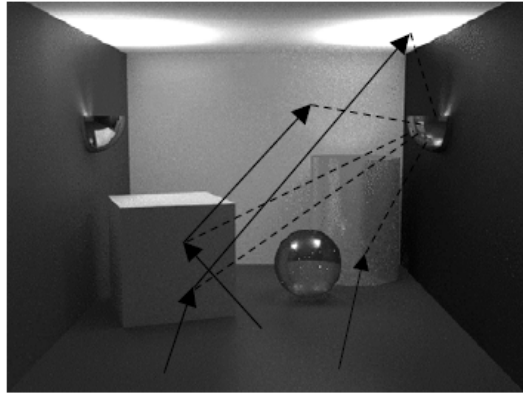
- Generate a path from the eye to the light source



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## When does it not work?

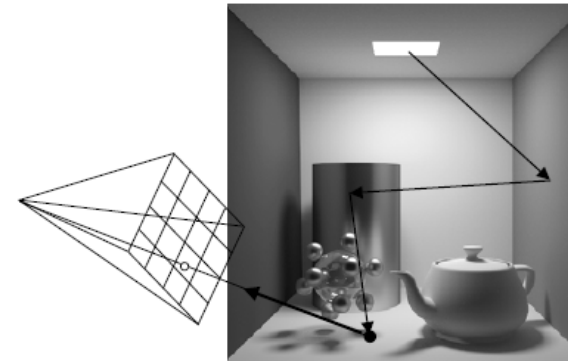
- Scenes in which indirect lighting dominates



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## Bidirectional Path Tracing

- So ... we can generate paths starting from the light sources!

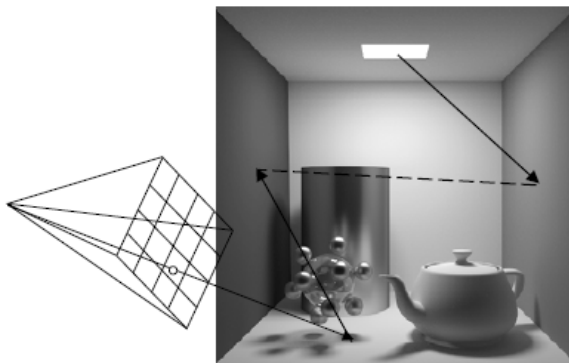


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- Shoot ray to camera to see what pixels get contributions

## Bidirectional Path Tracing

- Or paths generated from both camera and source at the same time ...!



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- Connect endpoints to compute final contribution

## Unbiased vs. Consistent

### • Unbiased

- No systematic error
- $E[I_{\text{estimator}}] = I$
- Better results with larger N

### • Consistent

- Converges to correct results with more samples
- $E[I_{\text{estimator}}] = I + \epsilon$ , where  $\lim_{n \rightarrow \infty} \epsilon = 0$

## Biased Methods

- MC methods
  - Too noisy and slow
  - Noise is objectionable
- Biased methods: store information (caching)
  - Irradiance caching
  - Photon mapping

29

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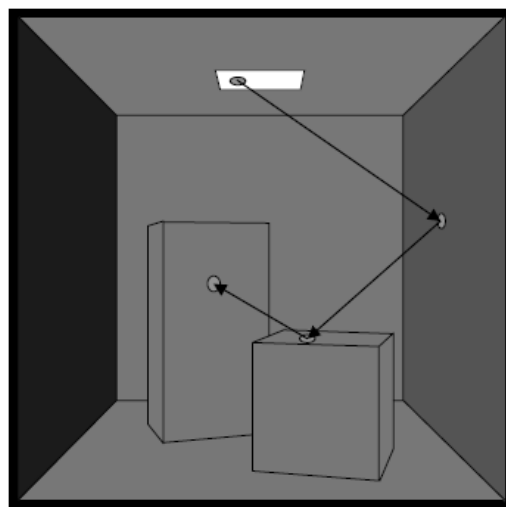
## Photon Mapping

- 2 passes:
  - Shoot "photons" (light-rays) and record any hit-points
  - Shoot viewing rays and collect information from stored photons

30

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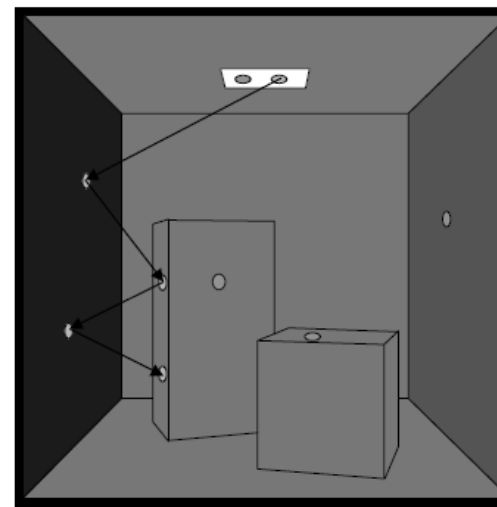
### Pass 1: shoot photons



- Light path generated using MC techniques and Russian Roulette
- Store:
  - position
  - incoming direction
  - color
  - ...

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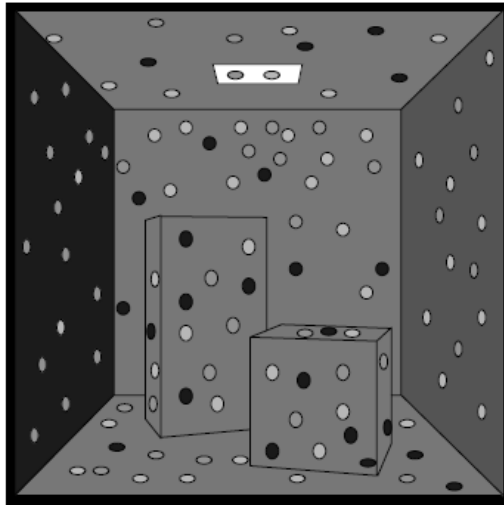
### Pass 1: shoot photons



- Light path generated using MC techniques and Russian Roulette
- Store: **Flux for each photon**
  - position
  - incoming direction
  - color
  - ...

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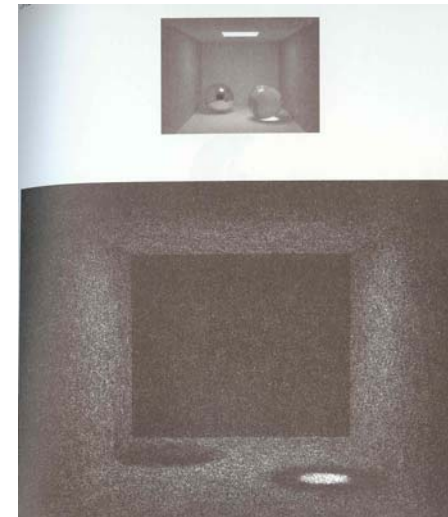
## Pass 1: shoot photons



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- Light path generated using MC techniques and Russian Roulette
- Store: **for diffuse materials**
  - position
  - incoming direction
  - color
  - ...

## Stored Photons

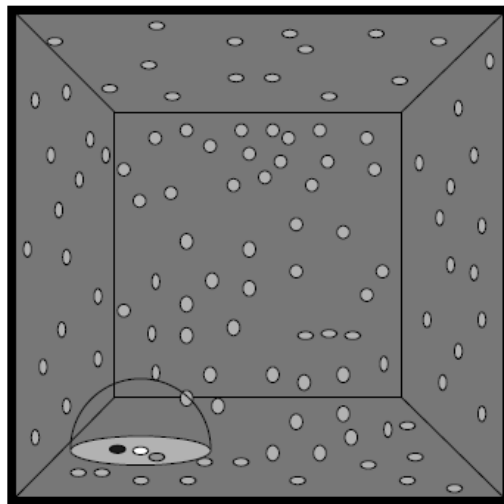


Generate a few hundreds of thousands of photons

34

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## Pass 2: viewing ray

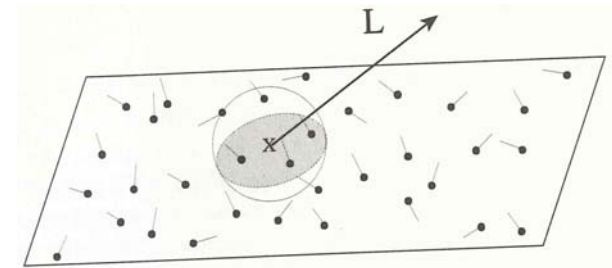


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- Search for  $N$  closest photons (+check normal)
- Assume these photons hit the point we're interested in
- Compute average radiance

## Radiance Estimation

- **Compute  $N$  nearest photons**
  - Consider a few hundreds of photons
  - Compute the radiance for each photon to outgoing direction
  - Consider BRDF and
  - Divided by area



36

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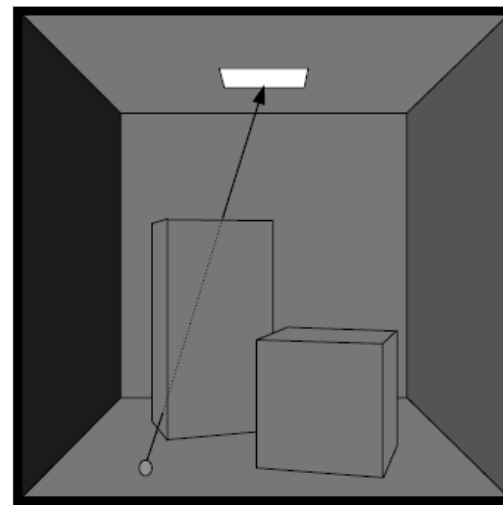
## Efficiency

- Want  $k$  nearest photons
  - Use kd-tree
- Using photon maps as it create noisy images
  - Need extremely large amount of photons

37

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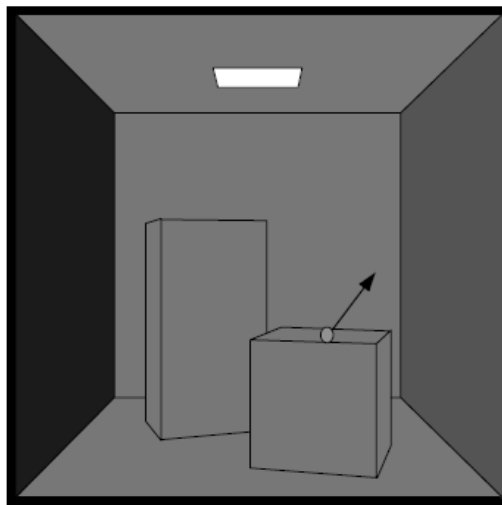
## Pass 2: Direct Illumination



Perform direct illumination for visible surface using regular MC sampling

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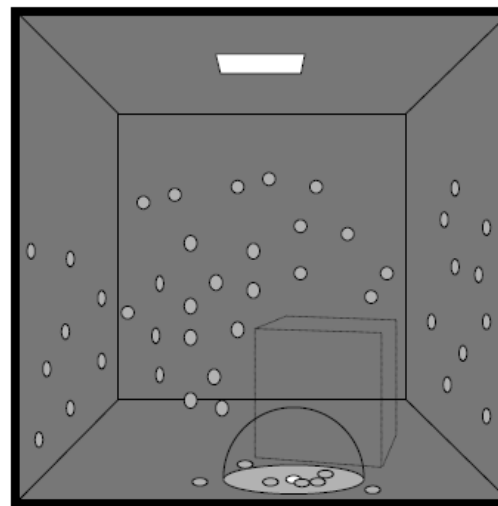
## Pass 2: Specular reflections



Specular reflection and transmission are ray traced

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## Pass 2: Caustics

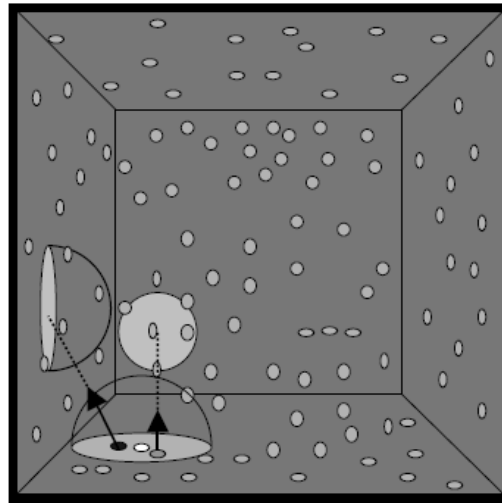


- Direct use of “caustic” maps
- The “caustic” map is similar to a photon map but treats  $LS^*D$  path
- Density of photons in caustic map usually high enough to use as is

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## Pass 2: Indirect Diffuse



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- Search for  $N$  closest photons
- Assume these photons hit the point
- Compute average radiance by importance sampling of hemisphere

## Result



350K photons  
for the caustic  
map

42

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## Class Objectives were:

- **Extensions to the basic MC path tracer**
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43

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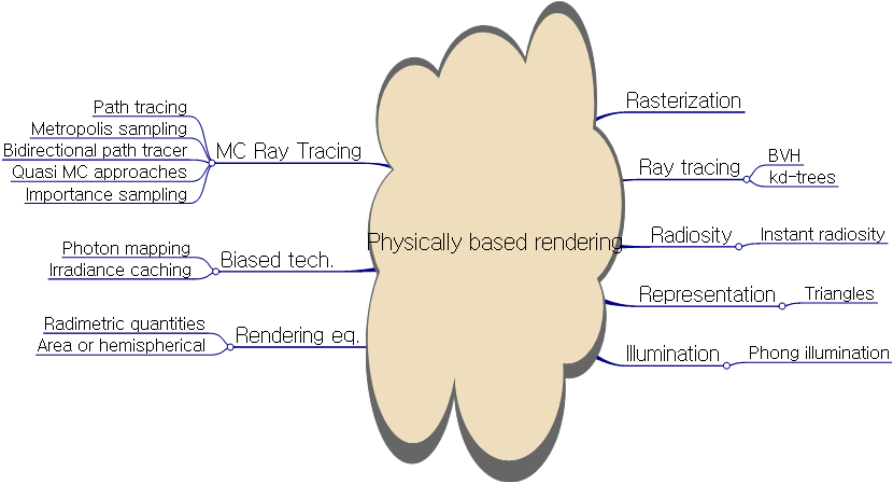
## Summary

- **Two basic building blocks**
- **Radiometry**
- **Rendering equation**
- **MC integration**
- **MC ray tracing**
  - Unbiased methods
  - Biased methods

44

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# Summary



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## Scalable Graphics Algorithms

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Sung-eui Yoon

Associate Professor  
KAIST

<http://sglab.kaist.ac.kr>



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## Acknowledgements

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- Collaborators

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- Funding sources

- Korea Research Foundation
- Ministry of Knowledge Economy
- Samsung, Microsoft Research Asia, Adobe, Boeing



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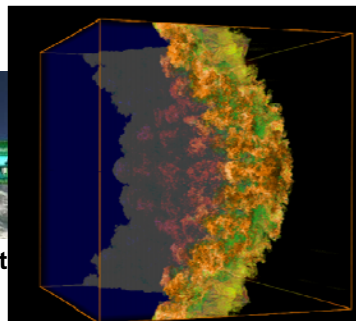
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## Massive Geometric Data

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- Due to advances of modeling, simulation, and data capture techniques



Over 3 Terabytes of geometric data

Large-scale virtual world, 83 M tri.



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## Possible Solutions?

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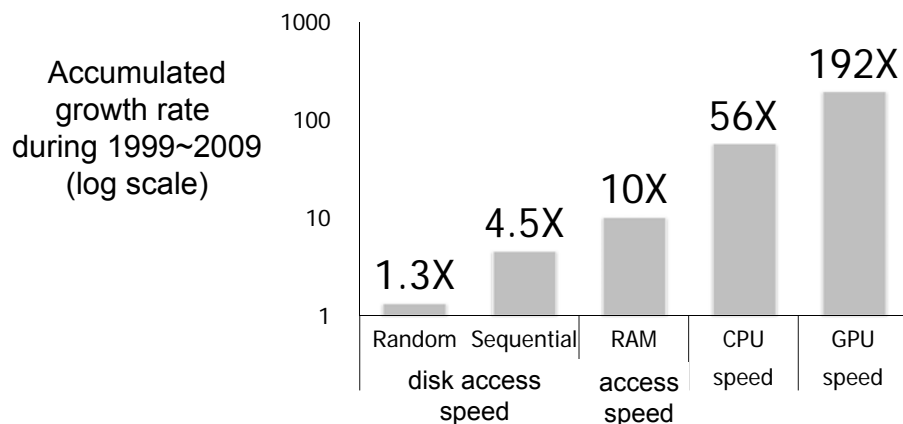
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- Hardware improvement will address the data avalanche?

- Moore's law: the number of transistor is roughly double every 18 months



## Current Architecture Trends



Data access time becomes the major computational bottleneck! **KAIST**

## Data Growth

- An observation
  - If we got higher performance, we attempt to produce bigger data to derive more useful information and handle such bigger data
- Amount of data is doubling every 18 ~ 24 months
  - "How Much Information," 2003, Lyman, Peter and Hal R.

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## Main Research Theme

- Designing *scalable graphics and geometric algorithms* to efficiently handle massive models on commodity hardware

- Multi-resolution methods
- Cache-coherent algorithms
- Culling techniques
- Data compression
- Parallel computation,
- Reducing data dimensions, etc

- Rendering
- Collision detection and path planning
- Image retrieval

Proximity queries

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## Rendering in 2002



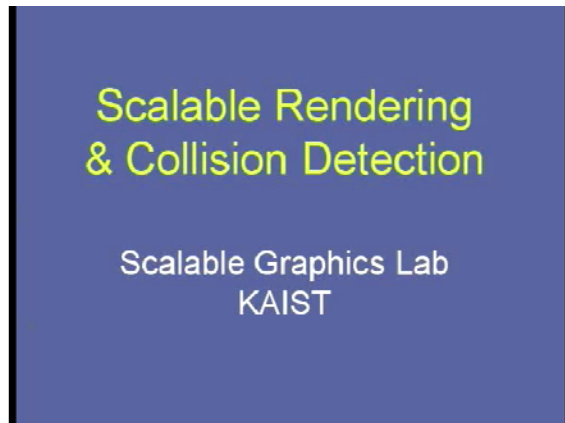
**SGI Reality Monster**  
(consisting of 40 processing units)

Running on SGI Reality Monster

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# Rendering Today

- Applied various algorithms that we designed for rendering and collision detection



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# Cache-Oblivious Ray Reordering [Moon et al., ToG 2010]



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# Adaptive Rendering based on Weighted Local Regression [Moon et al. ToG 14]

- Denoise Monte Carlo ray tracing results, and generate more rays on regions with higher reconstruction error
  - Uses a novel image-space reconstruction method
  - Derives a robust error estimation process
  - Uses it to drive our sampling process



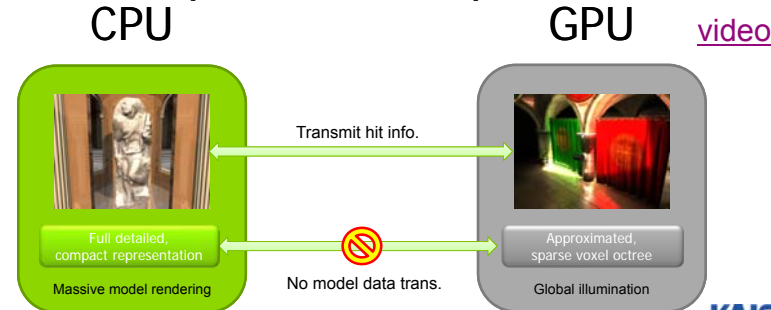
(a) Ours 106 spp (879 s) MSE 0.00446 (b) MC 128 spp (886 s) MSE 0.06308 (c) NLM 108 spp (892 s) MSE 0.01358 (d) SURE 103 spp (888 s) MSE 0.01621 (e) Ours 106 spp (879 s) MSE 0.00446 (f) Reference 16K spp

Video

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# T-ReX: Interactive Global Illumination of Massive Models on Heterogeneous Computing Resources

- Use compact, decoupled representations for CPUs and GPUs [Kim et al., TVCG 14]
  - Use HCCMesh, random-accessible compressed mesh and BVHs, at CPU
  - Use a compact volumetric rep. at GPU



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## Codes are available

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- <http://sglab.kaist.ac.kr/software.htm>
- T-Rex is available
- Adaptive sampling and reconstruction will be available
  - Will be discussed tomorrow at 신진연구자 세션